Electromagnetics:
Electromagnetic Field Theory

Scattering on a Transmission Line

1

Lecture Outline

- Scattering at an Impedance Discontinuity
- The full story about transmission line discontinuities
- Power on a Transmission Line
- Voltage Standing Wave Ratio (VSWR)

Slide 2

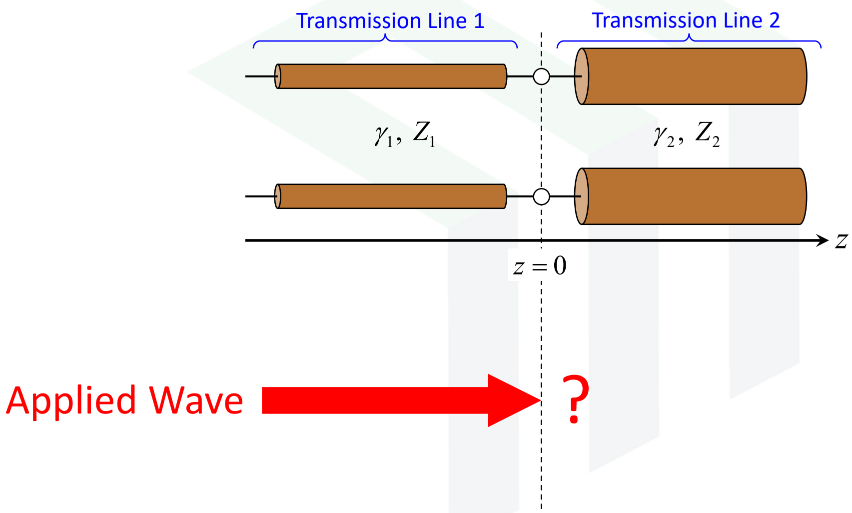
2

Scattering at an Impedance Discontinuity

Slide 3

3

Problem Setup



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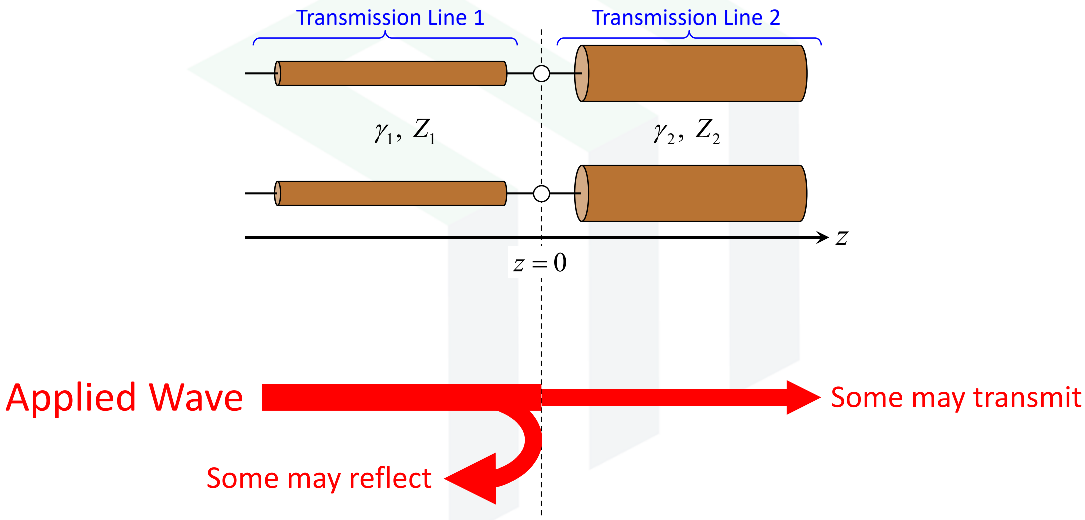
Slide 4

4

Problem Setup



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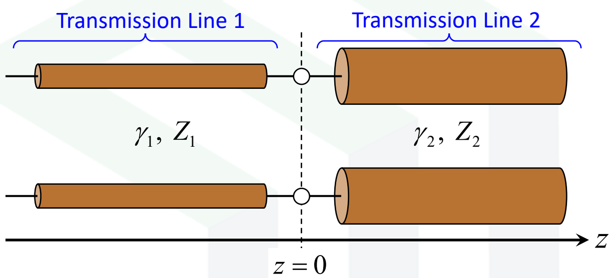
5

