Electromagnetics:
Electromagnetic Field Theory

Scattering From Multiple Interfaces

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Lecture Outline

- Scattering From a Dielectric Slab
- Anti-Reflection Layer
- Bragg Gratings
- Photonic Crystals

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Scattering From a Dielectric Slab

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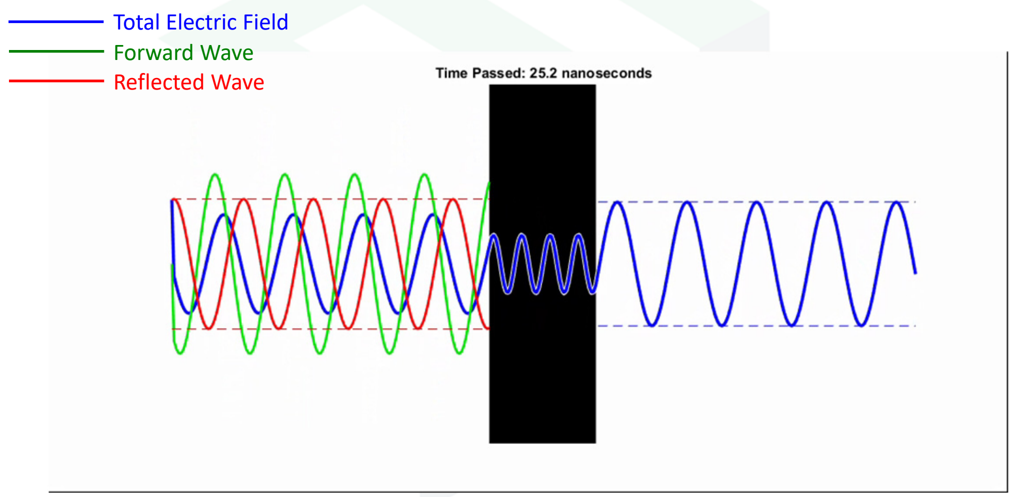
Time-Domain Simulation of Scattering From a Slab



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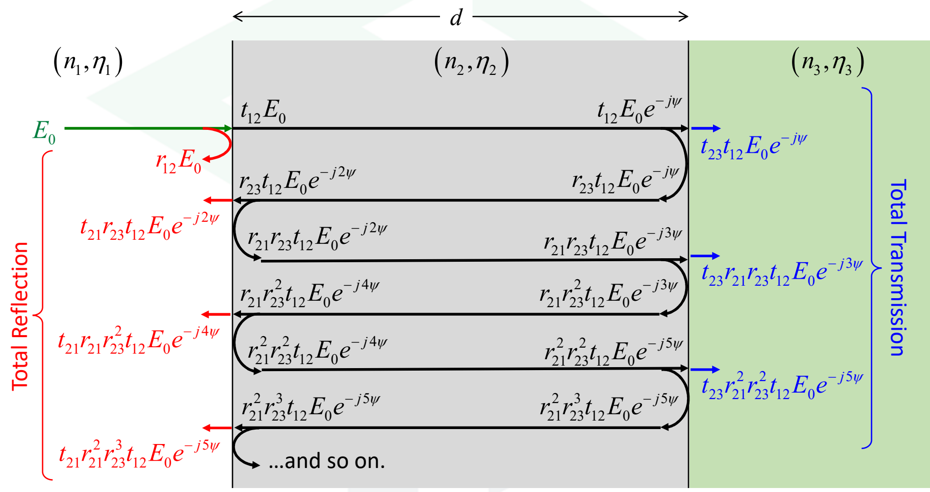
Analysis of a Dielectric Slab



$r_{12}=\frac{\eta_2-\eta_1}{\eta_2+\eta_1}$ $\quad t_{12}=1-r_{12}$

$r_{21}=\frac{\eta_1-\eta_2}{\eta_1+\eta_2}=-r_{12}$ $\quad t_{21}=1+r_{12}$

$\psi=k_0n_2d$

$r_{23}=\frac{\eta_3-\eta_2}{\eta_3+\eta_2}$

$t_{23}=1-r_{23}$

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Overall Reflection, r (1 of 3)

The overall reflection from the slab is the sum of all the individual reflected waves.

$$r=r_{12}+t_{21}r_{23}t_{12}e^{-j2\psi}+t_{21}r_{21}r_{23}^2t_{12}e^{-j4\psi}+t_{21}r_{21}^3r_{23}^3t_{12}e^{-j5\psi}+\cdots$$

All of these terms arise due to multiple reflections within the slab. They can be written as a summation.

$$\sum_{n=0}^{\infty}t_{21}r_{21}^nr_{23}^{n+1}t_{12}e^{-j2(n+1)\psi}$$

$$r_{23}t_{21}t_{12}e^{-j2\psi}\sum_{n=0}^{\infty}\left(r_{21}r_{23}e^{-j2\psi}\right)^n$$

Now put the summation back into the expression for overall reflection r.

$$r=r_{12}+r_{23}t_{21}t_{12}e^{-j2\psi}\sum_{n=0}^{\infty}\left(r_{21}r_{23}e^{-j2\psi}\right)^n$$

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Overall Reflection, r (2 of 3)

Recall the closed-form expression for a geometric series.

$$\sum_{n=0}^{\infty} ax^n = \frac{a}{1-x} \qquad \text{for } |x|<1$$

The summation in the expression for overall reflection is a geometric series that can be written in closed form.

$$\begin{aligned} r &= r_{12} + r_{23}t_{21}t_{12}e^{-j2\psi}\sum_{n=0}^{\infty}\left(r_{21}r_{23}e^{-j2\psi}\right)^n \\ &= r_{12} + r_{23}t_{21}t_{12}e^{-j2\psi}\frac{1}{1-r_{21}r_{23}e^{-j2\psi}} \\ &= r_{12} + \frac{r_{23}t_{21}t_{12}e^{-j2\psi}}{1-r_{21}r_{23}e^{-j2\psi}} \end{aligned}$$

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Overall Reflection, r (3 of 3)

Recall how the local reflection and transmission parameters were related.

$$r_{21} = -r_{12} \qquad t_{12} = 1 - r_{12} \qquad t_{21} = 1 + r_{12} \qquad t_{23} = 1 - r_{23}$$

This lets r be expressed just in terms of r_{12}, r_{23}, and θ.

$$\begin{aligned} r &= r_{12} + \frac{r_{23}t_{21}t_{12}e^{-j2\psi}}{1-r_{21}r_{23}e^{-j2\psi}} \\ &= r_{12} + \frac{r_{23}\left(1+r_{12}\right)\left(1-r_{12}\right)e^{-j2\psi}}{1-\left(-r_{12}\right)r_{23}e^{-j2\psi}} \\ &= \frac{r_{12}\left(1+r_{12}r_{23}e^{-j2\psi}\right)}{1+r_{12}r_{23}e^{-j2\psi}} + \frac{r_{23}e^{-j2\psi} - r_{12}^2 r_{23}e^{-j2\psi}}{1+r_{12}r_{23}e^{-j2\psi}} \quad \rightarrow \quad \boxed{r = \frac{r_{12}+r_{23}e^{-j2\psi}}{1+r_{12}r_{23}e^{-j2\psi}}} \end{aligned}$$

