Electromagnetics:
Electromagnetic Field Theory

Multi-Segment Transmission Line Devices

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Lecture Outline

- Quarter-Wave Transformer
- Impedance Matching
- Stubs
- Scattering Parameters

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Quarter-Wave Transformer

Quarter-Wave Transformer (1 of 2)

A quarter-wave transformer is a section of line that is a $\lambda/4$ long.

When the length of the line is $\lambda/4$, then we have

$$\beta\ell = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{\pi}{2}$$

This means the signal accumulates 90° of phase.
When told a TL is $\lambda/4$, usually no other information is needed.

When this is the case, our impedance transformation equation reduces to

$$Z_{\text{in}}(-\ell) = Z_0 \frac{Z_L + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_L \tan(\beta\ell)} = Z_0 \frac{Z_L + jZ_0 \tan(\pi/2)}{Z_0 + jZ_L \tan(\pi/2)}$$

$$= Z_0 \frac{Z_L + jZ_0 \cdot \infty}{Z_0 + jZ_L \cdot \infty}$$

$\tan(\pi/2) = \infty$.

$$= ?$$

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Quarter-Wave Transformer (2 of 2)

Since both the numerator and denominator are ∞, we must apply L'Hopital's rule.

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{f'(x_0)}{g'(x_0)}$$

Applying this to our impedance transformation equation, we get

$$Z_{\text{in}}(-\ell) = \lim_{\beta\ell \to \pi/2} Z_0 \frac{Z_{\text{L}} + jZ_0 \tan(\beta\ell)}{Z_0 + jZ_{\text{L}} \tan(\beta\ell)} = Z_0 \frac{jZ_0 \sec^2(\beta\ell)}{jZ_{\text{L}} \sec^2(\beta\ell)} = \frac{Z_0^2}{Z_{\text{L}}}$$

The final equation shows that the load impedance Z_{L} gets completely inverted. The input impedance becomes the input admittance.

$$\boxed{Z_{\text{in}}\left(-\frac{\lambda}{4}\right) = \frac{Z_0^2}{Z_{\text{L}}}}$$

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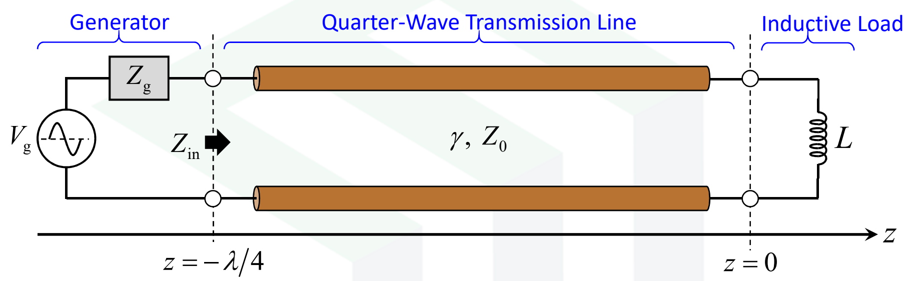
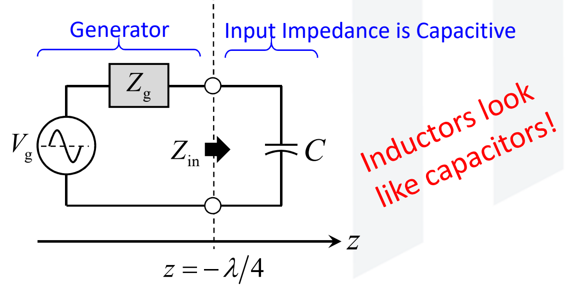
Impedance Inversion (1 of 5)