Incorporate Reflected Wave



$$V_1(z) = V_1^+ e^{-\gamma_1 z} + V_1^- e^{\gamma_1 z}$$

$$I_1(z) = \frac{V_1^+}{Z_1} e^{-\gamma_1 z} - \frac{V_1^-}{Z_1} e^{\gamma_1 z}$$

$$V_2(z) = V_2^+ e^{-\gamma_2 z}$$

$$I_2(z) = \frac{V_2^+}{Z_2} e^{-\gamma_2 z}$$

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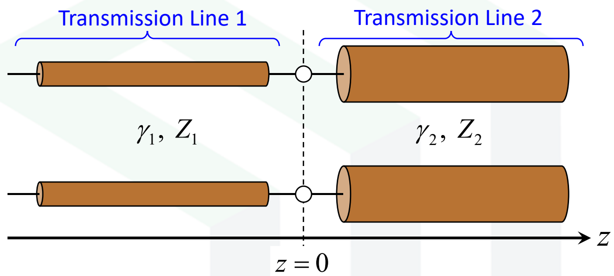
6

Enforce Boundary Conditions (1 of 2)



$$V_1(z) = V_2(z)$$

$$V_1^+ e^{-\gamma_1 z} + V_1^- e^{\gamma_1 z} = V_2^+ e^{-\gamma_2 z}$$

$$I_1(z) = I_2(z)$$

$$\frac{V_1^+}{Z_1} e^{-\gamma_1 z} - \frac{V_1^-}{Z_1} e^{\gamma_1 z} = \frac{V_2^+}{Z_2} e^{-\gamma_2 z}$$

Boundary conditions require the voltage and current on either side of the interface to be equal.

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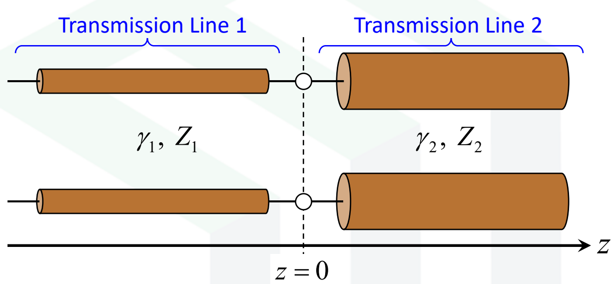
7

Enforce Boundary Conditions (2 of 2)



$$V_1(0) = V_2(0)$$

$$V_1^+ + V_1^- = V_2^+$$

$$I_1(0) = I_2(0)$$

$$\frac{V_1^+}{Z_1} - \frac{V_1^-}{Z_1} = \frac{V_2^+}{Z_2}$$

The interface occurs at $z = 0$ so the exponential terms all equal 1.

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Slide 8

8

Reflection Coefficient, Γ

Enforcing the boundary conditions at $z = 0$ gave

$$V_1^+ + V_1^- = V_2^+ \quad \text{Eq. (1)} \qquad \frac{V_1^+}{Z_1} - \frac{V_1^-}{Z_1} = \frac{V_2^+}{Z_2} \quad \text{Eq. (2)}$$

Substitute Eq. (1) into Eq. (2) to eliminate V_2^+.

$$\frac{V_1^+}{Z_1} - \frac{V_1^-}{Z_1} = \frac{V_1^+ + V_1^-}{Z_2}$$

Solve this new expression for V_1^- / V_1^+.

$$\frac{1}{Z_1}V_1^+ - \frac{1}{Z_1}V_1^- = \frac{1}{Z_2}V_1^+ + \frac{1}{Z_2}V_1^-$$

$$\frac{1}{Z_1}V_1^- + \frac{1}{Z_2}V_1^- = \frac{1}{Z_1}V_1^+ - \frac{1}{Z_2}V_1^+$$

$$\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)V_1^- = \left(\frac{1}{Z_1} - \frac{1}{Z_2}\right)V_1^+$$

$$(Z_2 + Z_1)V_1^- = (Z_2 - Z_1)V_1^+$$

$$\frac{V_1^-}{V_1^+} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\boxed{\Gamma = \frac{V_1^-}{V_1^+} = \frac{Z_2 - Z_1}{Z_2 + Z_1}}$$

