



Electromagnetics:
Electromagnetic Field Theory

Transmission Line Examples

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Lecture Outline

- RG-59 Coaxial Cable
- Microstrip Design

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Example:

Properties of RG-59 Coax

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The Lossless Circular Coax

Fundamental Parameters (derived in EE 3321)

$$C = \frac{2\pi\epsilon}{\ln(b/a)} \quad (\text{F/m})$$

$$L = \frac{\mu}{2\pi} \left[\frac{1}{4} + \ln\left(\frac{b}{a}\right) \right] \quad (\text{H/m})$$

Attenuation Coefficient, α

$$\alpha = 0$$

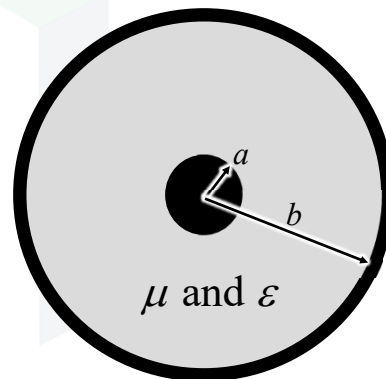
Phase Constant, β

$$\beta = \omega\sqrt{\mu\epsilon}$$

Characteristic Impedance, Z_0

$$Z_0 = R_0 + jX_0 = \frac{\eta}{2\pi} \ln\left(\frac{b}{a}\right) \quad a \ll b$$

$$R_0 = \frac{\eta}{2\pi} \ln\left(\frac{b}{a}\right) \quad X_0 = 0$$



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Typical $RLGC$ for RG-59 Coax at 2 GHz

The typical RG-59 coaxial cable operating at 2.0 GHz has the following $RLGC$ parameters:

$$R = 36 \text{ m}\Omega/\text{m}$$

$$L = 430 \text{ nH/m}$$

$$G = 10 \text{ }\mu\text{S/m}$$

$$C = 69 \text{ pF/m}$$

Calculate the transmission line parameters γ , α , β , and Z_0 .

Classify the line as lossless, weakly absorbing, distortionless, etc.

Solution (1 of 3)

The equations mostly utilize the angular frequency ω instead of the ordinary frequency f .

$$\omega = 2\pi f = 2\pi(2.0 \times 10^9 \text{ s}^{-1}) = \underline{12.5664 \times 10^9 \text{ rad/s}}$$

The characteristic impedance Z_0 is

$$\begin{aligned} Z_0 &= \sqrt{\frac{R + j\omega L}{G + j\omega C}} \\ &= \sqrt{\frac{(36 \text{ m}\Omega/\text{m}) + j(12.5664 \times 10^9 \text{ rad/s})(430 \text{ nH/m})}{(10 \text{ }\mu\text{S/m}) + j(12.5664 \times 10^9 \text{ rad/s})(69 \text{ pF/m})}} \\ &= \boxed{78.94 + j1.92 \times 10^{-4} \text{ }\Omega} \end{aligned}$$

Note the imaginary part of Z_0 is very small indicating that the line is very low loss.

Solution (2 of 3)

The complex propagation constant γ is

$$\begin{aligned}\gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\ &= \sqrt{\left[(36 \text{ m}\Omega/\text{m}) + j(12.5664 \times 10^9 \text{ rad/s})(430 \text{ nH/m}) \right]} \\ &= \sqrt{\left[(10 \text{ }\mu\text{S/m}) + j(12.5664 \times 10^9 \text{ rad/s})(69 \text{ pF/m}) \right]} \\ &= \boxed{6.23 \times 10^{-4} + j68.45 \text{ m}^{-1}}\end{aligned}$$

From this result, the attenuation coefficient α and phase constant β are read off as

$$\gamma = \alpha + j\beta = 6.23 \times 10^{-4} + j68.45 \text{ m}^{-1}$$

$$\alpha = 6.23 \times 10^{-4} \text{ Np/m}$$

Np is Nepers

$$\beta = 68.45 \text{ rad/m}$$

rad is radians

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Solution (3 of 3)

Is the line lossless? \rightarrow NO

No because $R \neq 0$ and $G \neq 0$.