Z_{in} of Quarter-Wave Line

$$Z_{\text{in}} = \frac{Z_0^2}{Z_{\text{L}}}$$

Equivalent Capacitance

$$C = \frac{L}{Z_0^2}$$

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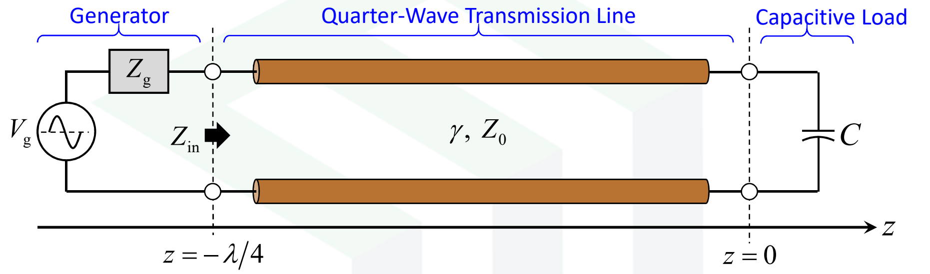
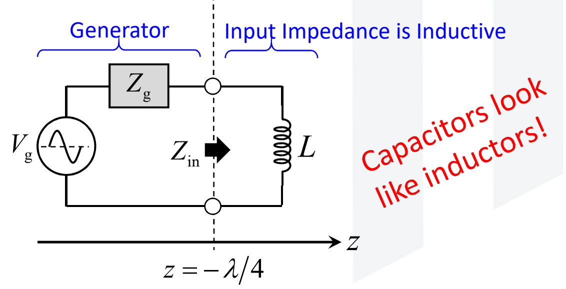
Impedance Inversion (2 of 5)





Z_{in} of Quarter-Wave Line

$$Z_{in}=\frac{Z_0^2}{Z_L}$$

Equivalent Inductance

$$L=CZ_0^2$$

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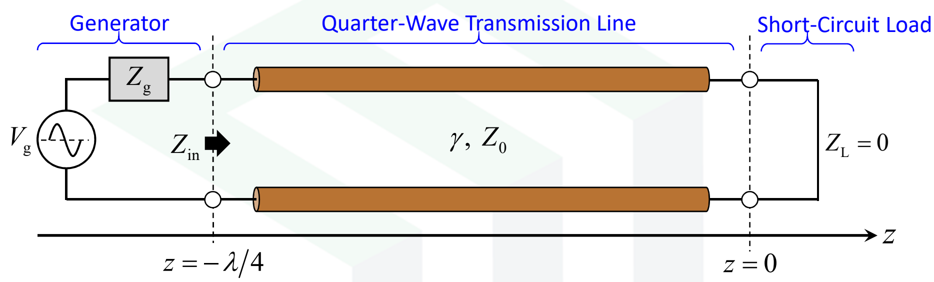
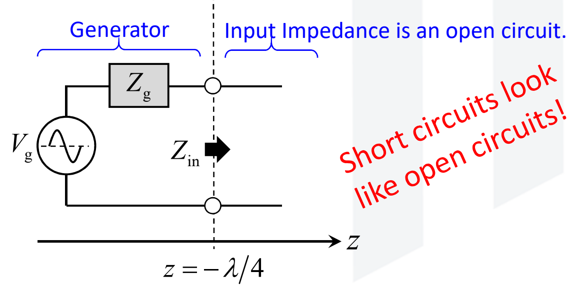
Impedance Inversion (3 of 5)





Z_{in} of Quarter-Wave Line

$$Z_{in}=\frac{Z_0^2}{Z_L}=\infty$$

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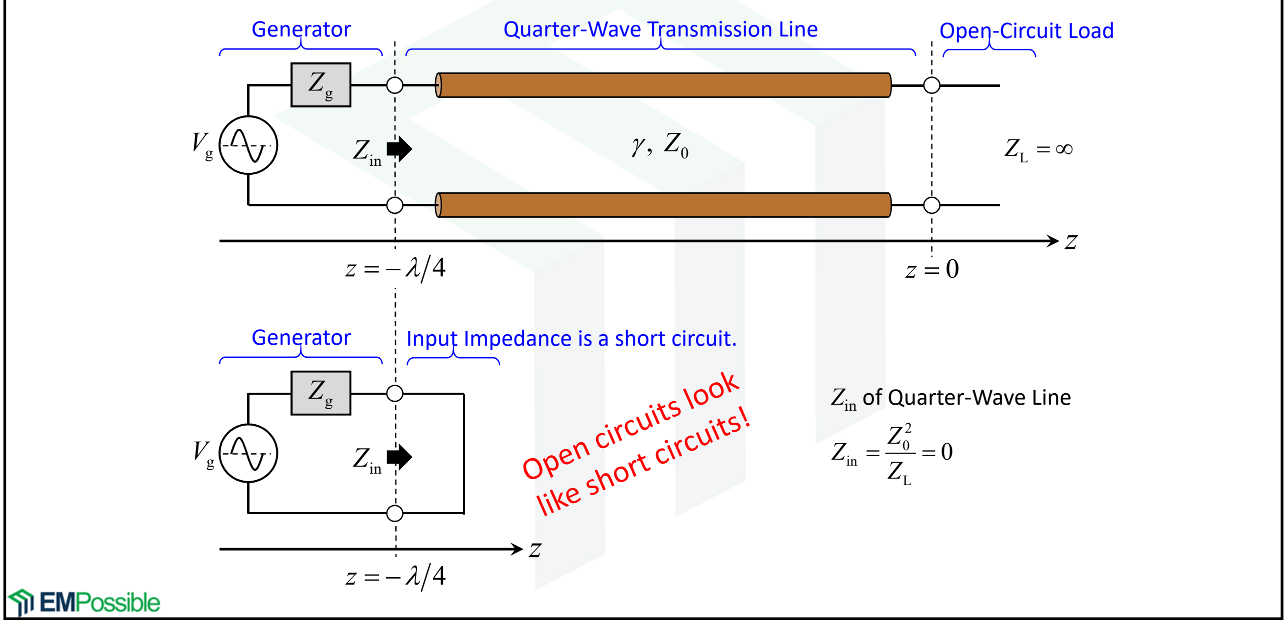
Impedance Inversion (4 of 5)



