Electromagnetics:
Electromagnetic Field Theory

Mathematical Preliminaries

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Outline

- Phasors
- Phasor Arithmetic
- Scalars & Vectors

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Phasors (1 of 2)

A time-harmonic function can be written as

$$y(t) = A\cos(\omega t + \theta)$$

Recall Euler's Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

This let's the function $y(t)$ be written as

$$y(t) = \text{Re}\left[Ae^{j(\omega t + \theta)}\right]$$
$$= \text{Re}\left[Ae^{j\omega t}e^{j\theta}\right]$$

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Phasors (2 of 2)

$$y(t) = \text{Re}\left[Ae^{j\omega t}e^{j\theta}\right]$$

In linear problems, frequency ω is constant. This means the $e^{j\omega t}$ term never changes. Therefore, it is usually dropped when writing harmonic functions as phasors.

$$y(t) = A\cos(\omega t + \theta) \quad \longleftrightarrow \quad \boxed{Y = Ae^{j\theta}}$$

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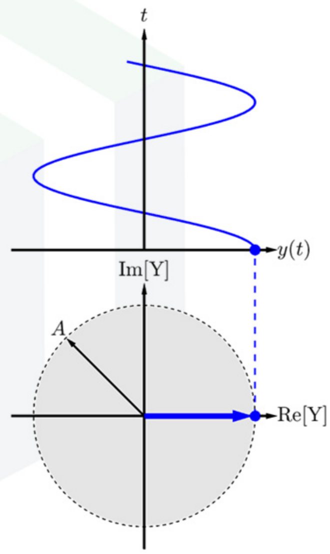
Visualization of a Phasor

$$y(t) = A\cos(\omega t + \theta)$$

$$\updownarrow$$

$$\boxed{Y = Ae^{j\theta}}$$



Polar Vs. Rectangular Form

A phasor in polar form is written as

$$Y = Ae^{j\theta} \text{ or } A\measuredangle\theta$$

The same phasor written in rectangular form is

$$Y = \alpha + j\beta$$

Rect → Polar

$$A = \sqrt{\alpha^2 + \beta^2}$$

$$\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$$

Polar → Rect

$$\alpha = A\cos\theta$$

$$\beta = A\sin\theta$$

Phasor Arithmetic

Addition

$$F_1 + F_2 = (\alpha_1 + \alpha_2) + j(\beta_1 + \beta_2)$$

Subtraction

$$F_1 - F_2 = (\alpha_1 - \alpha_2) + j(\beta_1 - \beta_2)$$

Multiplication

$$F_1 \cdot F_2 = (A_1 A_2) \measuredangle (\theta_1 + \theta_2)$$

Division

$$F_1 \div F_2 = (A_1 \div A_2) \measuredangle (\theta_1 - \theta_2)$$

Scalars & Vectors

Scalar Numbers

Scalars contain only one piece of information, magnitude. Scalars can be real or complex. Phasors are scalar quantities.

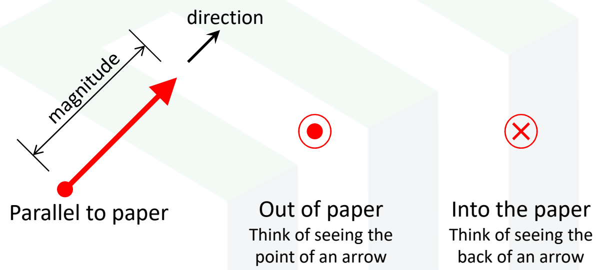
Examples: 7, π, -1.34, etc.

Vectors

Vectors have both a magnitude and a direction.

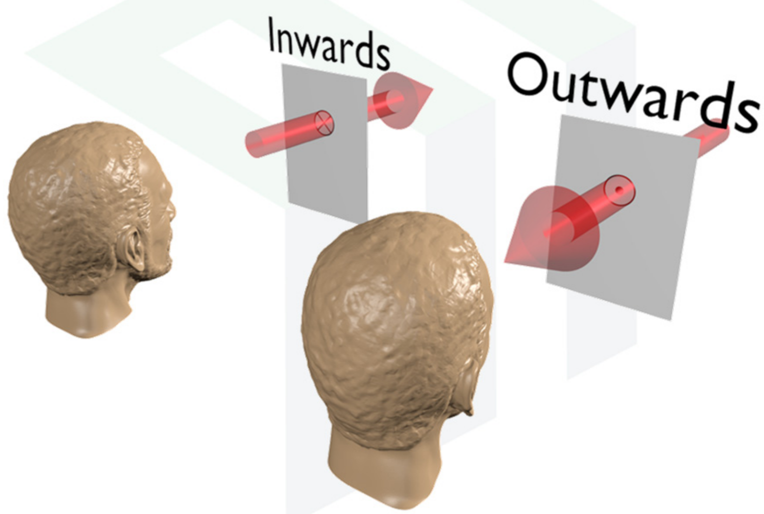
Examples: Velocity, force, electromagnetic fields

Vector Notation



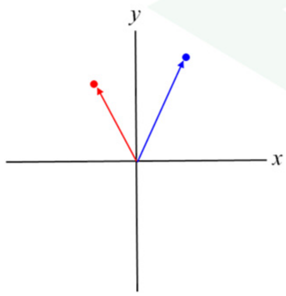
Note: Despite the arrow extending away from the point, a vector is describing something at that specific point and it does not actually extend outward.

Inward/Outward Notation



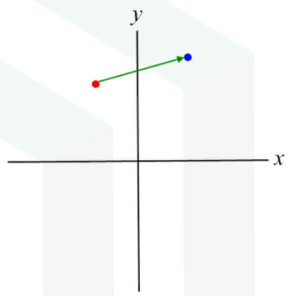
What Can Vectors Convey?

Position



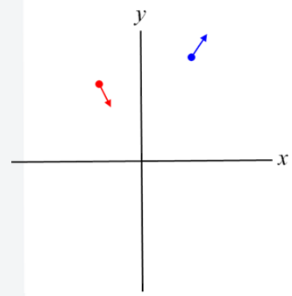
Position relative to the origin.

Distance



Vectors can indicate distance, but the origin is not given.

Disturbance



A vector can represent a directional disturbance. Think of this as a push.

Simple Vector Calculations

3D Vector

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

Vector Magnitude

$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Unit Vector

$$\hat{A} = \frac{\vec{A}}{\left|\vec{A}\right|}$$