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Overall Transmission, t (1 of 2)

The overall transmission through the slab is the sum of all the individual transmitted waves.

$$t = t_{23}t_{12}e^{-j\psi} + t_{23}r_{21}r_{23}t_{12}e^{-j3\psi} + t_{23}r_{21}^2r_{23}^2t_{12}e^{-j5\psi} + \cdots$$

This can be written as a summation.

$$t = \sum_{n=0}^{\infty} t_{23}r_{21}^n r_{23}^n t_{12}e^{-j(2n+1)\psi}$$

Now factor out some terms from the summation.

$$t = t_{23}t_{12}e^{-j\psi}\sum_{n=0}^{\infty}\left(r_{21}r_{23}e^{-j2\psi}\right)^n$$

The summation in this expression is a geometric series and can be written in closed form.

$$t = t_{23}t_{12}e^{-j\psi}\frac{1}{1-r_{21}r_{23}e^{-j2\psi}}$$

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Overall Transmission, t (2 of 2)

Recall how the local reflection and transmission parameters were related.

$$r_{21} = -r_{12} \qquad t_{12} = 1 - r_{12} \qquad t_{21} = 1 + r_{12} \qquad t_{23} = 1 - r_{23}$$

This let's t be expressed just in terms of r_{12}, r_{23}, and θ.

$$\begin{aligned} t &= \frac{t_{23}t_{12}e^{-j\psi}}{1-r_{21}r_{23}e^{-j2\psi}} \\ &= \frac{(1-r_{23})(1-r_{12})e^{-j\psi}}{1-(-r_{12})r_{23}e^{-j2\psi}} \end{aligned} \quad \rightarrow \quad \boxed{t = \frac{(1-r_{23})(1-r_{12})e^{-j\psi}}{1+r_{12}r_{23}e^{-j2\psi}}}$$

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Relation Between r and t

Solve the two expressions for r and t for $(1 + r_{12}r_{23}e^{-j2\theta})$.

$$r = \frac{r_{12} + r_{23}e^{-j2\psi}}{1 + r_{12}r_{23}e^{-j2\psi}} \qquad t = \frac{(1 - r_{23})(1 - r_{12})e^{-j\psi}}{1 + r_{12}r_{23}e^{-j2\psi}}$$

$$1 + r_{12}r_{23}e^{-j2\psi} = \frac{r_{12} + r_{23}e^{-j2\psi}}{r} \qquad 1 + r_{12}r_{23}e^{-j2\psi} = \frac{(1 - r_{23})(1 - r_{12})e^{-j\psi}}{t}$$

The expressions on the right-hand side of these equations must be equal.

$$\frac{(1 - r_{23})(1 - r_{12})e^{-j\psi}}{t} = \frac{r_{12} + r_{23}e^{-j2\psi}}{r}$$

The relation between r and t is therefore

$$\frac{t}{r} = \frac{(1 - r_{23})(1 - r_{12})}{r_{12} + r_{23}e^{-j\psi}}$$

Note: This is NOT the same relation that derived for a single interface.

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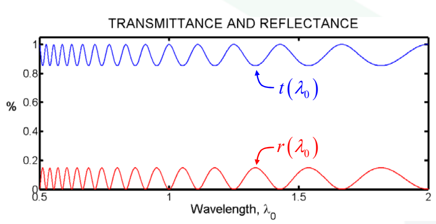
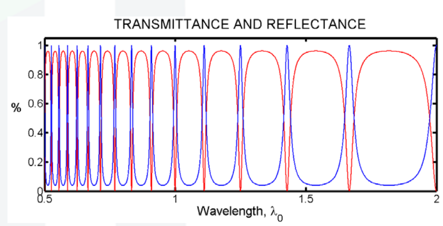
Plots of r and t

Small Reflections (low finesse)



The response resembles a cosine function and is usually approximated as such.

Large Reflections (high finesse)



The response resembles a comb filter.

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Low Finesse (1 of 2)

To understand low finesse, assume the slab is symmetric (i.e. $r_{12} = -r_{23}$) and that these reflection coefficients are small.

$$r = \frac{r_{12} + r_{23}e^{-j2\psi}}{1 + r_{12}r_{23}e^{-j2\psi}} = \frac{r_{12} + (-r_{12})e^{-j2\psi}}{1 + r_{12}(-r_{12})e^{-j2\psi}} = \frac{r_{12}\left(1 - e^{-j2\psi}\right)}{1 - r_{12}^2 e^{-j2\psi}}$$

$$t = \frac{(1 - r_{23})(1 - r_{12})e^{-j\psi}}{1 + r_{12}r_{23}e^{-j2\psi}} = \frac{\left(1 - (-r_{12})\right)(1 - r_{12})e^{-j\psi}}{1 + r_{12}(-r_{12})e^{-j2\psi}} = \frac{(1 + r_{12})(1 - r_{12})e^{-j\psi}}{1 - r_{12}^2 e^{-j2\psi}}$$

For small reflections, $1 - r_{12}^2 e^{-j2\psi} \approx 1$

The expressions for r and t reduce to

$$r = r_{12}\left(1 - e^{-j2\psi}\right)$$

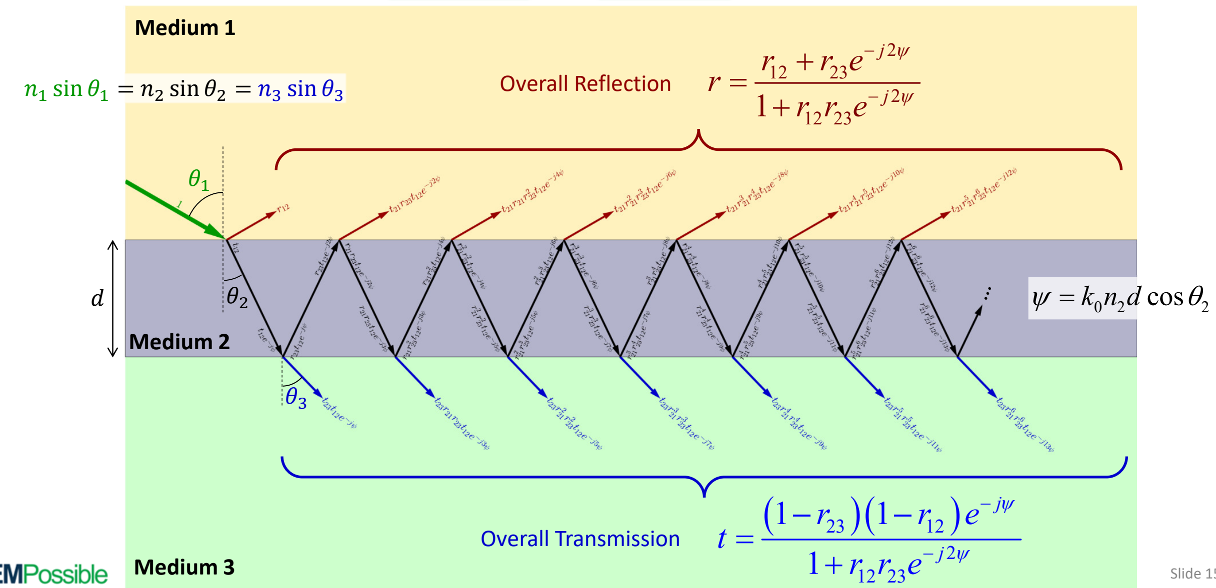
$$t = \left(1 - r_{12}^2\right)e^{-j\psi}$$