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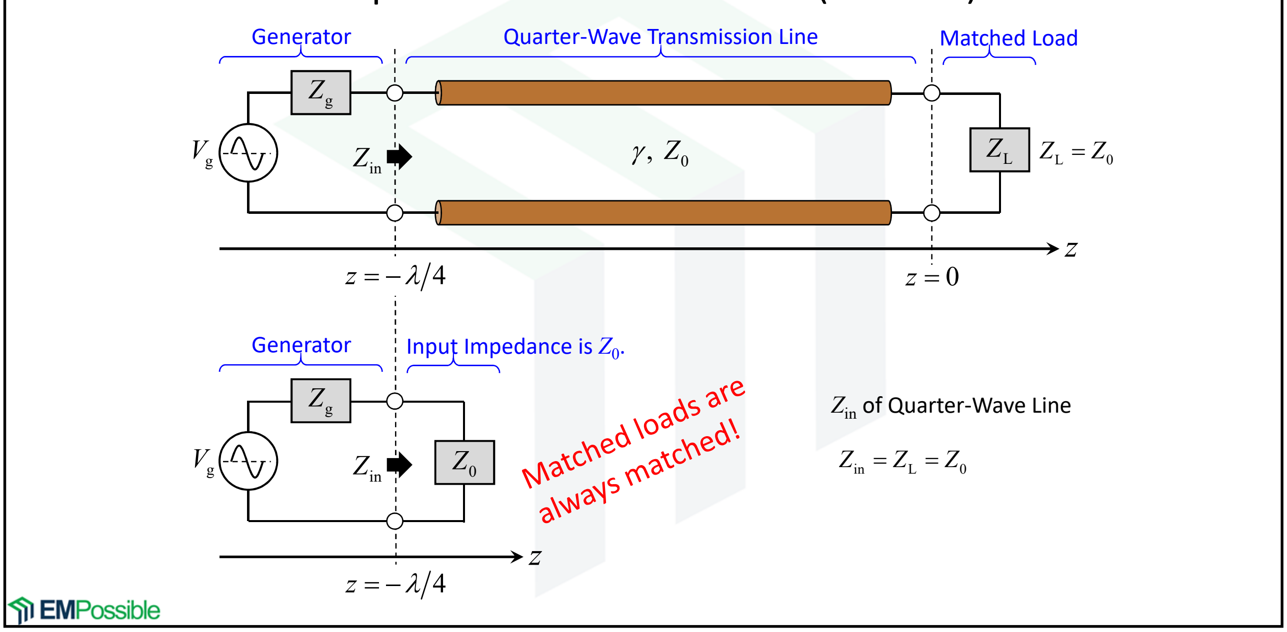
Impedance Inversion (5 of 5)



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Impedance Matching

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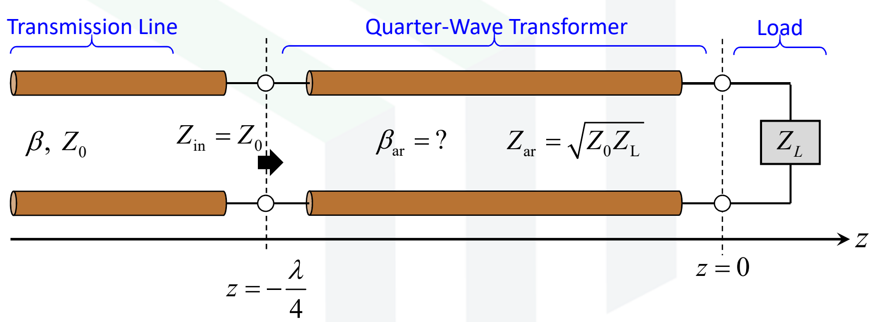
Impedance Matching

Similar to the anti-reflection layer for waves, we can match a transmission line to a load impedance by inserting a quarter-wave section of a second transmission line.



