



Electromagnetics:
Electromagnetic Field Theory

Multi-Segment Transmission Line Devices

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Lecture Outline

- Quarter-Wave Transformer
- Impedance Matching
- Stubs
- Scattering Parameters

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Quarter-Wave Transformer

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Quarter-Wave Transformer (1 of 2)

A quarter-wave transformer is a section of line that is $\lambda/4$ long.

When the length of the line is $\lambda/4$,

$$\beta\ell = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

This means the signal accumulates exactly 90° of phase through a quarter-wave section of line.
When told a TL is $\lambda/4$, usually no other information is needed (i.e. β and ℓ are not needed).

When this is the case, the impedance transformation equation reduces to

$$\begin{aligned} Z_{in}(-\ell) &= Z_0 \frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)} = Z_0 \frac{Z_L + jZ_0 \tan(\pi/2)}{Z_0 + jZ_L \tan(\pi/2)} \\ &= Z_0 \frac{Z_L + jZ_0 \cdot \infty}{Z_0 + jZ_L \cdot \infty} \quad \tan(\pi/2) = \infty. \\ &=? \end{aligned}$$

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Quarter-Wave Transformer (2 of 2)

Since both the numerator and denominator are ∞ , apply L'Hopital's rule.

$$\lim_{x \rightarrow x_0} \frac{f(x)}{g(x)} = \frac{f'(x_0)}{g'(x_0)}$$

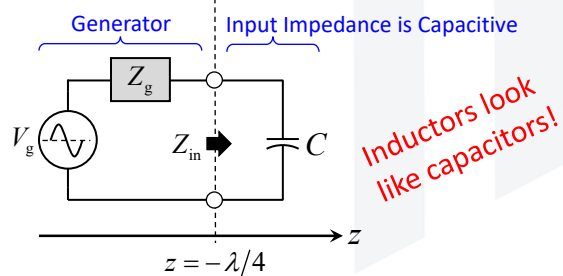
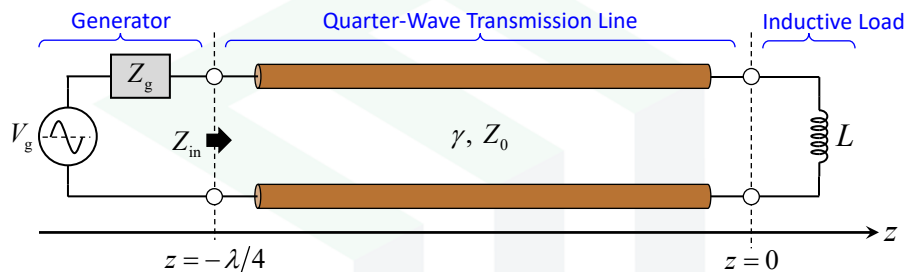
Applying this to the impedance transformation equation gives

$$Z_{in}(-\ell) = \lim_{\beta\ell \rightarrow \pi/2} Z_0 \frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)} = Z_0 \frac{jZ_0 \sec^2(\beta\ell)}{jZ_L \sec^2(\beta\ell)} = \frac{Z_0^2}{Z_L}$$

The final equation shows that the load impedance Z_L gets completely inverted. The input impedance becomes the input admittance.

$$Z_{in}\left(-\frac{\lambda}{4}\right) = \frac{Z_0^2}{Z_L}$$

Impedance Inversion (1 of 5)



