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Electromagnetics:
Electromagnetic Field Theory

Statics & Dynamics

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Outline

- Map of Electromagnetics 101
- Conditions to apply static approximation
- Consequences of static approximation

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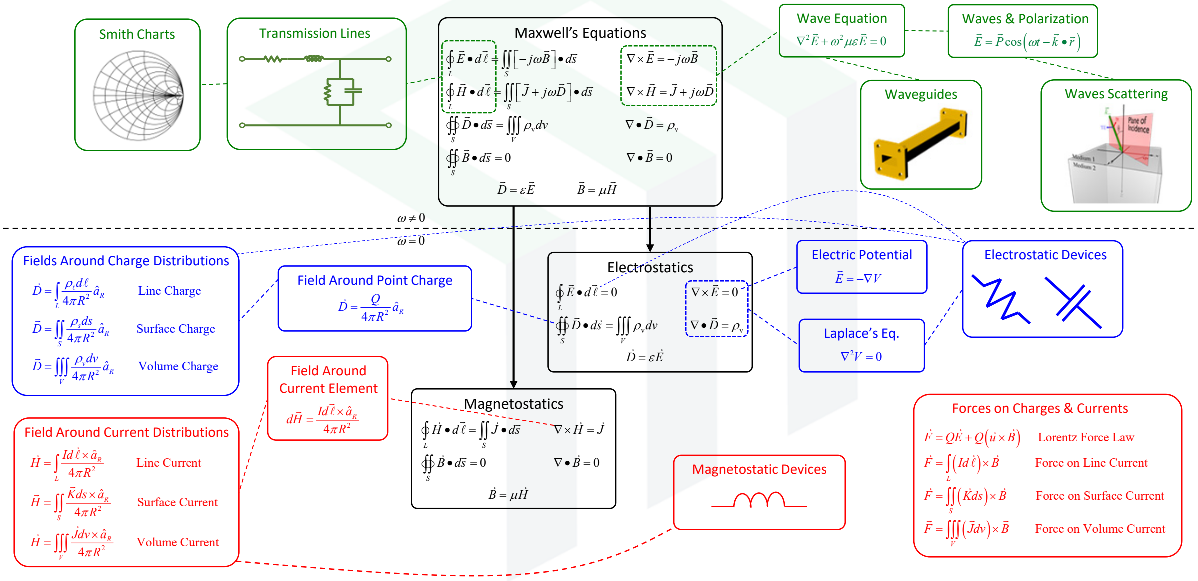
Map of Electromagnetics



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Conditions to Apply Static Approximation

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Conditions for Statics

Situation #1 – At $\omega = 0$

At DC (i.e. $\omega = 0$), nothing changes with time.

$$\omega \to 0$$

$$\frac{\partial}{\partial t} \to 0$$

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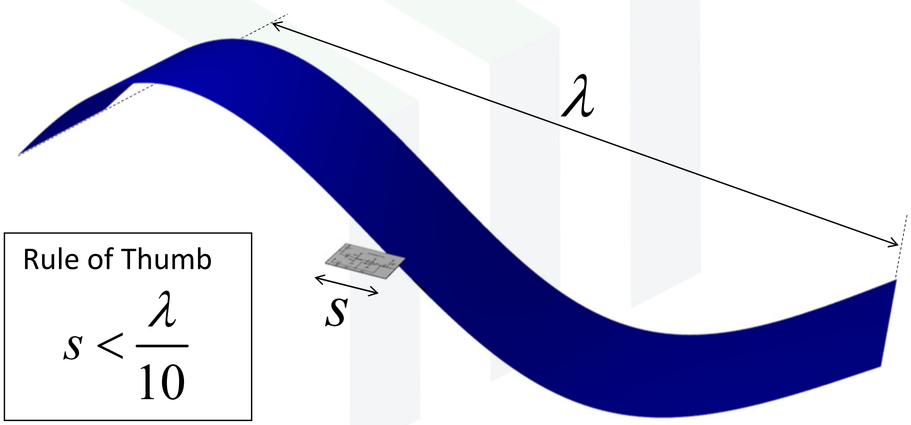
Conditions for Statics

Situation #2 – When size s of a problem is much less than the wavelength λ.



Rule of Thumb

$$s < \frac{\lambda}{10}$$

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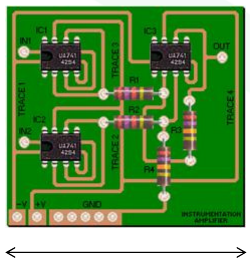
Example #1

Estimate the maximum frequency that this circuit can be analyzed using standard circuit theory?



4.0 cm

Note: This does not mean that circuits cannot be designed at higher frequencies. It is just that these designs cannot use ordinary circuit theory.