We must perform an electromagnetic analysis of the transmission line to determine β_{ar}.

$$\beta_{ar} \approx \omega\sqrt{\mu_r \varepsilon_r} \qquad \ell = \frac{\lambda}{4} = \frac{\pi}{2\beta_{ar}}$$

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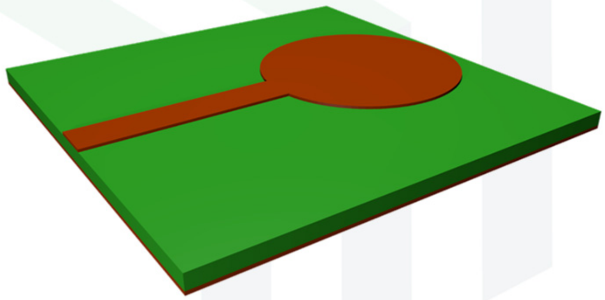
Example (1 of 3)

A 50 Ω microstrip line on FR-4 (ε_r = 4.4) operates at 2.4 GHz and is connected to patch antenna which has a 120 Ω input impedance. How much power is reflected? How can the circuit be improved?



Reflected Power: $\left|\Gamma_L\right|^2 = \left|\frac{Z_L - Z_0}{Z_L - Z_0}\right|^2 = \left|\frac{(120\ \Omega)-(50\ \Omega)}{(120\ \Omega)+(50\ \Omega)}\right|^2 = \left|0.4118\right|^2 = 17\%$

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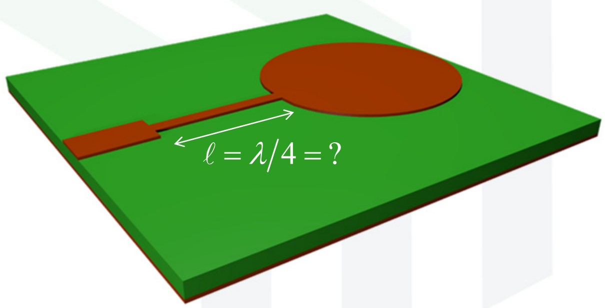
Example (2 of 3)

A 50 Ω microstrip line on FR-4 (ε_r = 4.4) operates at 2.4 GHz and is connected to patch antenna which has a 120 Ω input impedance. How much power is reflected? How can the circuit be improved?



Design: $Z_{ar} = \sqrt{Z_L Z_0} = \sqrt{(120\ \Omega)(50\ \Omega)} = 77.5\ \Omega$

Perform an EM analysis to determine TL dimensions to get 50 Ω. For TEM mode,

$$\beta = \omega\sqrt{\mu\varepsilon} = \frac{2\pi f}{c_0}\sqrt{\mu_r\varepsilon_r} = \frac{2\pi\left(2.4\times10^9\ \text{s}^{-1}\right)}{\left(3.0\times10^8\ \text{m/s}\right)}\sqrt{(1.0)(4.4)} = 105.44\ \text{rad/s}$$

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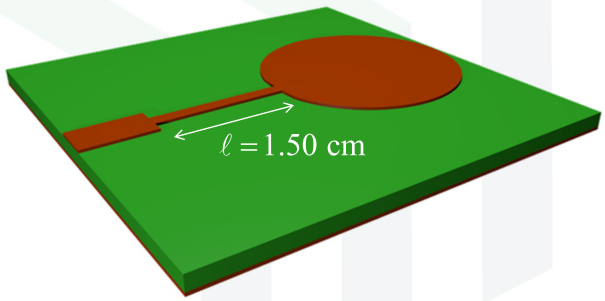
Example (3 of 3)

A 50 Ω microstrip line on FR-4 ($\varepsilon_r = 4.4$) operates at 2.4 GHz and is connected to patch antenna which has a 120 Ω input impedance. How much power is reflected? How can the circuit be improved?



