Electromagnetics:

Electromagnetic Field Theory

Transmission Line Examples

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Lecture Outline

- RG-59 Coaxial Cable
- Microstrip Design

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Example:

Properties of RG-59 Coax

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The Lossless Circular Coax

Fundamental Parameters (derived in EE 3321)

$$C = \frac{2\pi\varepsilon}{\ln(b/a)} \quad (\text{F/m})$$

$$L = \frac{\mu}{2\pi}\left[\frac{1}{4} + \ln\left(\frac{b}{a}\right)\right] \quad (\text{H/m})$$

Attenuation Coefficient, α

$$\alpha = 0$$

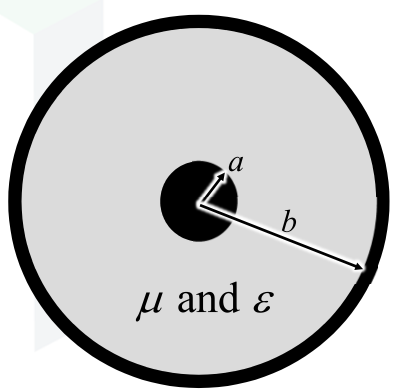
Phase Constant, β

$$\beta = \omega\sqrt{\mu\varepsilon}$$

Characteristic Impedance, Z_0

$$Z_0 = R_0 + jX_0 = \frac{\eta}{2\pi}\ln\left(\frac{b}{a}\right) \qquad a \ll b$$

$$R_0 = \frac{\eta}{2\pi}\ln\left(\frac{b}{a}\right) \qquad X_0 = 0$$



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Typical RLGC for RG-59 Coax at 2 GHz

The typical RG-59 coaxial cable operating at 2.0 GHz has the following RLGC parameters:

$$R = 36\ \text{m}\Omega/\text{m}$$
$$L = 430\ \text{nH/m}$$
$$G = 10\ \mu\mho/\text{m}$$
$$C = 69\ \text{pF/m}$$

Calculate the transmission line parameters γ, α, β, and Z_0.

Classify the line as lossless, weakly absorbing, distortionless, etc.

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Solution (1 of 3)

Our equations mostly utilize the angular frequency ω instead of the ordinary frequency f.

$$\omega = 2\pi f = 2\pi\left(2.0\times10^{9}\ \text{s}^{-1}\right) = \underline{12.5664\times10^{9}\ \text{rad/s}}$$

The characteristic impedance Z_0 is

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$
$$= \sqrt{\frac{\left(36\ \text{m}\Omega/\text{m}\right) + j\left(12.5664\times10^{9}\ \text{rad/s}\right)\left(430\ \text{nH/m}\right)}{\left(10\ \mu\mho/\text{m}\right) + j\left(12.5664\times10^{9}\ \text{rad/s}\right)\left(69\ \text{pF/m}\right)}}$$
$$= \boxed{78.94 + j1.92\times10^{-4}\ \Omega}$$

Note the imaginary part of Z_0 is very small indicating that our line is very low loss.

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Solution (2 of 3)

The complex propagation constant γ is

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{\begin{array}{l}\left[(36\ \text{m}\Omega/\text{m}) + j(12.5664\times10^{9}\ \text{rad/s})(430\ \text{nH/m})\right] \\ \cdot\left[(10\ \mu\mho/\text{m}) + j(12.5664\times10^{9}\ \text{rad/s})(69\ \text{pF/m})\right]\end{array}}$$

$$= \boxed{6.23\times10^{-4} + j68.45\ \text{m}^{-1}}$$

From this result, we read off α and β.

$$\gamma = \alpha + j\beta = 6.23\times10^{-4} + j68.45\ \text{m}^{-1}$$

$$\boxed{\alpha = 6.23\times10^{-4}\ \ \text{Np/m}}$$ Np is Nepers

$$\boxed{\beta = 68.45\ \ \text{rad/m}}$$ rad is radians

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Solution (3 of 3)

Is the line lossless? → NO

No because $R \neq 0$ and $G \neq 0$.
Also, we can determine this because $\alpha \neq 0$.