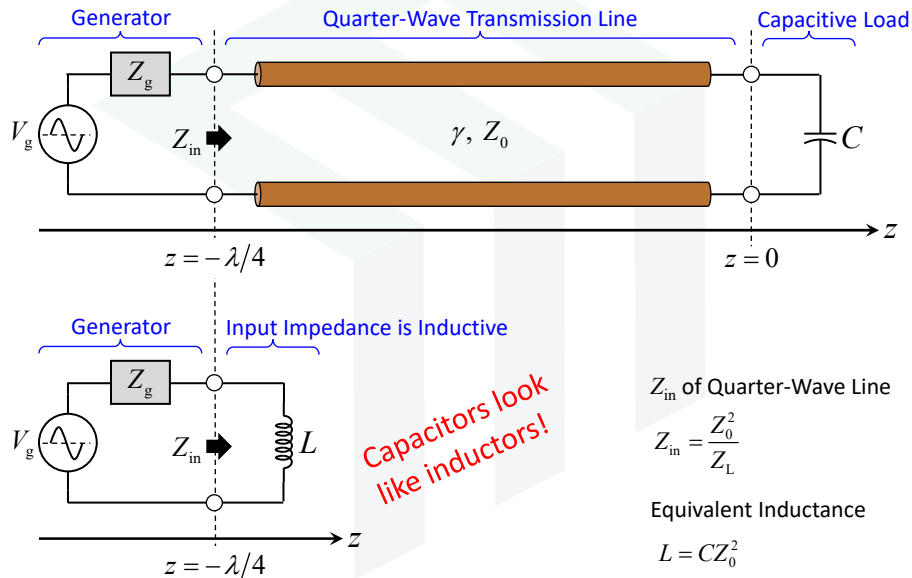
Z_{in} of Quarter-Wave Line

$$Z_{in} = \frac{Z_0^2}{Z_L}$$

Equivalent Capacitance

$$C = \frac{L}{Z_0^2}$$

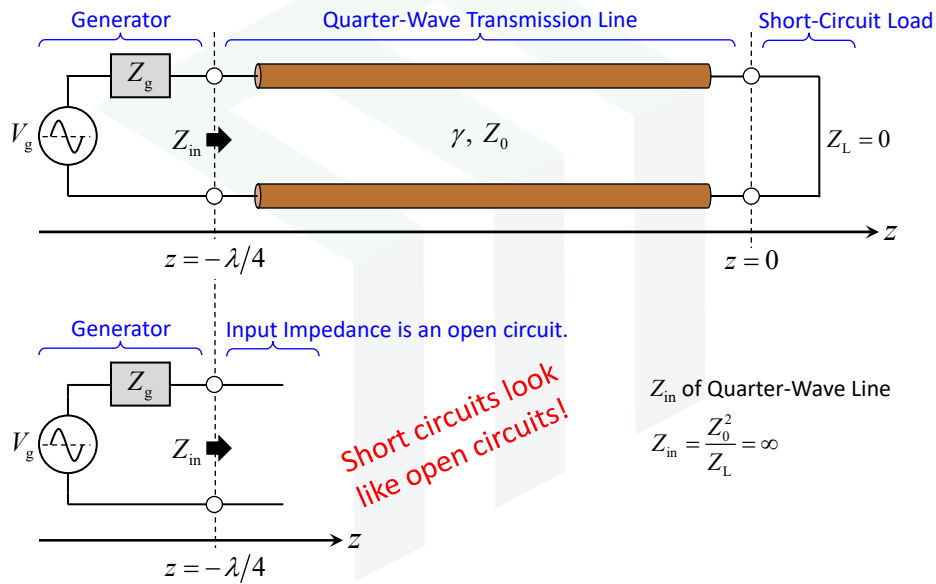
Impedance Inversion (2 of 5)



EMPossible

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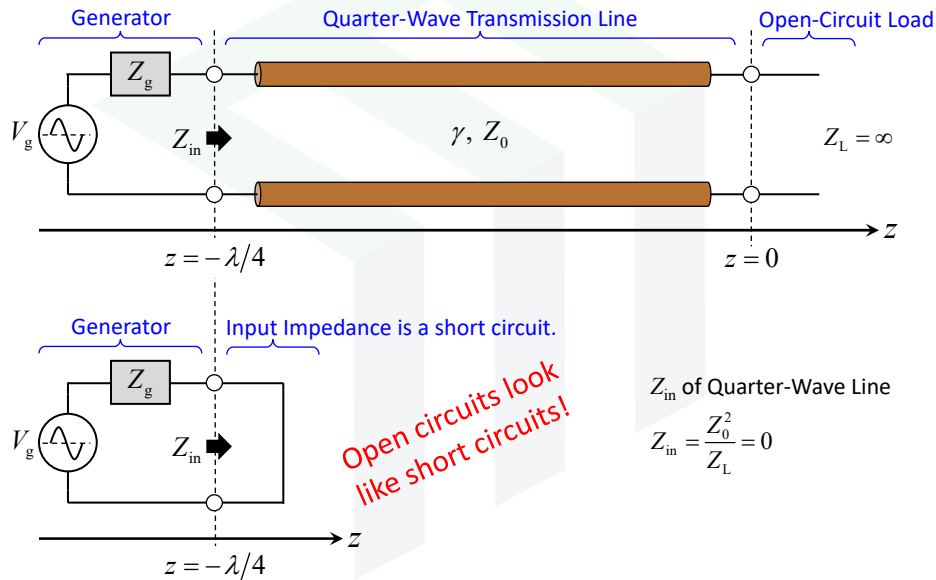
Impedance Inversion (3 of 5)



