



Electromagnetics:
Electromagnetic Field Theory

Mathematical Preliminaries

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Outline

- Phasors
- Phasor Arithmetic
- Scalars & Vectors

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Phasors (1 of 2)

A time-harmonic function can be written as

$$y(t) = A \cos(\omega t + \theta)$$

Recall Euler's Identity

$$e^{j\theta} = \cos \theta + j \sin \theta$$

This let's the function $y(t)$ be written as

$$\begin{aligned} y(t) &= \operatorname{Re} \left[A e^{j(\omega t + \theta)} \right] \\ &= \operatorname{Re} \left[A e^{j\omega t} e^{j\theta} \right] \end{aligned}$$

Phasors (2 of 2)

$$y(t) = \operatorname{Re} \left[A e^{j\omega t} e^{j\theta} \right]$$

In linear problems, frequency ω is constant. This means the $e^{j\omega t}$ term never changes. Therefore, it is usually dropped when writing harmonic functions as phasors.

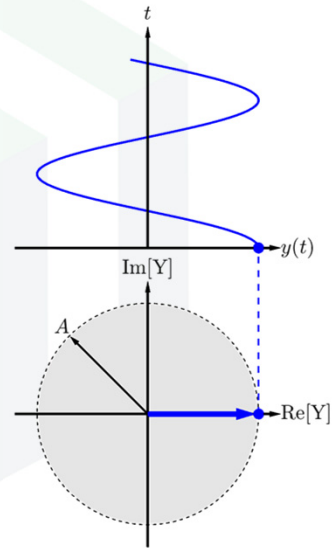
$$y(t) = A \cos(\omega t + \theta) \quad \longleftrightarrow \quad Y = A e^{j\theta}$$

Visualization of a Phasor

$$y(t) = A \cos(\omega t + \theta)$$



$$Y = Ae^{j\theta}$$



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Polar Vs. Rectangular Form

A phasor in polar form is written as

$$Y = Ae^{j\theta} \text{ or } A \angle \theta$$

The same phasor written in rectangular form is

$$Y = \alpha + j\beta$$

Rect \rightarrow Polar

$$A = \sqrt{\alpha^2 + \beta^2}$$

$$\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$$

Polar \rightarrow Rect

$$\alpha = A \cos \theta$$

$$\beta = A \sin \theta$$

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Phasor Arithmetic

Addition

$$F_1 + F_2 = (\alpha_1 + \alpha_2) + j(\beta_1 + \beta_2)$$

Subtraction

$$F_1 - F_2 = (\alpha_1 - \alpha_2) + j(\beta_1 - \beta_2)$$

Multiplication

$$F_1 \cdot F_2 = (A_1 A_2) \angle (\theta_1 + \theta_2)$$

Division

$$F_1 \div F_2 = (A_1 \div A_2) \angle (\theta_1 - \theta_2)$$

Scalars & Vectors

Scalar Numbers

Scalars contain only one piece of information, magnitude. Scalars can be real or complex. Phasors are scalar quantities.

Examples: 7, π , -1.34, etc.

Vectors


Vectors have both a magnitude and a direction.

Examples: Velocity, force, electromagnetic fields

Vector Notation

The diagram illustrates vector notation. On the left, a red arrow points upwards and to the right. A double-headed arrow labeled "magnitude" is drawn along the arrow's length. A smaller arrow labeled "direction" points along the arrow's path. Below this, the text "Parallel to paper" is written. To the right, there are two symbols: a red circle with a dot inside (representing a vector pointing out of the paper) and a red circle with an 'X' inside (representing a vector pointing into the paper). Below these symbols are the labels "Out of paper" and "Into the paper" respectively, with subtext: "Think of seeing the point of an arrow" and "Think of seeing the back of an arrow".


Note: Despite the arrow extending away from the point, a vector is describing something at that specific point and it does not actually extend outward.

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Inward/Outward Notation

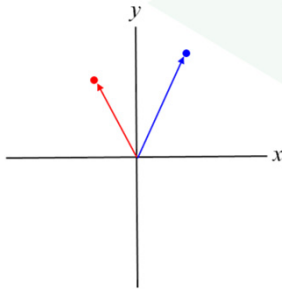
The diagram shows two human head models. The top model is shown in profile, facing right. A red arrow points towards it from the right, with the word "Inwards" written above it. The bottom model is shown from the back. A red arrow points away from it towards the right, with the word "Outwards" written above it. The background features a large, faint green geometric shape.

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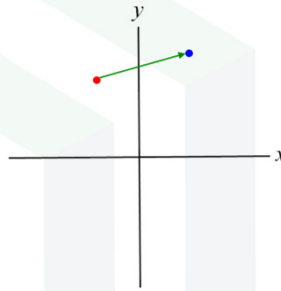
What Can Vectors Convey?

Position



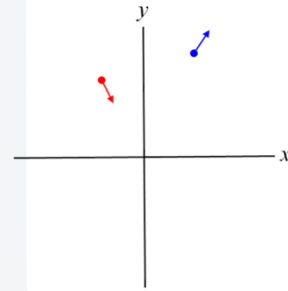
Position relative to the origin.

Distance



Vectors can indicate distance, but the origin is not given.

Disturbance



A vector can represent a directional disturbance. Think of this as a push.

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Simple Vector Calculations

3D Vector

$$\vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

Vector Magnitude

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Unit Vector

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

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