Low Finesse (2 of 2)

The magnitude of r gives the sine wave response that was expected.

$$|r| = \left|r_{12}\left(1 - e^{-j2\psi}\right)\right| = 2|r_{12}|\sin\psi$$

Oblique Incidence



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Generalizations

Normal incidence, no loss: $\psi = k_0 n_2 d$

Oblique incidence, no loss: $\psi = k_0 n_2 d \cos\theta_2$

Normal incidence, lossy: $\psi = k_0 \tilde{n}_2 d, \;\; \tilde{n}_2 = n_o - j\kappa$

Oblique incidence, lossy: $\psi = k_0 \tilde{n}_2 d \cos\theta_2, \;\; \tilde{n}_2 = n_o - j\kappa$

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Example: Do Windows Block Wifi? (1 of 2)

Windows are typically made of fused silica (n = 1.52) and are around 3 mm thick.

Solution

Transmission through a slab of dielectric is calculated using

$$t = \frac{(1-r_{23})(1-r_{12})e^{-j\psi}}{1+r_{12}r_{23}e^{-j2\psi}}$$

The parameters in this equation are

$$r_{12} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{n_1-n_2}{n_1+n_2} = \frac{1.0-1.52}{1.0+1.52} = -0.2063$$

$$r_{23} = \frac{\eta_3-\eta_2}{\eta_3+\eta_2} = \frac{n_2-n_3}{n_2+n_3} = \frac{1.52-1.0}{1.52+1.0} = +0.2063$$

$$\psi = k_0 nd = \frac{2\pi f}{c_0} nd = \frac{2\pi\left(2.4\times10^9 \text{ Hz}\right)}{\left(3.0\times10^8 \text{ m/s}\right)}(1.52)(0.003 \text{ m}) = 0.2292$$

Example: Do Windows Block Wifi? (2 of 2)

Windows are typically made of fused silica (n = 1.52) and are around 3 mm thick.

Solution cont'd

Substituting our values into the transmission equation gives

$$t = \frac{(1-0.2063)(1+0.2063)e^{-j0.2292}}{1+(-0.2063)(0.2063)e^{-j2(0.2292)}} = 0.9646 - j0.2450$$

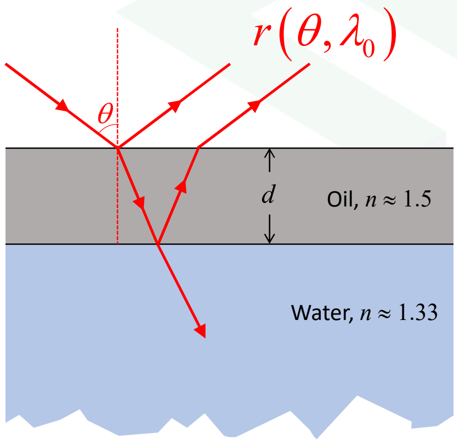
Total power transmitted is

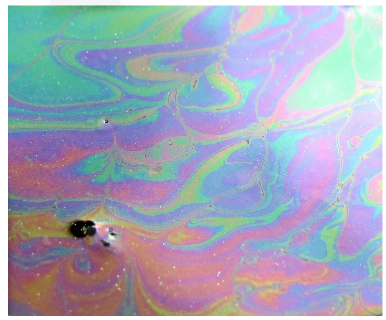
$$T = |t|^2 = |0.9646 - j0.2450|^2 = 99.52\%$$

CONCLUSION → Windows do almost nothing to block Wifi.

Example: Oil on Water

Oil on water is an example of *thin film interference*.





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Antireflection Layer

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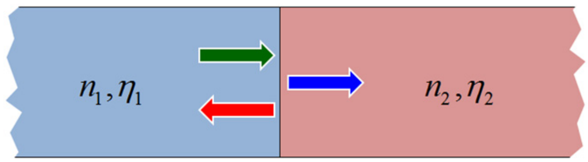
Problem Setup (1 of 2)

Let there be an interface between two materials.



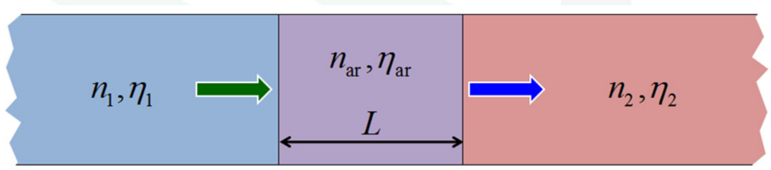
This will produce reflections according to

$$r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

How can reflection be prevented at this interface?

Problem Setup (2 of 2)

Insert an intermediate layer that will be called an *anti-reflection layer*.



How can n_{ar}, η_{ar}, and L be chosen to get exactly zero reflections from this interface?

How to Get $r = 0$ From a Slab (1 of 4)

Recall the overall reflection from a dielectric slab.

$$r = \frac{r_{12} + r_{23}e^{-j2\psi}}{1 + r_{12}r_{23}e^{-j2\psi}}$$

To get $r = 0$, the numerator of this expression must be zero.

$$r_{12} + r_{23}e^{-j2\psi} = 0$$

The reflection coefficients r_{12} and r_{23} arise from the materials in the problem that cannot be changed. The trick must be in the $e^{-j2\psi}$ term. Solving this term for ψ gives

$$e^{-j2\psi} = -\frac{r_{12}}{r_{23}} \quad \rightarrow \quad \psi = \frac{1}{j2}\ln\left(\frac{r_{23}}{r_{12}}\right) - \pi m$$

$$m = \text{any integer}$$

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How to Get $r = 0$ From a Slab (2 of 4)

Recall that

$$r_{12} = \frac{\eta_{\text{ar}} - \eta_1}{\eta_{\text{ar}} + \eta_1} \qquad r_{23} = \frac{\eta_2 - \eta_{\text{ar}}}{\eta_2 + \eta_{\text{ar}}}$$

The expression for ψ becomes

$$\psi = \frac{1}{j2}\ln\left[\frac{(\eta_{\text{ar}} + \eta_1)(\eta_{\text{ar}} - \eta_2)}{(\eta_{\text{ar}} - \eta_1)(\eta_{\text{ar}} + \eta_2)}\right] - \pi m$$

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How to Get $r = 0$ From a Slab (3 of 4)

Recall that $\psi = k_0 n_{\text{ar}} d$. Substitute this into the design equation from the previous slide.

$$k_0 n_{\text{ar}} d = \frac{1}{j2} \ln\left[\frac{(\eta_{\text{ar}} + \eta_1)(\eta_{\text{ar}} - \eta_2)}{(\eta_{\text{ar}} - \eta_1)(\eta_{\text{ar}} + \eta_2)}\right] - \pi m$$

Using $k_0 = 2\pi/\lambda_0$ and solving this for d gives

$$k_0 n_{\text{ar}} d = \frac{1}{j2} \ln\left[\frac{(\eta_{\text{ar}} + \eta_1)(\eta_{\text{ar}} - \eta_2)}{(\eta_{\text{ar}} - \eta_1)(\eta_{\text{ar}} + \eta_2)}\right] - \pi m$$

$$\downarrow$$

$$\boxed{d = \frac{\lambda_0}{4 n_{\text{ar}}} \frac{1}{j\pi} \ln\left[\frac{(\eta_{\text{ar}} + \eta_1)(\eta_{\text{ar}} - \eta_2)}{(\eta_{\text{ar}} - \eta_1)(\eta_{\text{ar}} + \eta_2)}\right] - m \frac{\lambda_0}{2 n_{\text{ar}}}}$$

This is the most general design equation and provides more freedom than the simpler one about to be derived.