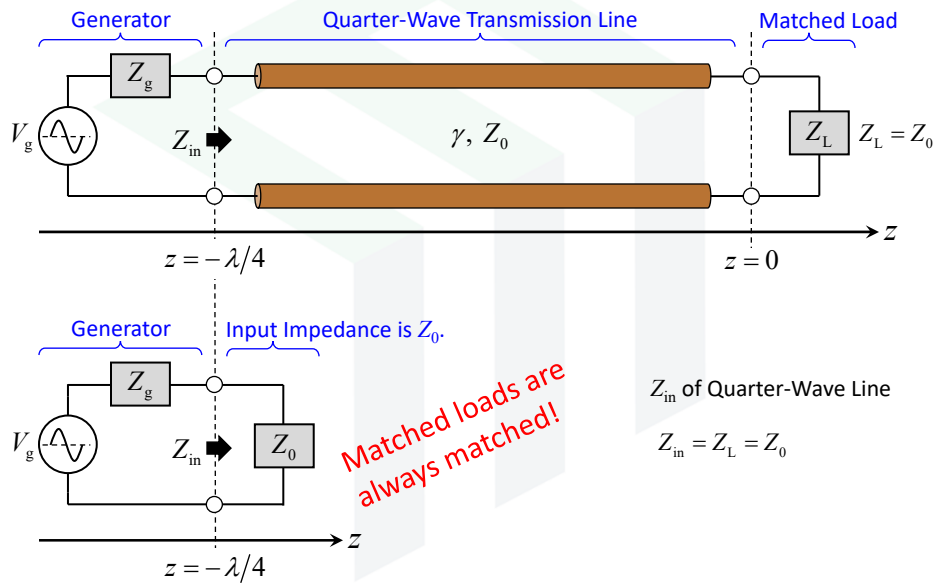
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Impedance Inversion (4 of 5)



Impedance Inversion (5 of 5)

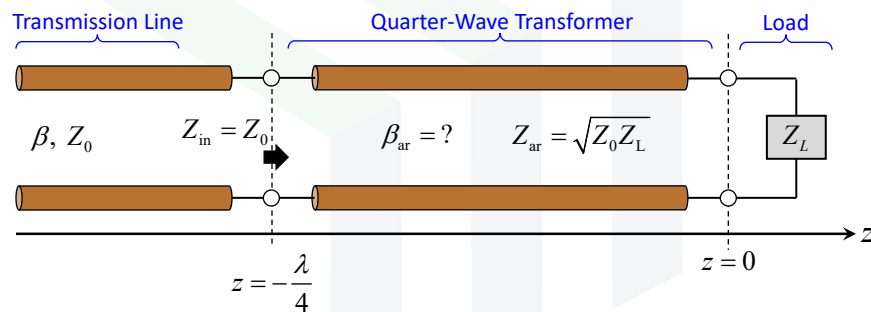


Impedance Matching

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Impedance Matching

Similar to the anti-reflection layer for waves, a transmission line can be matched to a load impedance by inserting a quarter-wave section of a second transmission line.



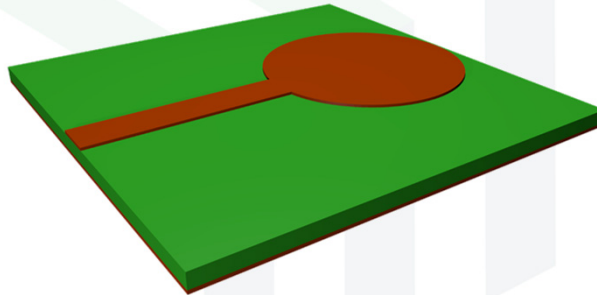
An electromagnetic analysis of the transmission line must be performed to determine the exact phase constant β_{ar} .

$$\beta_{ar} \approx \omega \sqrt{\mu_r \epsilon_r} \quad \ell = \frac{\lambda}{4} = \frac{\pi}{2\beta_{ar}}$$

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Example (1 of 3)

A $50\ \Omega$ microstrip line on FR-4 ($\epsilon_r = 4.4$) operates at 2.4 GHz and is connected to patch antenna which has a $120\ \Omega$ input impedance. How much power is reflected? How can the circuit be improved?