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Slide 9

9

Revised Equations for $V(z)$ and $I(z)$

The total voltage and current in any section of line was written as

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

Using the concept of the reflection coefficient Γ, these equations can now be written as

$$V(z) = V_0^+ e^{-\gamma z} + \Gamma V_0^+ e^{\gamma z} = V_0^+ \left(e^{-\gamma z} + \Gamma e^{\gamma z}\right)$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{\Gamma V_0^+}{Z_0} e^{\gamma z} = \frac{V_0^+}{Z_0}\left(e^{-\gamma z} - \Gamma e^{\gamma z}\right)$$

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

Reflection coefficient at the load

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Slide 10

10

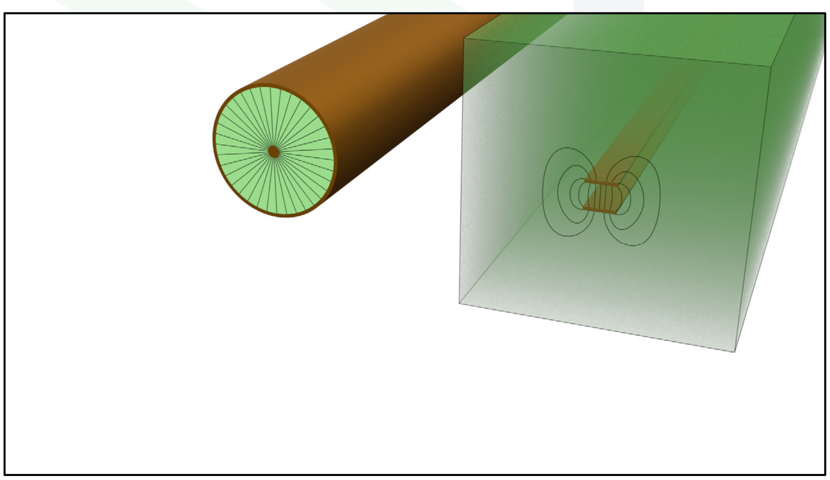
The Full Story About Transmission Line Discontinuities

Slide 11

11

Connecting Two Different Transmission Line Types

What happens when you connect a $50\ \Omega$ coaxial transmission line directly to a $50\ \Omega$ parallel plate transmission line?



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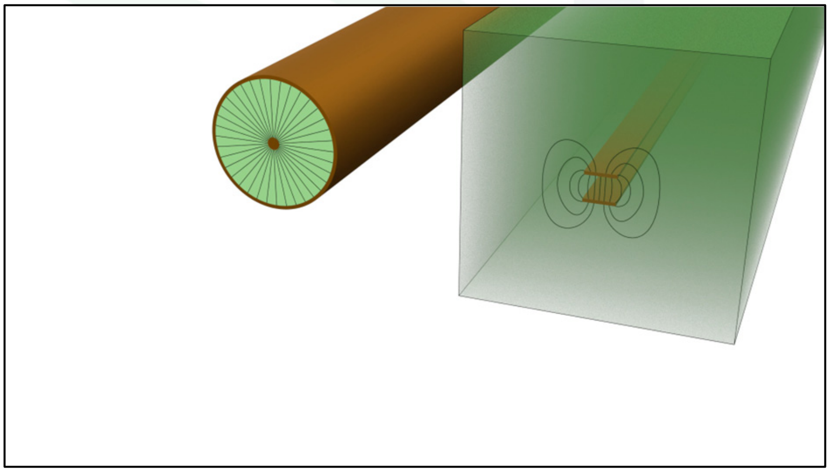
12

Connecting Two Different Transmission Line Types

What happens when you connect a 50 Ω coaxial transmission line directly to a 50 Ω parallel plate transmission line?