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How to Get $r = 0$ From a Slab (3 of 4)

The general design equation on the previous slide is complicated. Is it possible to simplify?

To figure out a way to do this, multiply out the expression inside of the natural logarithm function.

$$\frac{(\eta_{\text{ar}} + \eta_1)(\eta_{\text{ar}} - \eta_2)}{(\eta_{\text{ar}} - \eta_1)(\eta_{\text{ar}} + \eta_2)} = \frac{\eta_{\text{ar}}^2 - \eta_{\text{ar}}(\eta_2 - \eta_1) - \eta_1 \eta_2}{\eta_{\text{ar}}^2 + \eta_{\text{ar}}(\eta_2 - \eta_1) - \eta_1 \eta_2}$$

This simplifies when $\eta_{\text{ar}}^2 = \eta_1 \eta_2$. In fact, it reduces to just -1.

Recognizing that $\ln(-1) = j\pi$, the simplified expression for d becomes

$$\boxed{d = \frac{\lambda_0}{4 n_{\text{ar}}} - m \frac{\lambda_0}{2 n_{\text{ar}}} \quad \text{for } \eta_{\text{ar}}^2 = \eta_1 \eta_2}$$

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Interpretation of Design Equation

When $\eta_{\text{ar}}^2 = \eta_1 \eta_2$

$$d = \frac{\lambda_0}{4n_{\text{ar}}} - m\frac{\lambda_0}{2n_{\text{ar}}} \qquad m = \text{any integer}$$

The second term tells us that the length d can be adjusted by any integer multiple of a half-wavelength.

$$m\frac{\lambda_0}{2n_{\text{ar}}} = m\frac{\lambda}{2}$$

Interpret this first terms as a quarter wavelength slab of dielectric.

$$\frac{\lambda_0}{4n_{\text{ar}}} = \frac{\lambda}{4}$$

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Design Procedure

Step 1 – Choose an antireflection material such that

$$\eta_{\text{ar}} = \sqrt{\eta_1 \eta_2}$$

There is a bit of freedom here.

However, when only dielectric materials are used (i.e. $\mu_r \approx 1$), only one choice is possible.

$$\varepsilon_{\text{ar}} = \sqrt{\varepsilon_1 \varepsilon_2} \quad \text{or} \quad n_{\text{ar}} = \sqrt{n_1 n_2}$$

Step 2 – Calculate thickness based on refractive index.

$$d = \frac{\lambda_0}{4n_{\text{ar}}} - m\frac{\lambda_0}{2n_{\text{ar}}} \qquad m = \text{any integer}$$

$m = 0$ is the most common choice.

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Example

It is desired to maximize the light through a lens. The lens is made of glass with n = 1.52 and resides in air with n = 1.0. Design an anti-reflection coating to maximize transmission at the center of the visible spectrum, λ_0 = 500 nm.

Solution

Step 1: At optical frequencies, materials cannot have a significant magnetic response. Therefore, we will design the anti-reflection layer through the refractive index n_{ar}.

$$n_{\text{ar}} = \sqrt{n_1 n_2} = \sqrt{(1.0)(1.52)} \quad \rightarrow \quad \boxed{n_{\text{ar}} = 1.2329}$$

Step 2: The thickness of the anti-reflection layer is

$$d = \frac{500\text{ nm}}{4(1.2329)} - m\frac{500\text{ nm}}{2(1.2329)} = 101.4\text{ nm} - m(202.8\text{ nm})$$

Choose the $m = 0$ solution.

$$\boxed{d = 101.4\text{ nm}}$$

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Anti-Transmission Layer?

Recall the general expression for transmission.

$$t = \frac{t_{23}t_{12}e^{-j\psi}}{1 - r_{21}r_{23}e^{-j2\psi}}$$

How can $t = 0$?

$$t_{23} = 0 \longrightarrow \quad t_{23} = \frac{2\eta_3}{\eta_2 + \eta_3} = 0 \quad \rightarrow \eta_3 = 0 \ \text{ not a valid choice}$$

$$t_{12} = 0 \longrightarrow \quad t_{12} = \frac{2\eta_2}{\eta_1 + \eta_2} = 0 \quad \rightarrow \eta_2 = 0 \ \text{ choose a metal}$$

$$e^{-j\psi} = 0 \longrightarrow \quad -j\psi = \ln 0 \quad \rightarrow d = \infty \ \text{ impossible thickness}$$

Unless an extreme material is used to block all transmission at the first reflection, there really is no such thing as an anti-transmission layer.

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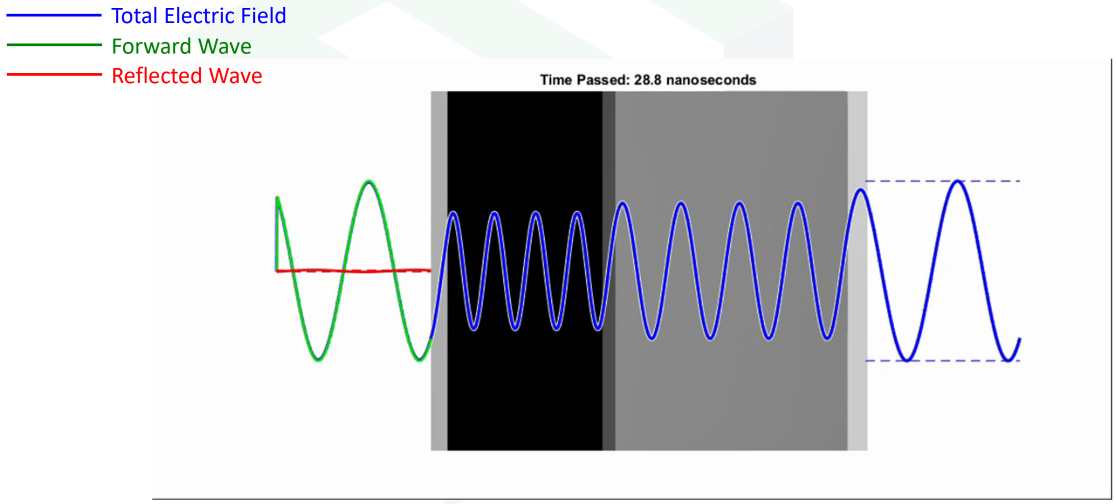
Simulation of Antireflection Layers



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Bragg Gratings

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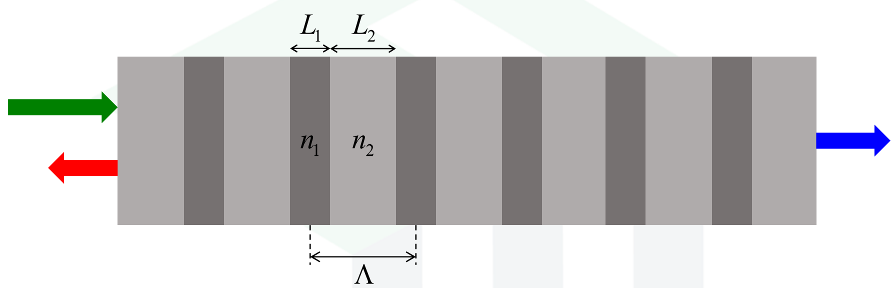
What is a Bragg Grating?