Also, the attenuation coefficient is not zero $\alpha \neq 0$.

Is the line weakly absorbing? \rightarrow YES

$$R \stackrel{?}{\leq} \omega L$$

$$(36 \text{ m}\Omega/\text{m}) \stackrel{?}{\leq} (12.5664 \times 10^9 \text{ rad/s})(430 \text{ nH/m})$$

$$0.036 \stackrel{?}{\leq} 5403.5$$

Yes

$$G \stackrel{?}{\leq} \omega C$$

$$(10 \text{ }\mu\text{S/m}) \stackrel{?}{\leq} (12.5664 \times 10^9 \text{ rad/s})(69 \text{ pF/m})$$

$$10 \times 10^{-6} \stackrel{?}{\leq} 0.8671$$

Yes

Is the line distortionless? \rightarrow NO, but close

$$RC \stackrel{?}{=} LG$$

$$(36 \text{ m}\Omega/\text{m})(69 \text{ pF/m}) \stackrel{?}{\leq} (430 \text{ nH/m})(10 \text{ }\mu\text{S/m})$$

$$2.48 \times 10^{-12} \stackrel{?}{\leq} 4.30 \times 10^{-12}$$

No, but close

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Example:

Microstrip Design

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The Lossless Microstrip

Attenuation Coefficient, α

$$\alpha = 0$$

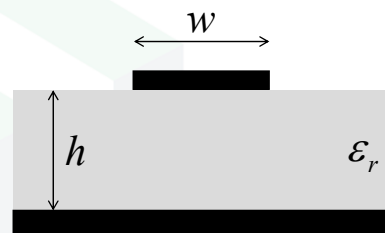
Phase Constant, β

$$\epsilon_{r,\text{eff}} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2\sqrt{1 + 12h/w}}$$

$$\beta \approx k_0 \sqrt{\epsilon_{r,\text{eff}}}$$

Characteristic Impedance, Z_0

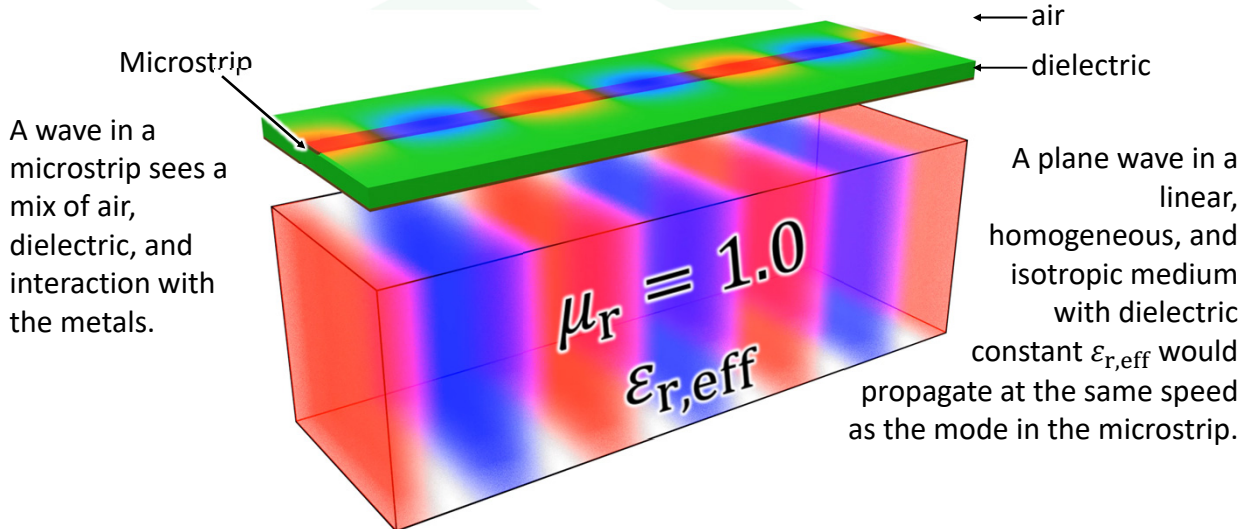
$$Z_0 = R_0 + jX_0 \cong \begin{cases} \frac{60}{\sqrt{\epsilon_{r,\text{eff}}}} \ln\left(\frac{8h}{w} + \frac{w}{4h}\right) & w/h \leq 1 \text{ thin lines} \\ \frac{120\pi}{\sqrt{\epsilon_{r,\text{eff}}}} \left[\frac{1}{w/h + 1.393 + 0.667 \ln(w/h + 1.444)} \right] & w/h > 1 \text{ wide lines} \end{cases}$$