50 Ω Coaxial Transmission Line



50 Ω Parallel Plate Transmission Line

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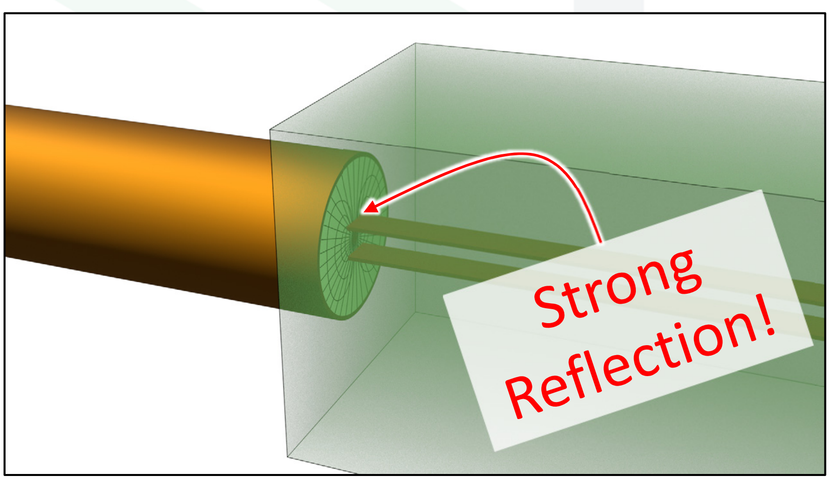
13

Connecting Two Different Transmission Line Types

Despite the impedance match, there will be a strong reflection because the field profile on either side of the interface is very different.

50 Ω Coaxial Transmission Line



50 Ω Parallel Plate Transmission Line

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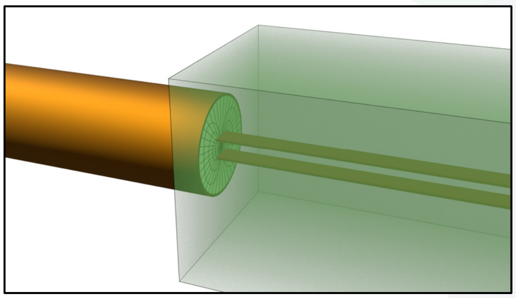
14

How to Resolve Reflections

General Case

The general case requires a field analysis to determine reflection. There may also be spurious radiation at a very discontinuous interface!



Special Case

This simple formula really only applies when the two transmission lines are very similar.

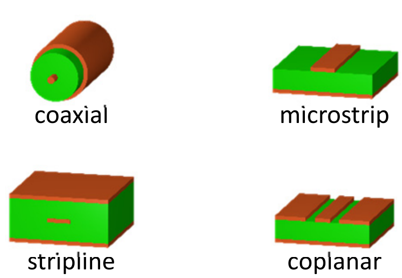
$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

15

Balanced Vs. Single-Ended Lines

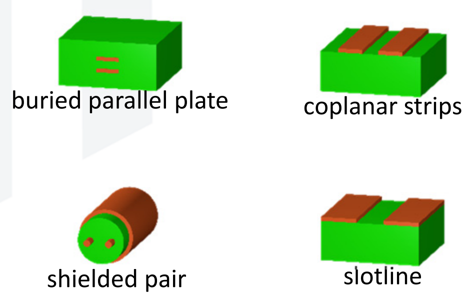
Single-Ended Lines

Single-ended (unbalanced) transmission lines use one signal wire and a ground that acts as the reference.



Balanced Lines

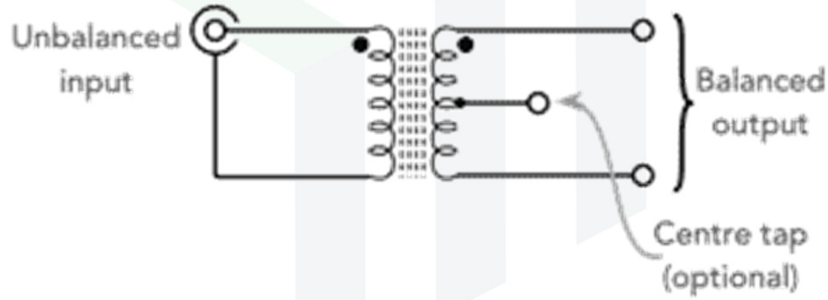
Balanced transmission lines have two signal wires. The two signal wires are 180° out of phase so they are exactly inverted.



16

Interfacing Balanced and Single-Ended Lines

Interfacing balanced and unbalanced lines is particularly difficult. The devices that connects an unbalanced line to a balanced line (or load) is called a *balun* (Balanced to Unbalanced)



https://www.electronics-notes.com/articles/antennas-propagation/balun-unun/what-is-rf-antenna-balun.php

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Slide 17

17

Power on a Transmission Line

Slide 18

18

Power Flowing Along Length of Line

The RMS power flowing at a distance z from the load is

$$P_{\text{avg}}(z)=\frac{1}{2}\operatorname{Re}\left[V(z)I^{*}(z)\right]$$

* is complex conjugate

This equation is valid for any line, even those with loss.