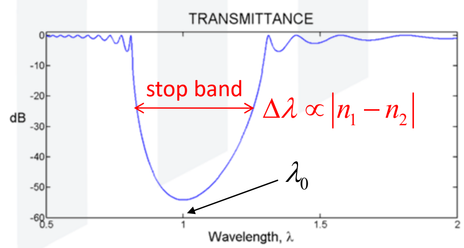


Design

$$L_1 = \frac{\lambda_0}{4n_1}$$

$$L_2 = \frac{\lambda_0}{4n_2}$$

Quarter-wave layers



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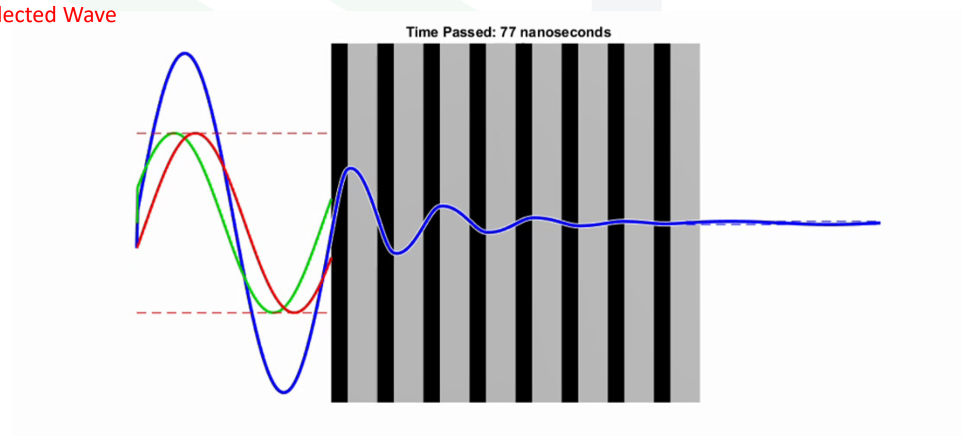
Simulation of a Bragg Grating

Total Electric Field
Forward Wave
Reflected Wave



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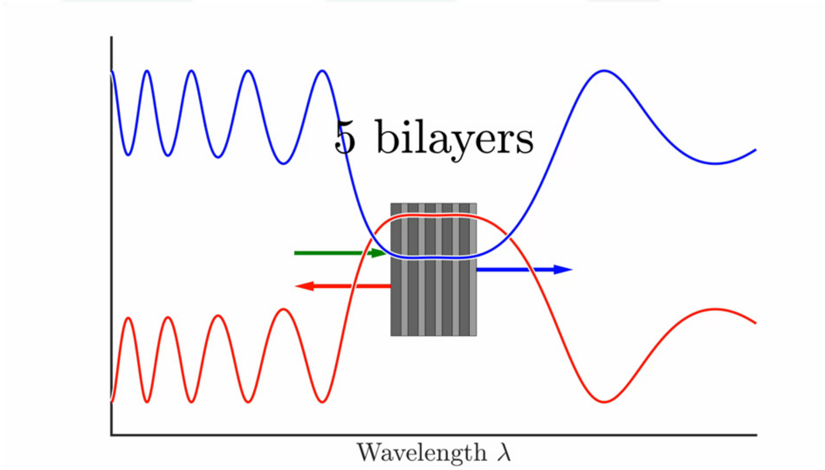
Effect of Number of Layers

The number of layers in a Bragg grating primarily effects the strength of reflection and transmission within the reflection band.



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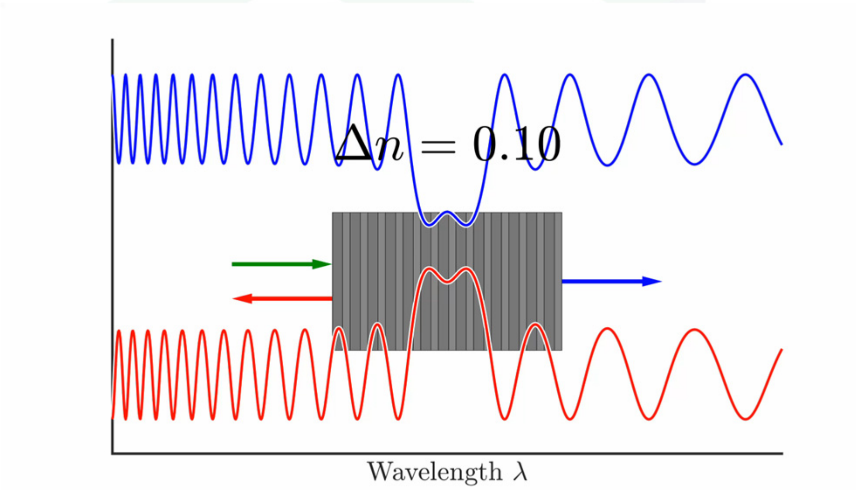
Effect of Index Contrast

The dielectric contrast of a Bragg grating primarily effects the bandwidth of the reflection band.



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Example

Design a Bragg grating to reflect 980 nm light using silicon dioxide (n_{SiO2} = 1.52) and silicon nitride (n_{SiN} = 1.9) to provide 20 dB suppression.

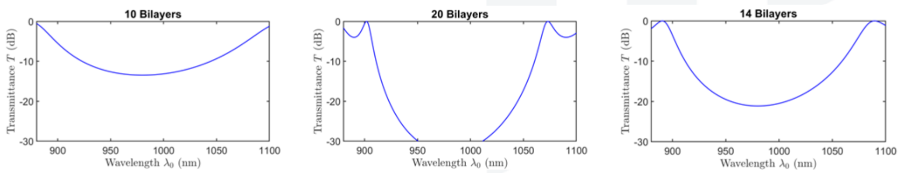
Solution

Step 1: Given refractive indices, calculate layer thicknesses.