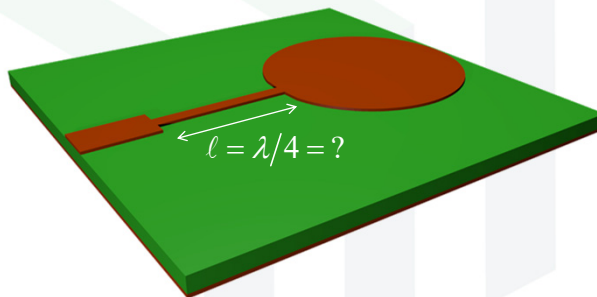
$$\text{Reflected Power: } |\Gamma_L|^2 = \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right|^2 = \left| \frac{(120\ \Omega) - (50\ \Omega)}{(120\ \Omega) + (50\ \Omega)} \right|^2 = |0.4118|^2 = 17\%$$



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Example (2 of 3)

A $50\ \Omega$ microstrip line on FR-4 ($\epsilon_r = 4.4$) operates at 2.4 GHz and is connected to patch antenna which has a $120\ \Omega$ input impedance. How much power is reflected? How can the circuit be improved?



$$\text{Design: } Z_{\text{in}} = \sqrt{Z_L Z_0} = \sqrt{(120\ \Omega)(50\ \Omega)} = 77.5\ \Omega$$

Perform an EM analysis to determine TL dimensions to get $50\ \Omega$. For TEM mode,

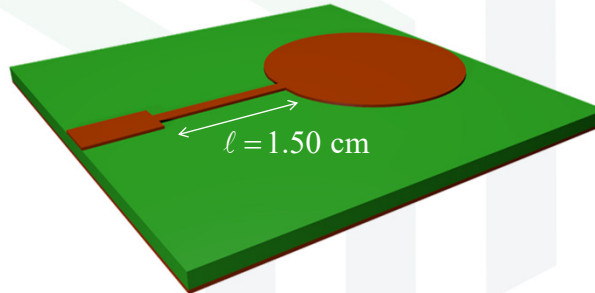
$$\beta = \omega\sqrt{\mu\epsilon} = \frac{2\pi f}{c_0} \sqrt{\mu_r \epsilon_r} = \frac{2\pi(2.4 \times 10^9\ \text{s}^{-1})}{(3.0 \times 10^8\ \text{m/s})} \sqrt{(1.0)(4.4)} = 105.44\ \text{rad/s}$$



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Example (3 of 3)

A $50\ \Omega$ microstrip line on FR-4 ($\epsilon_r = 4.4$) operates at 2.4 GHz and is connected to patch antenna which has a $120\ \Omega$ input impedance. How much power is reflected? How can the circuit be improved?



Design: Given β , the length of the line should be

$$\beta_{\text{at}} \ell = \frac{\pi}{2} \rightarrow \ell = \frac{\pi}{2\beta_{\text{at}}} = \frac{\pi}{2(105.44 \text{ rad/s})} = 1.4898 \times 10^{-2} \text{ m}$$



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What About Purely Reactive Loads?

The reflection coefficient Γ from a purely reactive load $Z_L = jX_L$ is

$$\Gamma = \frac{Z_L - Z_{\text{in}}}{Z_L + Z_{\text{in}}} = \frac{jX_L - Z_{\text{in}}}{jX_L + Z_{\text{in}}}$$

The magnitude of the reflection coefficient is

$$|\Gamma| = \left| \frac{jX_L - Z_{\text{in}}}{jX_L + Z_{\text{in}}} \right| = \frac{|jX_L - Z_{\text{in}}|}{|jX_L + Z_{\text{in}}|} = \frac{\sqrt{(-Z_{\text{in}})^2 + X_L^2}}{\sqrt{Z_{\text{in}}^2 + X_L^2}} = \frac{\sqrt{Z_{\text{in}}^2 + X_L^2}}{\sqrt{Z_{\text{in}}^2 + X_L^2}} = 1$$

Conclusion: A purely reactive load will always reflect 100%. That is because a purely reactive load cannot dissipate any power. The only thing for incident power to do is to reflect.