For lossless lines (not lossless loads), these equations can be written as

$$V(z)=V_0^{+}\left(e^{-j\beta z}+\Gamma_L e^{j\beta z}\right) \qquad I(z)=\frac{V_0^{+}}{Z_0}\left(e^{-j\beta z}-\Gamma_L e^{j\beta z}\right)$$

Substituting these equations into the expression for $P_{\text{avg}}(z)$ gives

$$P_{\text{avg}}(z)=\frac{1}{2}\operatorname{Re}\left[V_0^{+}\left(e^{-j\beta z}+\Gamma_L e^{j\beta z}\right)\cdot\frac{\left(V_0^{+}\right)^{*}}{Z_0}\left(e^{-j\beta z}-\Gamma_L e^{j\beta z}\right)^{*}\right]$$

$$\boxed{P_{\text{avg}}=\frac{\left|V_0^{+}\right|^{2}}{2Z_0}\left(1-\left|\Gamma_L\right|^{2}\right)}$$

Notice that the z dependence vanished. This is because power flows uniformly without decay in lossless lines.

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Slide 19

19

Voltage Standing Wave Ratio (VSWR)

Slide 20

20

Voltage Standing Wave Ratio (VSWR)

The voltage standing wave ratio (VSWR) is essentially the same concept as the standing wave ratio (SWR) discussed with waves. The only difference is that VSWR describes voltage and current instead of electromagnetic fields.

$$\text{VSWR} = \frac{\max|V(z)|}{\min|V(z)|} = \frac{\max|I(z)|}{\min|I(z)|}$$

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21

Derivation of VSWR (1 of 2)

Start with the expression for waves travelling in opposite directions on a transmission line. Assume the line is lossless (i.e. $\alpha = 0$).

$$V(z) = V_0^+\left(e^{-j\beta z} + \Gamma_\text{L} e^{j\beta z}\right) \qquad I(z) = \frac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma_\text{L} e^{j\beta z}\right)$$

The magnitude of the voltage signal $V(z)$ is

$$|V(z)| = |V_0^+|\left|e^{-j\beta z} + \Gamma_\text{L} e^{j\beta z}\right| = |V_0^+|\left|1 + \Gamma_\text{L} e^{j2\beta z}\right|$$

By inspection of this equation, the maximum and minimum values of this function are determined to be

$$V_\text{max} = \max|V(z)| = |V_0^+|\left(1 + |\Gamma_\text{L}|\right)$$
$$V_\text{min} = \min|V(z)| = |V_0^+|\left(1 - |\Gamma_\text{L}|\right)$$

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22

Derivation of VSWR (2 of 2)

The VSWR is therefore

$$\text{VSWR} = \frac{\max\left|V(z)\right|}{\min\left|V(z)\right|} = \frac{\left|V_0^+\right|\left(1+\left|\Gamma_\text{L}\right|\right)}{\left|V_0^+\right|\left(1-\left|\Gamma_\text{L}\right|\right)} \quad \rightarrow \quad \boxed{\text{VSWR} = \frac{1+\left|\Gamma_\text{L}\right|}{1-\left|\Gamma_\text{L}\right|}}$$

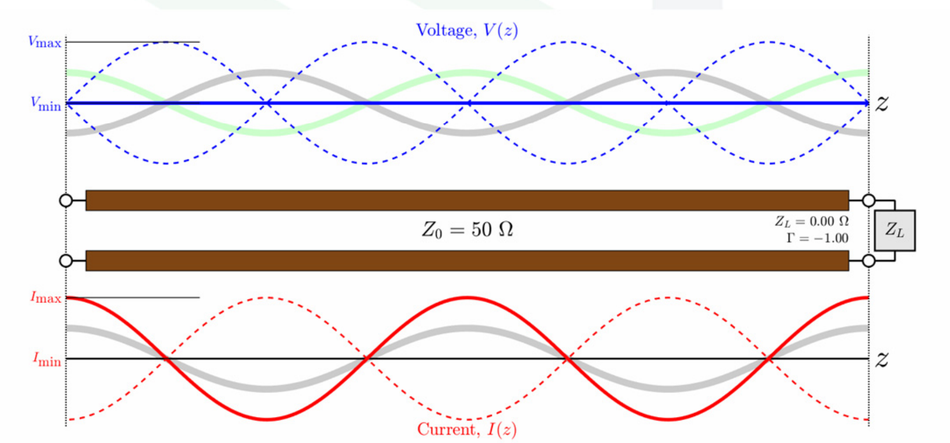
The VSWR is an easily measured quantity. In addition, the magnitude of the reflection coefficient at the load $|\Gamma_\text{L}|$ can be calculated directly from the VSWR.