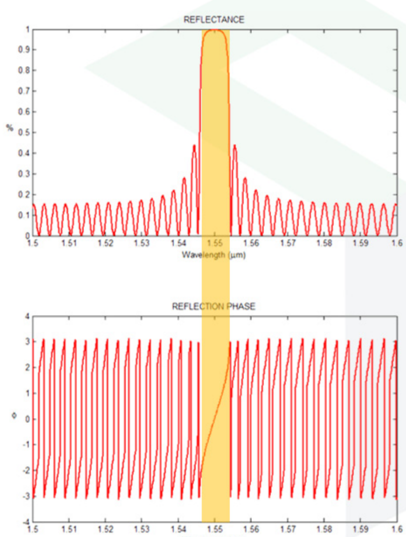
$$d_{\text{SiO}_2} = \frac{\lambda}{4} = \frac{\lambda_0}{4n_{\text{SiO}_2}} = \frac{980\text{ nm}}{4(1.52)} \boxed{=161.2\text{ nm}}$$

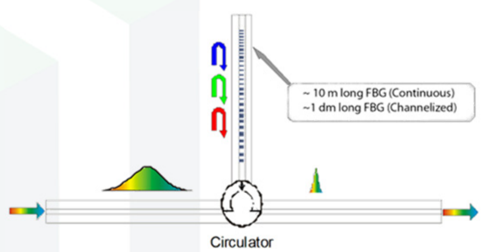
$$d_{\text{SiN}} = \frac{\lambda}{4} = \frac{\lambda_0}{4n_{\text{SiN}}} = \frac{980\text{ nm}}{4(1.9)} \boxed{=128.9\text{ nm}}$$

Step 2: Determine number of layers



Dispersion Compensation Using Chirped Bragg Gratings





http://electronicdesign.com/communications/fiber-bragg-gratings-dispersion-compensation-technology-40g-and-100g-optical-transpor

Photonic Crystals

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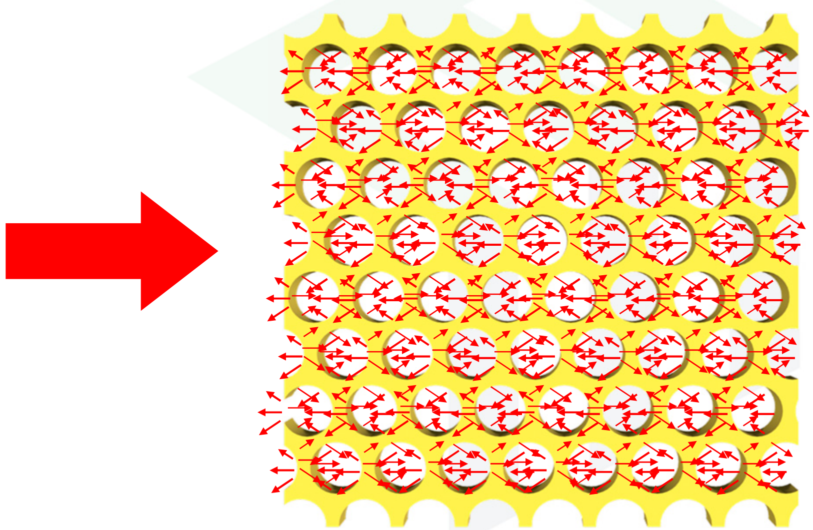
Very Complicated Multiple Scattering!



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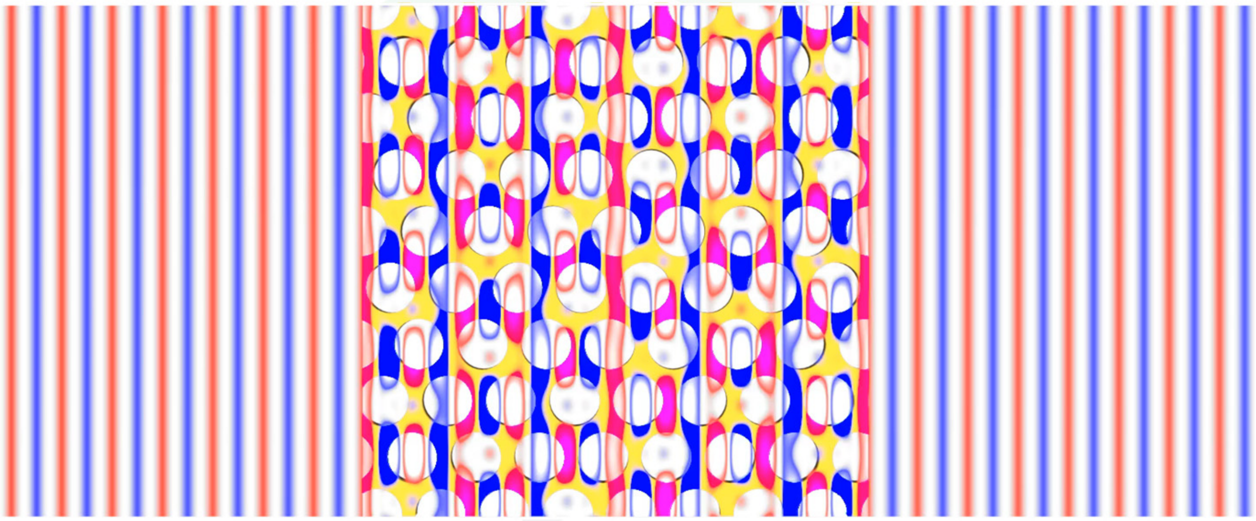
Bloch Waves Replace Plane Waves



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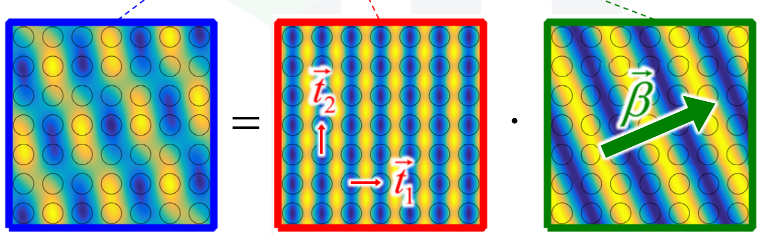
The Bloch Theorem

Waves inside of a periodic structure are analogous to plane waves, but they are modulated by an envelope function. It is the envelope function that takes on the same symmetry and periodicity as the structure.

$$\vec{E}(\vec{r}) = \vec{A}(\vec{r})\, e^{j\vec{\beta}\bullet\vec{r}}$$



Overall field is the combination of the envelope and plane wave term.

Envelope function has the same symmetry and periodicity as the periodic structure.

Plane-wave like phase "tilt" term.

$\vec{\beta} \equiv$ Bloch wave vector

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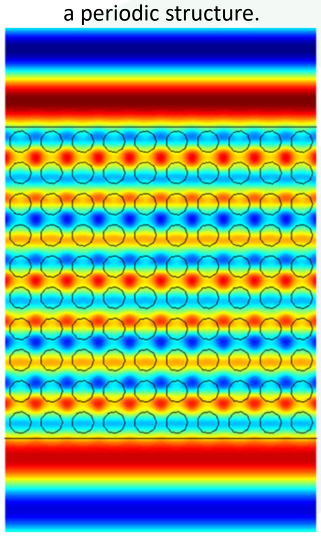
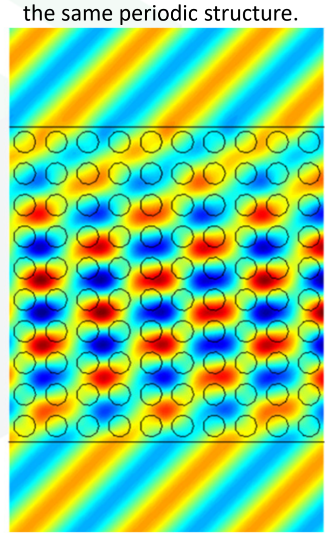
Example Waves in a Periodic Lattice

Wave normally incident onto a periodic structure.