Is the line weakly absorbing? → YES

$$R \overset{?}{\leq} \omega L$$

$$(36\ \text{m}\Omega/\text{m}) \overset{?}{\leq} (12.5664\times10^{9}\ \text{rad/s})(430\ \text{nH/m})$$

$$0.036 \overset{?}{\leq} 5403.5$$

Yes

$$G \overset{?}{\ll} \omega C$$

$$(10\ \mu\mho/\text{m}) \overset{?}{\leq} (12.5664\times10^{9}\ \text{rad/s})(69\ \text{pF/m})$$

$$10\times10^{-6} \overset{?}{\leq} 0.8671$$

Yes

Is the line distortionless? → NO, but close

$$RC \overset{?}{=} LG$$

$$(36\ \text{m}\Omega/\text{m})(69\ \text{pF/m}) \overset{?}{\leq} (430\ \text{nH/m})(10\ \mu\mho/\text{m})$$

$$2.48\times10^{-12} \overset{?}{\leq} 4.30\times10^{-12}$$

No, but close

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Example:

Microstrip Design

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The Lossless Microstrip

Attenuation Coefficient, α

$$\alpha = 0$$

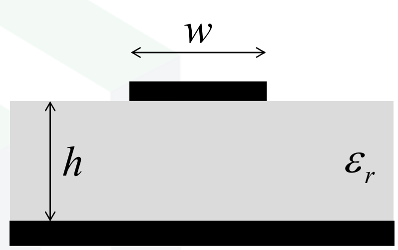
Phase Constant, β

$$\varepsilon_{\text{r,eff}} = \frac{\varepsilon_{\text{r}}+1}{2} + \frac{\varepsilon_{\text{r}}-1}{2\sqrt{1+12\,h/w}}$$

$$\beta \approx k_0\sqrt{\varepsilon_{\text{r,eff}}}$$

Characteristic Impedance, Z_0

$$Z_0 = R_0 + jX_0 \cong \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{\text{eff}}}}\ln\left(\dfrac{8h}{w}+\dfrac{w}{4h}\right) & w/h \leq 1 \quad \text{thin lines} \\ \dfrac{1}{\sqrt{\varepsilon_{\text{r,eff}}}}\dfrac{120\pi}{\left[w/h+1.393+0.667\ln\left(w/h+1.444\right)\right]} & w/h > 1 \quad \text{wide lines} \end{cases}$$



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Problem Description

Typically, the manufacturing process fixes the value of dielectric constant ε_r. This means the impedance of microstrips is controlled solely through the ratio w/h.

For this example, design a 50 Ω microstrip transmission line in FR-4, which as a dielectric constant of 4.5, to operate at 2.4 GHz.

$$\frac{w}{h} = ?$$

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Design Equations

To solve this problem, we must first derive some design equations. To do this, we solve our microstrip equations for w/h. This gives

$$A = \frac{Z_0}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

$$B = \frac{60\pi^2}{Z_0\sqrt{\varepsilon_r}}$$

$$\frac{w}{h} = \begin{cases} \dfrac{8e^A}{e^{2A} - 2} & w/h \le 2 \quad \text{thin lines} \\ \dfrac{2}{\pi}\left\{B - 1 - \ln(2B - 1) + \dfrac{\varepsilon_r - 1}{2\varepsilon_r}\left[\ln(B - 1) + 0.39 - \dfrac{0.61}{\varepsilon_r}\right]\right\} & w/h > 2 \quad \text{wide lines} \end{cases}$$

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Design Solution (1 of 2)

Applying our design equations, we get

$$A = 1.5438$$
$$B = 5.5831$$
$$\frac{w}{h} = \begin{cases} 1.8799 & w/h \le 2 \quad \text{thin lines} \\ 1.8812 & w/h > 2 \quad \text{wide lines} \end{cases}$$

Since the above numbers for w/h are essentially the same, we conclude that

$$\frac{w}{h} \cong 1.88$$

Design Solution (2 of 2)

We learn from our manufacturing engineer that a convenient choice for substrate thickness h is 0.5 mm. From this, to get 50 Ω the width w of the microstrip should be

$$w \cong 1.88h = 1.88(0.5 \text{ mm}) = \boxed{0.94 \text{ mm}}$$

The phase constant for this line will be

$$\varepsilon_{\text{r,eff}} = 3.3941$$
$$k_0 = \frac{\omega}{c_0} = \frac{2\pi f}{c_0} = \frac{2\pi\left(2.4\times10^9 \text{ s}^{-1}\right)}{299792458 \text{ m/s}} = 50.3 \text{ m}^{-1}$$
$$\beta \cong k_0\sqrt{\varepsilon_{\text{r,eff}}} = \left(50.3 \text{ m}^{-1}\right)\sqrt{3.3941} = \boxed{92.67 \text{ m}^{-1}}$$