Design: Given β, the length of the line should be

$$\beta_{ar}\ell = \frac{\pi}{2} \quad \rightarrow \quad \ell = \frac{\pi}{2\beta_{ar}} = \frac{\pi}{2(105.44 \ \text{rad/s})} = 1.4898 \times 10^{-2} \ \text{m}$$

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Stubs

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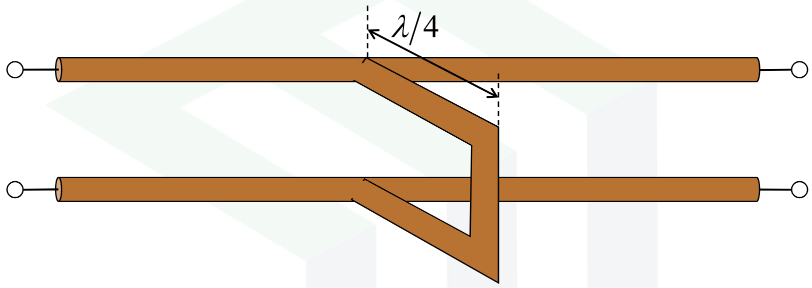
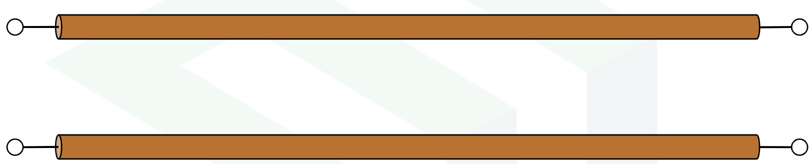
What is a Stub? (1 of 3)



What do short circuits look like $\lambda/4$ away?

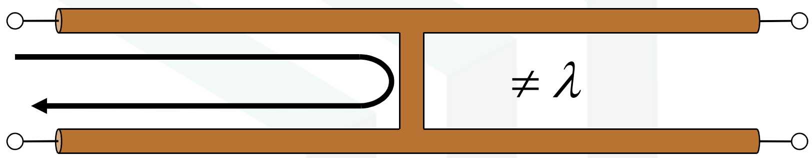
What is a Stub? (2 of 2)



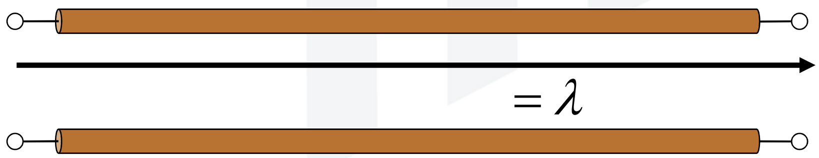
What do short circuits look like $\lambda/4$ away? Open circuits!

The Shorted Stub is a Band Pass Filter

The circuit is actually shorted for all frequencies other than whatever frequency has wavelength λ inside the line. The short circuit blocks all signals.



At the frequency with wavelength λ, the circuit is not shorted and signals are allowed to pass.



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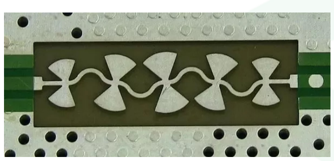
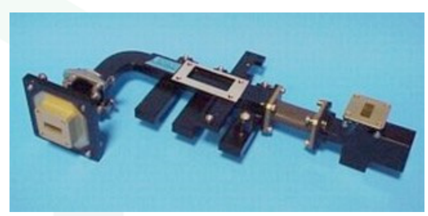
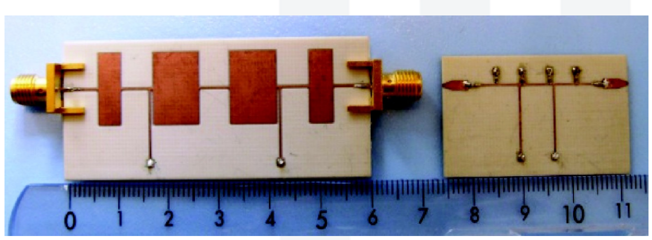
Stubs in Practice







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Scattering Parameters

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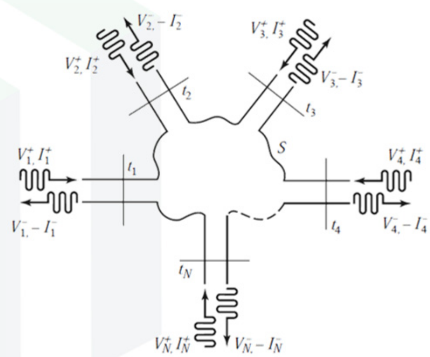
Definition of a Scattering Matrix

The scattering matrix relates the amplitudes of the input waves to the amplitudes of the output waves.

$$\begin{bmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \cdots & S_{2N} \\ \vdots & \vdots & \ddots & \\ S_{N1} & S_{N2} & & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

$$S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{\text{no other applied voltages}}$$

N-Port Network



Any linear system can be reduced to a single scattering matrix that describes how it behaves.