Wave incident at 45° onto the same periodic structure.



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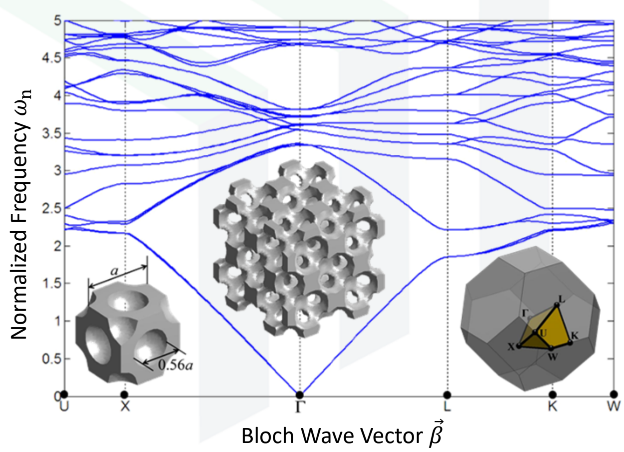
Photonic Crystals

Photonic crystals are periodic structures that control photons in analogous ways to how electrons are controlled inside of semiconductors.



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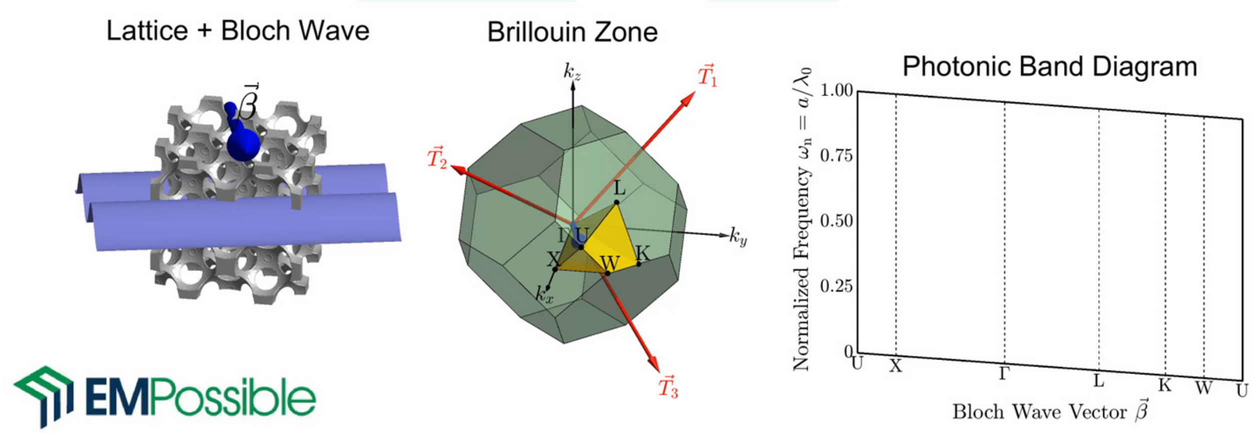
Photonic Band Diagrams



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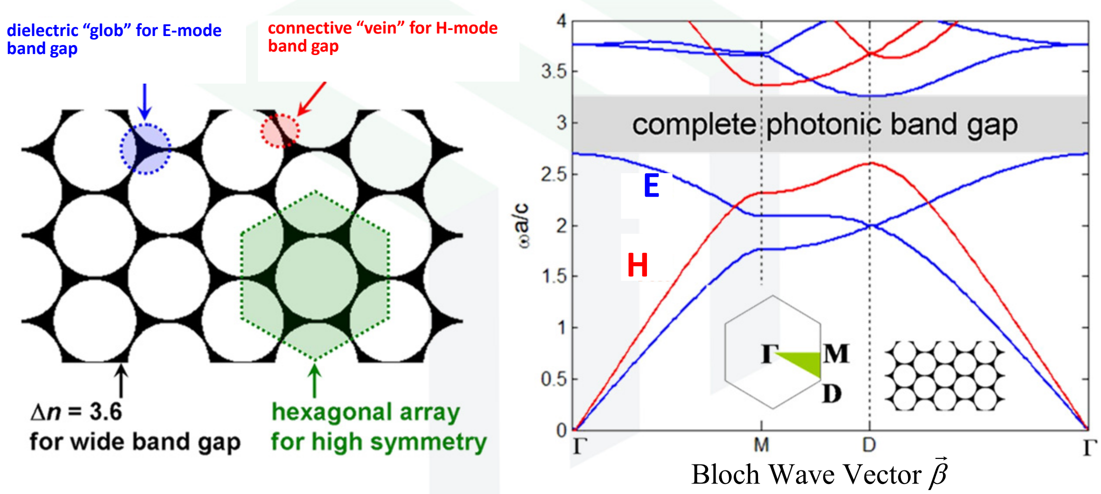
Photonic Band Gaps



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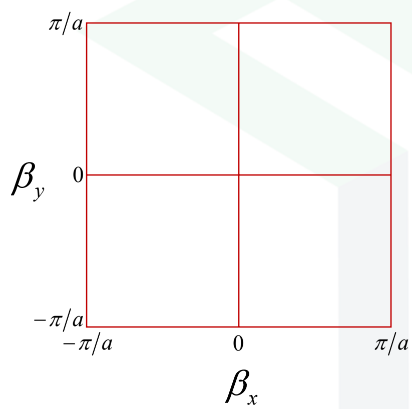
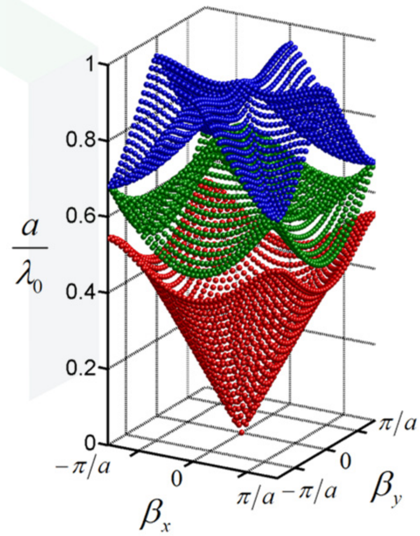
The Complete Band Diagram

The Full Brillouin Zone





There is an infinite set of eigen-frequencies associated with each point in the Brillouin zone. These form "sheets" as shown at right.