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Meaning of Effective Dielectric Constant $\epsilon_{r,eff}$



A wave in a microstrip sees a mix of air, dielectric, and interaction with the metals.

A plane wave in a linear, homogeneous, and isotropic medium with dielectric constant $\epsilon_{r,eff}$ would propagate at the same speed as the mode in the microstrip.

$\mu_r = 1.0$
 $\epsilon_{r,eff}$

$\epsilon_{r,eff}$

$\epsilon_{r,eff}$

EMPossible

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Problem Description

Typically, the manufacturing process fixes the value of the dielectric constant ϵ_r . This means the impedance of microstrips is usually controlled solely through the ratio w/h .

For this example, design a 50Ω microstrip transmission line in FR-4, which has a dielectric constant of 4.5, to operate at 2.4 GHz.

$$\frac{w}{h} = ?$$

EMPossible

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Design Equations

To solve this problem, it is best to start by deriving some design equations. To do this, the microstrip equations are solved for w/h . This gives

$$A = \frac{Z_0}{60} \sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1} \left(0.23 + \frac{0.11}{\epsilon_r} \right)$$

$$B = \frac{60\pi^2}{Z_0 \sqrt{\epsilon_r}}$$

$$\frac{w}{h} = \begin{cases} \frac{8e^A}{e^{2A} - 2} & w/h \leq 2 \text{ thin lines} \\ \frac{2}{\pi} \left[B - 1 - \ln(2B - 1) + \frac{\epsilon_r - 1}{2\epsilon_r} \left[\ln(B - 1) + 0.39 - \frac{0.61}{\epsilon_r} \right] \right] & w/h > 2 \text{ wide lines} \end{cases}$$

Design Solution (1 of 2)

Applying the design equations gives

$$A = 1.5438$$

$$B = 5.5831$$

$$\frac{w}{h} = \begin{cases} 1.8799 & w/h \leq 2 \text{ thin lines} \\ 1.8812 & w/h > 2 \text{ wide lines} \end{cases}$$

Since the above numbers for w/h are essentially the same, it can be concluded that

$$\frac{w}{h} \cong 1.88$$

Design Solution (2 of 2)

It was learned from the manufacturing engineer that a convenient choice for substrate thickness h is 0.5 mm. From this, to get 50Ω the width w of the microstrip should be

$$w \cong 1.88h = 1.88(0.5 \text{ mm}) = \boxed{0.94 \text{ mm}}$$

The phase constant β for this line will be

$$\epsilon_{r,\text{eff}} = 3.3941$$

$$k_0 = \frac{\omega}{c_0} = \frac{2\pi f}{c_0} = \frac{2\pi(2.4 \times 10^9 \text{ s}^{-1})}{299792458 \text{ m/s}} = 50.3 \text{ m}^{-1}$$

$$\beta \cong k_0 \sqrt{\epsilon_{r,\text{eff}}} = (50.3 \text{ m}^{-1}) \sqrt{3.3941} = \boxed{92.67 \text{ m}^{-1}}$$