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Electromagnetics:
Electromagnetic Field Theory

Statics & Dynamics

1

Outline

- Conditions to apply static approximation
- Consequences of static approximation

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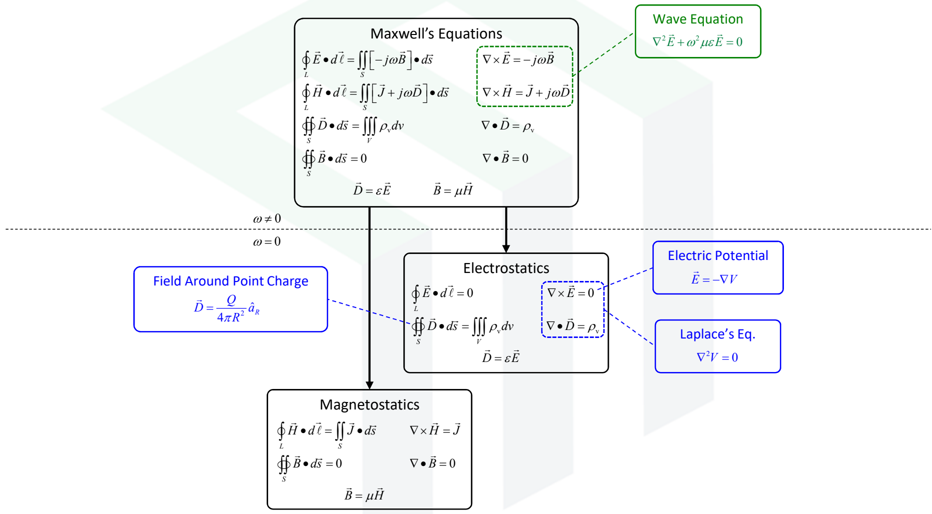
Map of Electromagnetics



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Conditions to Apply Static Approximation

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Conditions for Statics

Situation #1 – At $\omega = 0$

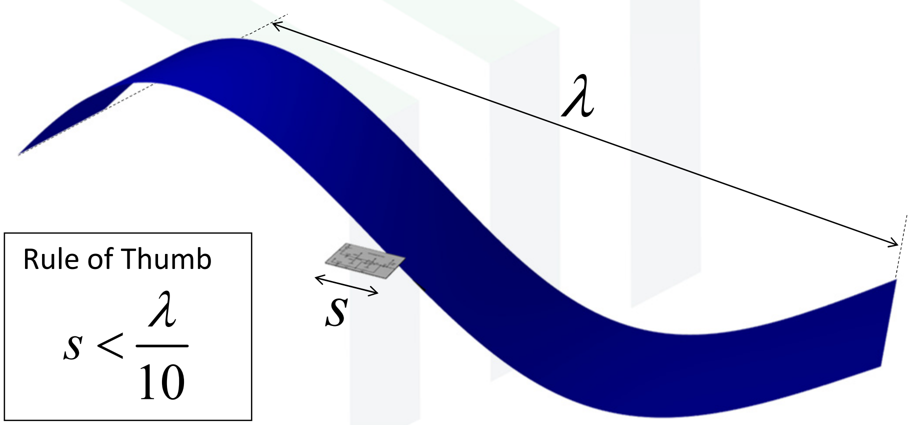
At DC (i.e. $\omega = 0$), nothing changes with time.

$$\omega \to 0$$

$$\frac{\partial}{\partial t} \to 0$$

Conditions for Statics

Situation #2 – When size s of problem is much less than a wavelength λ.

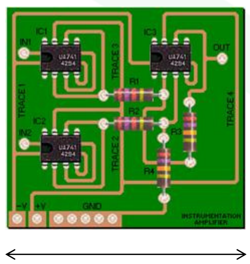


Rule of Thumb

$$s < \frac{\lambda}{10}$$

Example #1

Estimate the maximum frequency that this circuit can be analyzed using standard circuit theory?



4.0 cm

Note: This does not mean that circuits cannot be designed at higher frequencies. It is just that these designs cannot use ordinary circuit theory.