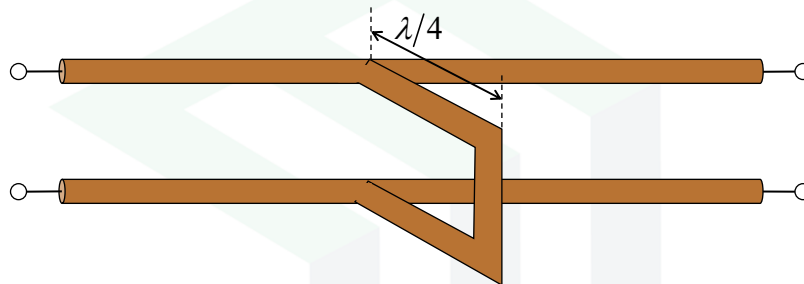


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Stubs

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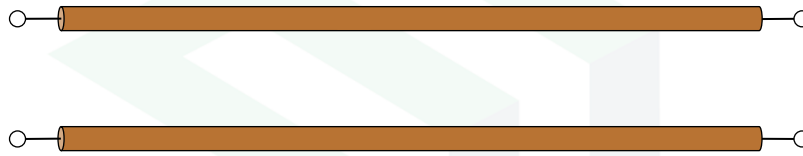
What is a Stub? (1 of 3)



What do short circuits look like $\lambda/4$ away?

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What is a Stub? (2 of 2)

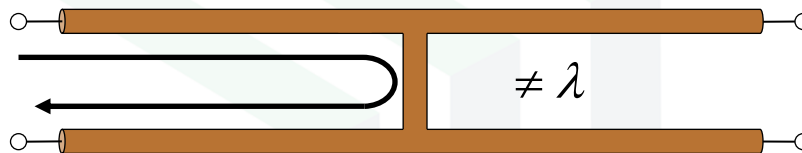


What do short circuits look like $\lambda/4$ away?

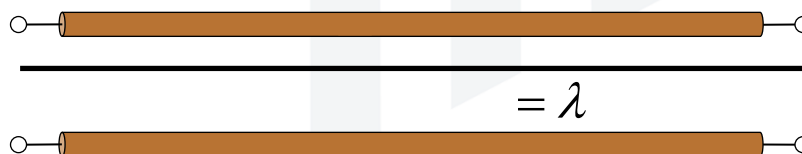
Open circuits!

The Shorted Stub is a Band Pass Filter

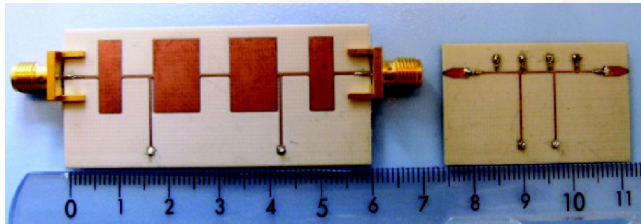
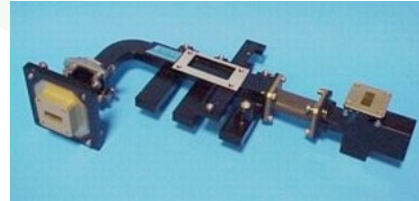
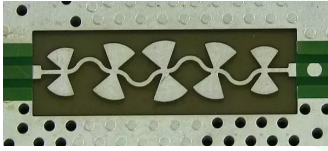
The circuit is actually shorted for all frequencies other than whatever frequency has wavelength λ inside the line. It is blocking these frequencies.



At the frequency with wavelength λ , the circuit is not shorted and signals are allowed to pass. A stub is a simple bandpass filter.



Stubs in Practice



EMPossible

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Scattering Parameters

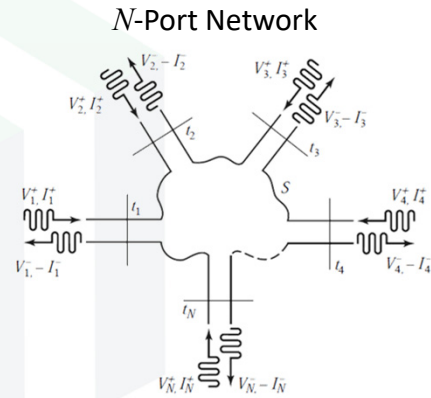
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Definition of a Scattering Matrix

The scattering matrix relates the amplitudes of the input waves to the amplitudes of the output waves.