$$\left|\Gamma_\text{L}\right| = \frac{\text{VSWR}-1}{\text{VSWR}+1}$$

Animation of VSWR (1 of 6)

Case 1: 50 Ω transmission line terminated with a short-circuit load.

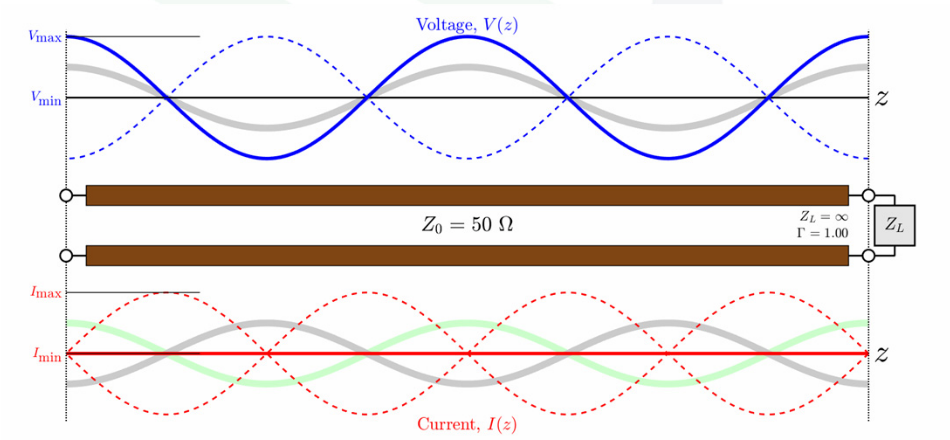
$$\Gamma_\text{L} = -1$$



Animation of VSWR (2 of 6)

Case 2: 50 Ω transmission line terminated with an open-circuit load.

$$\Gamma_L = +1$$



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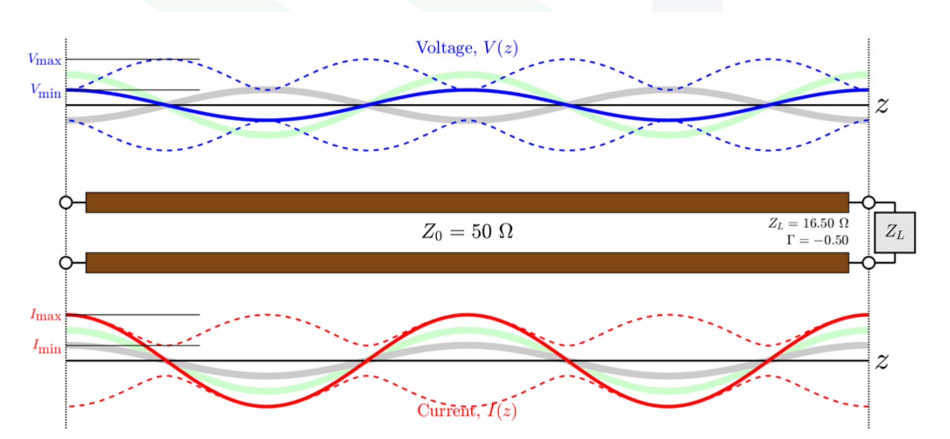
25

Animation of VSWR (3 of 6)

Case 3: 50 Ω transmission line terminated with a 16.5 Ω load.