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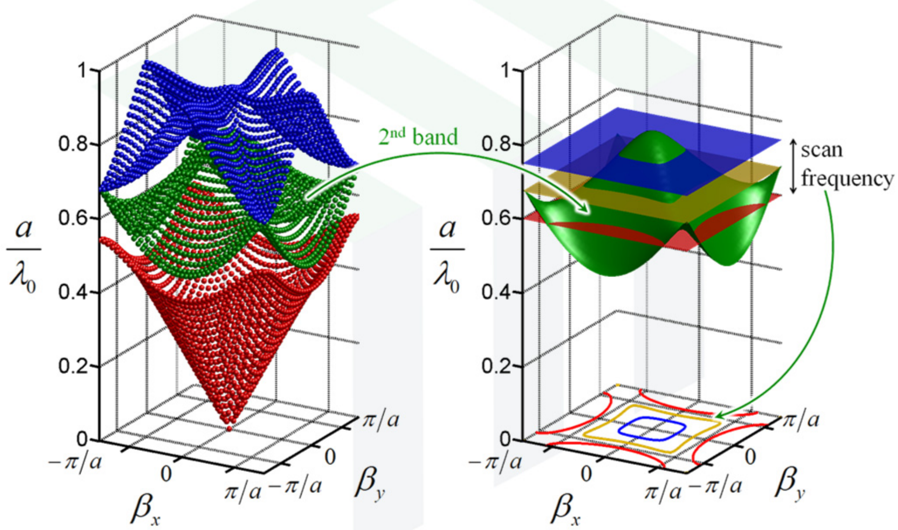
Constructing Isofrequency Contours (Index Ellipsoids)



Index ellipsoids are "isofrequency contours" in k-space.

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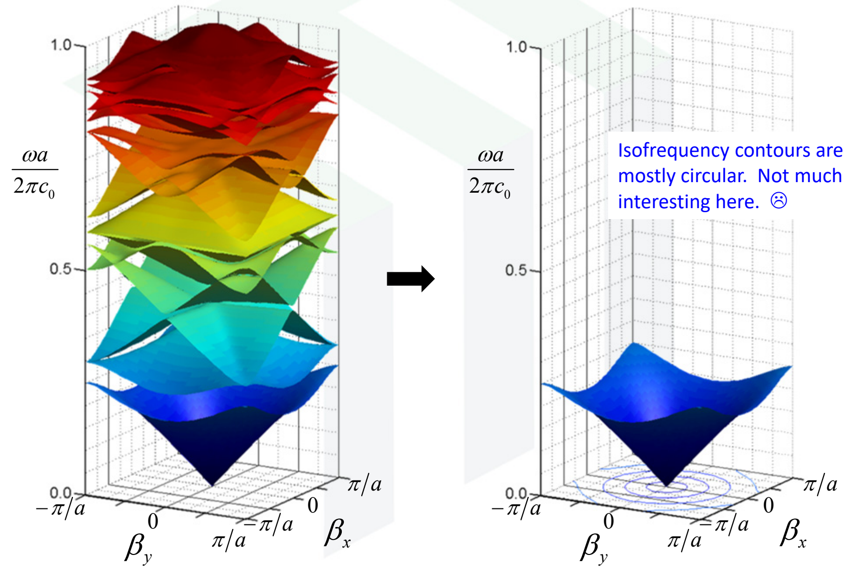
Isofrequency Contours From First-Order Band



Isofrequency contours are mostly circular. Not much interesting here. ☹

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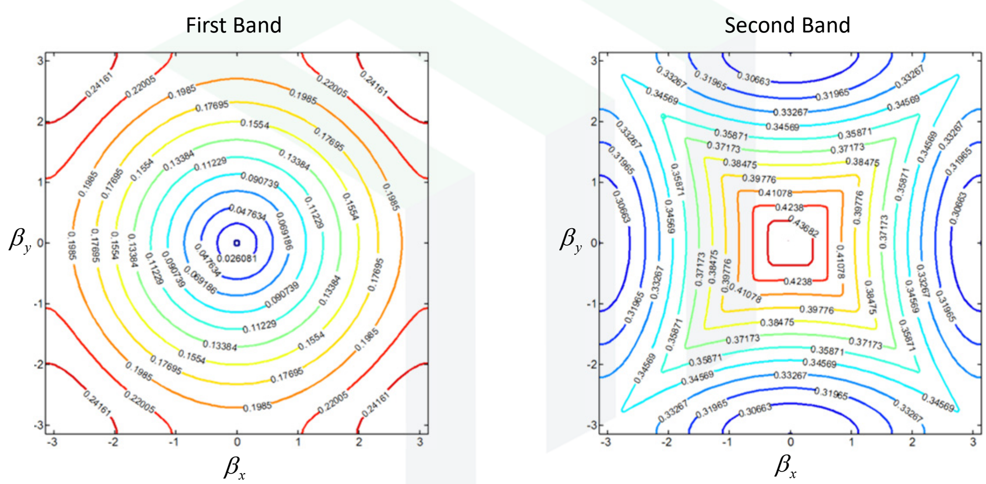
Standard View of Isofrequency Contours



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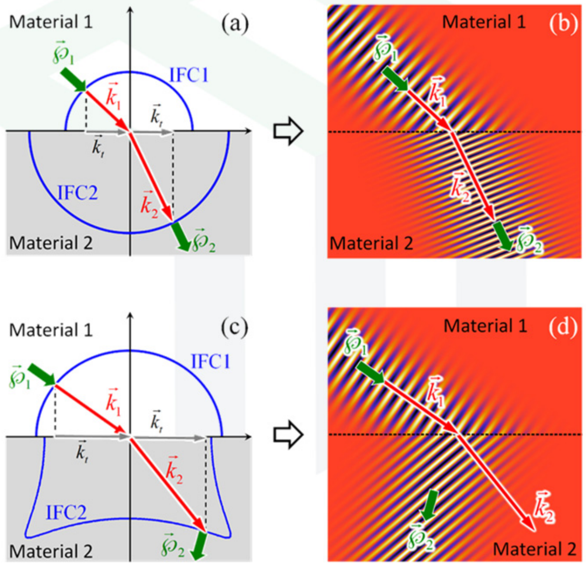
Negative Refraction Without Negative Refractive Index



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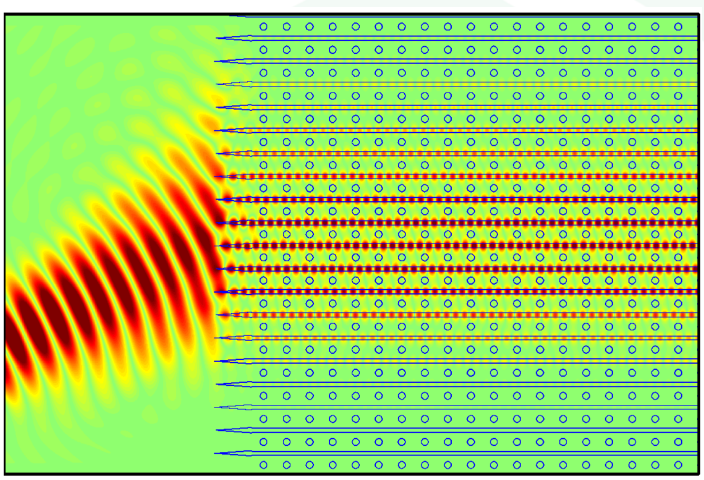
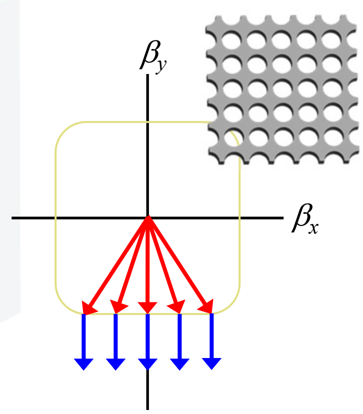
Self-Collimating Photonic Crystals





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