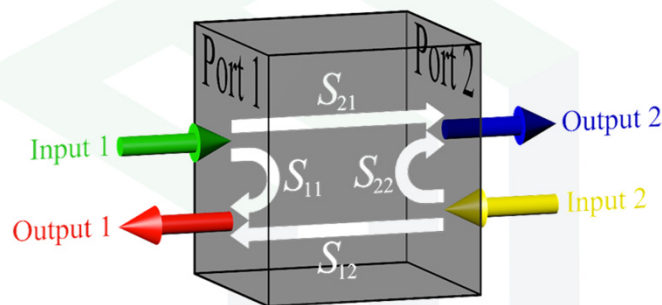
$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1} & S_{N2} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

$$S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{\text{no other applied voltages}}$$



Any linear system can be reduced to a single scattering matrix that describes how it behaves.

S-Matrix for Two-Port Networks



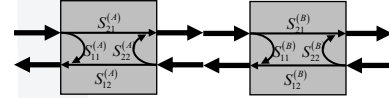
$$\begin{bmatrix} \text{Output 1} \\ \text{Output 2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \text{Input 1} \\ \text{Input 2} \end{bmatrix}$$

S_{11} is synonymous with reflection coefficient.
 S_{21} is synonymous with transmission coefficient.

Very often, engineers will say "S-1-1" instead of saying "reflection," and say "S-2-1" instead of saying transmission.

Combining S-Matrices

Suppose there are two circuits, A and B, described by scattering matrices that placed in series. What is the scattering matrix of the combined network?



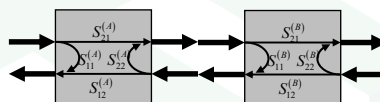
The answer is NOT matrix multiplication!!!

$$\begin{bmatrix} S_{11}^{(AB)} & S_{12}^{(AB)} \\ S_{21}^{(AB)} & S_{22}^{(AB)} \end{bmatrix} \neq \begin{bmatrix} S_{11}^{(A)} & S_{12}^{(A)} \\ S_{21}^{(A)} & S_{22}^{(A)} \end{bmatrix} \begin{bmatrix} S_{11}^{(B)} & S_{12}^{(B)} \\ S_{21}^{(B)} & S_{22}^{(B)} \end{bmatrix}$$

Instead, it is a Redheffer star product.

$$\begin{bmatrix} S_{11}^{(AB)} & S_{12}^{(AB)} \\ S_{21}^{(AB)} & S_{22}^{(AB)} \end{bmatrix} = \begin{bmatrix} S_{11}^{(A)} & S_{12}^{(A)} \\ S_{21}^{(A)} & S_{22}^{(A)} \end{bmatrix} \otimes \begin{bmatrix} S_{11}^{(B)} & S_{12}^{(B)} \\ S_{21}^{(B)} & S_{22}^{(B)} \end{bmatrix}$$

Redheffer Star Product



$$\begin{bmatrix} S_{11}^{(AB)} & S_{12}^{(AB)} \\ S_{21}^{(AB)} & S_{22}^{(AB)} \end{bmatrix} = \begin{bmatrix} S_{11}^{(A)} & S_{12}^{(A)} \\ S_{21}^{(A)} & S_{22}^{(A)} \end{bmatrix} \otimes \begin{bmatrix} S_{11}^{(B)} & S_{12}^{(B)} \\ S_{21}^{(B)} & S_{22}^{(B)} \end{bmatrix}$$

$$S_{11}^{(AB)} = \frac{S_{11}^{(A)} - S_{11}^{(A)} S_{22}^{(A)} S_{11}^{(B)} + S_{12}^{(A)} S_{21}^{(A)} S_{11}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}$$

$$S_{12}^{(AB)} = \frac{S_{12}^{(A)} S_{12}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}$$

$$S_{22}^{(AB)} = \frac{S_{22}^{(B)} - S_{22}^{(A)} S_{11}^{(B)} S_{22}^{(B)} + S_{21}^{(A)} S_{12}^{(A)} S_{22}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}$$

$$S_{21}^{(AB)} = \frac{S_{21}^{(A)} S_{21}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}$$