$$Z_0 > Z_L \quad \rightarrow \quad \Gamma_L = -0.5$$



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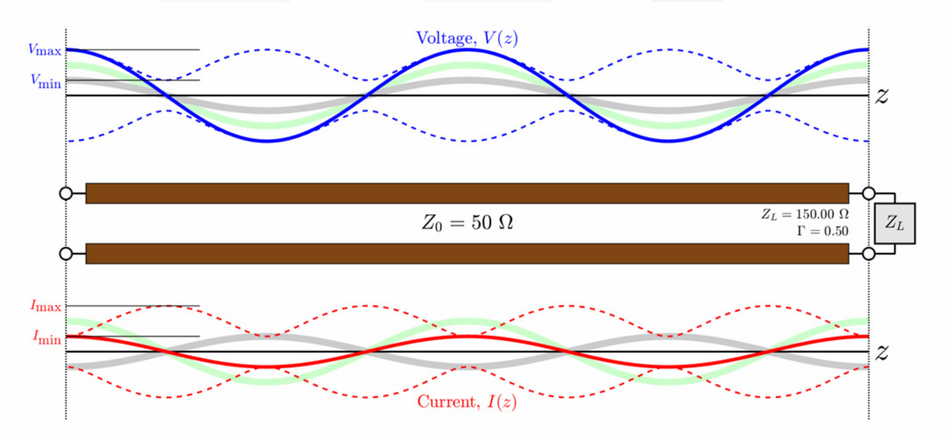
26

Animation of VSWR (4 of 6)

Case 4: 50 Ω transmission line terminated with a 150 Ω load.

$$Z_0 < Z_L \quad \rightarrow \quad \Gamma_L = +0.5$$



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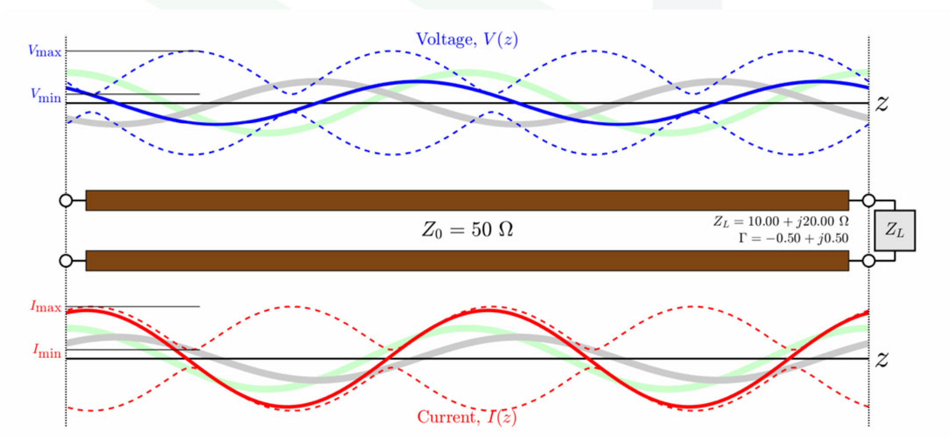
27

Animation of VSWR (5 of 6)

Case 5: 50 Ω transmission line terminated with an RL load.



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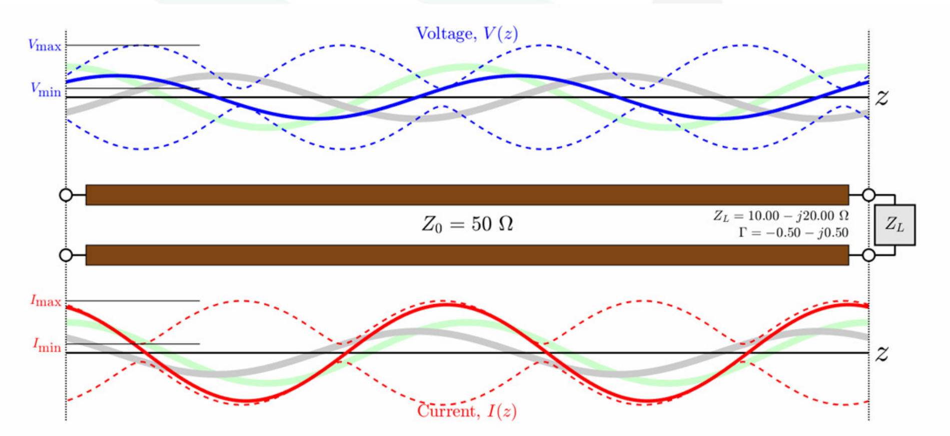
28

Animation of VSWR (6 of 6)

Case 6: 50 Ω transmission line terminated with an RC load.



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29