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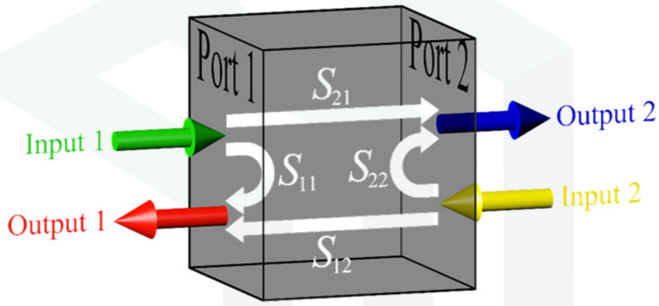
S-Matrix for Two-Port Networks



$$\begin{bmatrix} \text{Output 1} \\ \text{Output 2} \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} \text{Input 1} \\ \text{Input 2} \end{bmatrix}$$

S_{11} is synonymous with reflection coefficient.
S_{21} is synonymous with transmission coefficient.

Very often, engineers will say “S-1-1” instead of saying “reflection,” and say “S-2-1” instead of saying transmission.

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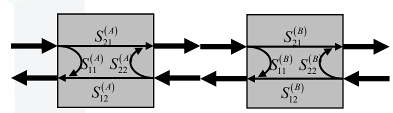
Combining S-Matrices

Suppose there are two circuits, A and B, described by scattering matrices that placed in series. What is the scattering matrix of the combined network?



The answer is NOT matrix multiplication!!!

$$\begin{bmatrix} S_{11}^{(AB)} & S_{12}^{(AB)} \\ S_{21}^{(AB)} & S_{22}^{(AB)} \end{bmatrix} \neq \begin{bmatrix} S_{11}^{(A)} & S_{12}^{(A)} \\ S_{21}^{(A)} & S_{22}^{(A)} \end{bmatrix} \begin{bmatrix} S_{11}^{(B)} & S_{12}^{(B)} \\ S_{21}^{(B)} & S_{22}^{(B)} \end{bmatrix}$$

Instead, it is a Redheffer star product.

$$\begin{bmatrix} S_{11}^{(AB)} & S_{12}^{(AB)} \\ S_{21}^{(AB)} & S_{22}^{(AB)} \end{bmatrix} = \begin{bmatrix} S_{11}^{(A)} & S_{12}^{(A)} \\ S_{21}^{(A)} & S_{22}^{(A)} \end{bmatrix} \otimes \begin{bmatrix} S_{11}^{(B)} & S_{12}^{(B)} \\ S_{21}^{(B)} & S_{22}^{(B)} \end{bmatrix}$$

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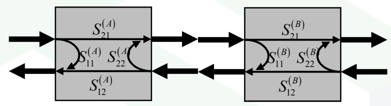
Redheffer Star Product



$$\begin{bmatrix} S_{11}^{(AB)} & S_{12}^{(AB)} \\ S_{21}^{(AB)} & S_{22}^{(AB)} \end{bmatrix} = \begin{bmatrix} S_{11}^{(A)} & S_{12}^{(A)} \\ S_{21}^{(A)} & S_{22}^{(A)} \end{bmatrix} \otimes \begin{bmatrix} S_{11}^{(B)} & S_{12}^{(B)} \\ S_{21}^{(B)} & S_{22}^{(B)} \end{bmatrix}$$

$$S_{11}^{(AB)} = \frac{S_{11}^{(A)} - S_{11}^{(A)} S_{22}^{(A)} S_{11}^{(B)} + S_{12}^{(A)} S_{21}^{(A)} S_{11}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}$$

$$S_{12}^{(AB)} = \frac{S_{12}^{(A)} S_{12}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}$$

$$S_{22}^{(AB)} = \frac{S_{22}^{(B)} - S_{22}^{(A)} S_{11}^{(B)} S_{22}^{(B)} + S_{22}^{(A)} S_{12}^{(B)} S_{21}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}$$

$$S_{21}^{(AB)} = \frac{S_{21}^{(A)} S_{21}^{(B)}}{1 - S_{22}^{(A)} S_{11}^{(B)}}$$

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