Solution

The maximum dimension s of the circuit was given to be

$$s = 4.0 \text{ cm}$$

The minimum wavelength $\lambda_{\min}$ is

$$\lambda_{\min} \approx 10s = 10(4.0 \text{ cm}) = 40 \text{ cm}$$

It follows that the maximum frequency $f_{\max}$ is

$$f_{\max} = \frac{c_0}{\lambda_{\min}} = \frac{3.0\times10^{8} \text{ m/s}}{0.40 \text{ m}} = \boxed{750 \text{ MHz}}$$

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Consequences of Static Approximation

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Maxwell's Equations for Static Fields

	Integral Form	Differential Form
Time-Domain	**Most general form** $\oiint_S \vec{D} \bullet d\vec{s} = \iiint_V \rho_v dv$ $\oiint_S \vec{B} \bullet d\vec{s} = 0$ $\oint_L \vec{E} \bullet d\vec{\ell} = -\iint_S \left[\cancel{\frac{\partial \vec{B}}{\partial t}}\right] \bullet d\vec{s}$ $\oint_L \vec{H} \bullet d\vec{\ell} = \iint_S \left[\vec{J} + \cancel{\frac{\partial \vec{D}}{\partial t}}\right] \bullet d\vec{s}$	$\nabla \bullet \vec{D} = \rho_v$ $\nabla \bullet \vec{B} = 0$ $\nabla \times \vec{E} = -\cancel{\frac{\partial \vec{B}}{\partial t}}$ $\nabla \times \vec{H} = \vec{J} + \cancel{\frac{\partial \vec{D}}{\partial t}}$
Frequency-Domain	$\oiint_S \vec{D} \bullet d\vec{s} = \iiint_V \rho_v dv$ $\oiint_S \vec{B} \bullet d\vec{s} = 0$ $\oint_L \vec{E} \bullet d\vec{\ell} = -\iint_S \left[\cancel{j\omega \vec{B}}\right] \bullet d\vec{s}$ $\oint_L \vec{H} \bullet d\vec{\ell} = \iint_S \left[\vec{J} + \cancel{j\omega \vec{D}}\right] \bullet d\vec{s}$	**Most common form** $\nabla \bullet \vec{D} = \rho_v$ $\nabla \bullet \vec{B} = 0$ $\nabla \times \vec{E} = -\cancel{j\omega \vec{B}}$ $\nabla \times \vec{H} = \vec{J} + \cancel{j\omega \vec{D}}$

Constitutive Relations: $\vec{D} = \varepsilon \vec{E}$ $\vec{B} = \mu \vec{H}$

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Maxwell's Equations for Static Fields

	Integral Form	Differential Form
Time-Domain	**Most general form** $\oiint_S \vec{D} \bullet d\vec{s} = \iiint_V \rho_v dv$ $\oiint_S \vec{B} \bullet d\vec{s} = 0$ $\oint_L \vec{E} \bullet d\vec{\ell} = 0$ $\oint_L \vec{H} \bullet d\vec{\ell} = \iint_S \vec{J} \bullet d\vec{s}$	$\nabla \bullet \vec{D} = \rho_v$ $\nabla \bullet \vec{B} = 0$ $\nabla \times \vec{E} = 0$ $\nabla \times \vec{H} = \vec{J}$
Frequency-Domain	$\oiint_S \vec{D} \bullet d\vec{s} = \iiint_V \rho_v dv$ $\oiint_S \vec{B} \bullet d\vec{s} = 0$ $\oint_L \vec{E} \bullet d\vec{\ell} = 0$ $\oint_L \vec{H} \bullet d\vec{\ell} = \iint_S \vec{J} \bullet d\vec{s}$	**Most common form** $\nabla \bullet \vec{D} = \rho_v$ $\nabla \bullet \vec{B} = 0$ $\nabla \times \vec{E} = 0$ $\nabla \times \vec{H} = \vec{J}$

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Electric and Magnetic Fields are Decoupled

After observing the results on the previous slide, we see that Maxwell's equations have decoupled into two independent sets of equations. One set describes electrostatics and the other describes magnetostatics.

Electrostatics

$$\oint\!\!\oint_S \vec{D} \bullet d\vec{s} = \iiint_V \rho_v dv \qquad \nabla \bullet \vec{D} = \rho_v$$

$$\oint_L \vec{E} \bullet d\vec{\ell} = 0 \qquad \nabla \times \vec{E} = 0$$

$$\vec{D} = \varepsilon \vec{E}$$

- Charges must be present to get a nontrivial solution.
- $\nabla \times \vec{E} = 0$ says $\vec{E}$ is irrotational and forms mostly straight lines.
- The electric fields are not affected by permeability.

Magnetostatics

$$\oint\!\!\oint_S \vec{B} \bullet d\vec{s} = 0 \qquad \nabla \bullet \vec{B} = 0$$

$$\oint_L \vec{H} \bullet d\vec{\ell} = \iint_S \vec{J} \bullet d\vec{s} \qquad \nabla \times \vec{H} = \vec{J}$$

$$\vec{B} = \mu \vec{H}$$

- Current must be present to get a nontrivial solution.
- $\nabla \cdot \vec{B} = 0$ says $\vec{B}$ is solenoidal and forms loops.
- The magnetic fields are not affected by permittivity.

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