Solution

The maximum dimension s of the circuit was given to be

$$s = 4.0 \text{ cm}$$

The minimum wavelength λ_{min} is

$$\lambda_{\text{min}} \approx 10s = 10(4.0 \text{ cm}) = 40 \text{ cm}$$

It follows that the maximum frequency f_{max} is

$$f_{\text{max}} = \frac{c_0}{\lambda_{\text{min}}} = \frac{3.0\times10^8 \text{ m/s}}{0.40 \text{ m}} = \boxed{750 \text{ MHz}}$$

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Consequences of Static Approximation

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Maxwell's Equations for Static Fields

	Integral Form	Differential Form
Time-Domain	**Most general form** $\oiint_S \vec{D}\bullet d\vec{s} = \iiint_V \rho_v dv$ $\oiint_S \vec{B}\bullet d\vec{s} = 0$ $\oint_L \vec{E}\bullet d\vec{\ell} = -\iint_S \left[\cancel{\frac{\partial \vec{B}}{\partial t}}\right]\bullet d\vec{s}$ $\oint_L \vec{H}\bullet d\vec{\ell} = \iint_S \left[\vec{J} + \cancel{\frac{\partial \vec{D}}{\partial t}}\right]\bullet d\vec{s}$	$\nabla\bullet\vec{D} = \rho_v$ $\nabla\bullet\vec{B} = 0$ $\nabla\times\vec{E} = -\cancel{\frac{\partial \vec{B}}{\partial t}}$ $\nabla\times\vec{H} = \vec{J} + \cancel{\frac{\partial \vec{D}}{\partial t}}$
Frequency-Domain	$\oiint_S \vec{D}\bullet d\vec{s} = \iiint_V \rho_v dv$ $\oiint_S \vec{B}\bullet d\vec{s} = 0$ $\oint_L \vec{E}\bullet d\vec{\ell} = -\iint_S \left[\cancel{j\omega\vec{B}}\right]\bullet d\vec{s}$ $\oint_L \vec{H}\bullet d\vec{\ell} = \iint_S \left[\vec{J} + \cancel{j\omega\vec{D}}\right]\bullet d\vec{s}$	**Most common form** $\nabla\bullet\vec{D} = \rho_v$ $\nabla\bullet\vec{B} = 0$ $\nabla\times\vec{E} = -\cancel{j\omega\vec{B}}$ $\nabla\times\vec{H} = \vec{J} + \cancel{j\omega\vec{D}}$

Constitutive Relations: $\vec{D} = \varepsilon\vec{E}$ $\quad \vec{B} = \mu\vec{H}$

Lorentz Force Law: $\vec{F} = Q\vec{E} + Q\left(\vec{u}\times\vec{B}\right)$

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Maxwell's Equations for Static Fields

	Integral Form	Differential Form
Time-Domain	**Most general form** $\oiint_S \vec{D}\bullet d\vec{s} = \iiint_V \rho_v dv$ $\oiint_S \vec{B}\bullet d\vec{s} = 0$ $\oint_L \vec{E}\bullet d\vec{\ell} = 0$ $\oint_L \vec{H}\bullet d\vec{\ell} = \iint_S \vec{J}\bullet d\vec{s}$	$\nabla\bullet\vec{D} = \rho_v$ $\nabla\bullet\vec{B} = 0$ $\nabla\times\vec{E} = 0$ $\nabla\times\vec{H} = \vec{J}$
Frequency-Domain	$\oiint_S \vec{D}\bullet d\vec{s} = \iiint_V \rho_v dv$ $\oiint_S \vec{B}\bullet d\vec{s} = 0$ $\oint_L \vec{E}\bullet d\vec{\ell} = 0$ $\oint_L \vec{H}\bullet d\vec{\ell} = \iint_S \vec{J}\bullet d\vec{s}$	**Most common form** $\nabla\bullet\vec{D} = \rho_v$ $\nabla\bullet\vec{B} = 0$ $\nabla\times\vec{E} = 0$ $\nabla\times\vec{H} = \vec{J}$

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Electric and Magnetic Fields are Decoupled

After observing the results on the previous slide, Maxwell's equations have decoupled into two independent sets of equations. One set describes *electrostatics* and the other describes *magnetostatics*.

Electrostatics

$$\oiint_S \vec{D} \bullet d\vec{s} = \iiint_V \rho_v dv \qquad \nabla \bullet \vec{D} = \rho_v$$

$$\oint_L \vec{E} \bullet d\vec{\ell} = 0 \qquad \nabla \times \vec{E} = 0$$

$$\vec{D} = \varepsilon \vec{E} \qquad \vec{F} = Q\vec{E}$$

- Charges must be present to get a nontrivial solution.
- $\nabla \times \vec{E} = 0$ says $\vec{E}$ is irrotational and forms mostly straight lines.
- The electric fields are not affected by permeability.

Magnetostatics

$$\oiint_S \vec{B} \bullet d\vec{s} = 0 \qquad \nabla \bullet \vec{B} = 0$$

$$\oint_L \vec{H} \bullet d\vec{\ell} = \iint_S \vec{J} \bullet d\vec{s} \qquad \nabla \times \vec{H} = \vec{J}$$

$$\vec{B} = \mu \vec{H} \qquad \vec{F} = Q\left(\vec{u} \times \vec{B}\right)$$

- Current must be present to get a nontrivial solution.
- $\nabla \cdot \vec{B} = 0$ says $\vec{B}$ is solenoidal and forms loops.
- The magnetic fields are not affected by permittivity.

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