Electromagnetics:
Electromagnetic Field Theory

Wave Parameters

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Lecture Outline

- Fundamental Vs. Intuitive Parameters
- Velocity v, Frequency ω and Wavelength λ
- Refractive Index n
- Wave Number k and Wave Vector $\vec{k}$
- Impedance η

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Fundamental Vs. Intuitive Parameters

Fundamental Parameters

These parameters are fundamental to solving Maxwell's equations, but it is difficult to specify how they affect a wave. This is because all of they all affect all properties of a wave.

Magnetic Permeability, μ
Electric Permittivity, ε
Electrical Conductivity, σ

Intuitive Parameters

These parameters consolidate specific information about a wave from the fundamental parameters.

Refractive index, n
Impedance, η
Wavelength, λ
Velocity, v
Wave Number, k
Propagation Constant, γ
Attenuation Coefficient, α
Phase Constant, β

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Wave Velocity, v

The scalar wave equation has been known since the 1700's to be

$$\nabla^2\psi + \left(\frac{\omega}{v}\right)^2 \psi = 0$$

$\psi \equiv$ wave disturbance
$\omega \equiv$ angular frequency
$v \equiv$ wave velocity

If electromagnetic wave equation is compared to the historical wave equation, an expression for wave velocity can be obtained.

$$\nabla^2\vec{E} + \omega^2\mu\varepsilon\vec{E} = 0$$

$$\nabla^2\psi + \left(\frac{\omega}{v}\right)^2 \psi = 0 \quad \rightarrow \quad \frac{\omega}{v} = \omega\sqrt{\mu\varepsilon} \quad \rightarrow \quad \boxed{v = \frac{1}{\sqrt{\mu\varepsilon}}}$$

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Speed of Light in Vacuum, c_0

In a vacuum, $\mu = \mu_0$ and $\varepsilon = \varepsilon_0$ and the velocity v becomes what is commonly called the "speed of light c_0."

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \Rightarrow \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$\boxed{c_0 = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 299{,}792{,}458\ \text{m/s}}$$

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Refractive Index, n

When not in a vacuum, $\mu = \mu_0\mu_\mathrm{r}$ and $\varepsilon = \varepsilon_0\varepsilon_\mathrm{r}$ and the velocity is reduced by a factor n called the *refractive index*.

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\mu_\mathrm{r}\varepsilon_0\varepsilon_\mathrm{r}}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\frac{1}{\sqrt{\mu_\mathrm{r}\varepsilon_\mathrm{r}}} = c_0 \cdot \frac{1}{n}$$

This term is the speed of light in vacuum.

$$c_0 = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

This is the factor by which a wave slows down inside a medium.

It is called the *refractive index*.

$$\boxed{n = \sqrt{\mu_\mathrm{r}\varepsilon_\mathrm{r}}}$$

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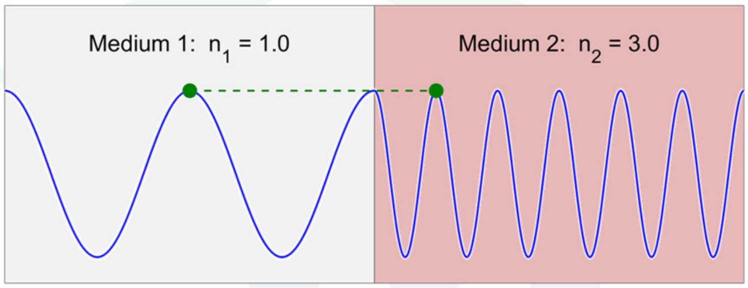
Frequency is Constant, Speed & Wavelength Change



Frequency is the most fundamental constant about a wave. It never changes in linear materials.

When a wave enters a different material, its speed and thus its wavelength change.

$$v = \frac{c_0}{n} \qquad \lambda = \frac{\lambda_0}{n}$$

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Speed v, Frequency f & Wavelength λ

The speed of a wave v, its frequency f, and its wavelength λ are related through

$$v = f\lambda \qquad c_0 = f\lambda_0$$

It is now possible to derive an expression for wavelength.

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = f\lambda \quad \rightarrow \quad \lambda = \frac{1}{f\sqrt{\mu\varepsilon}} = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}}$$

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Wavelength λ & Wave Number k

Recall that the wave number k was defined as as

$$k = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r} = \omega\cdot\sqrt{\mu_0\varepsilon_0}\cdot\sqrt{\mu_r\varepsilon_r} = \omega\cdot\frac{1}{c_0}\cdot n$$

The angular frequency ω is related to wavelength λ through the ordinary frequency f.

$$\omega = 2\pi f = 2\pi\frac{c_0}{\lambda_0} = 2\pi\frac{c_0}{n\lambda}$$

Substituting this into the first equation gives

$$k = \omega\cdot\frac{1}{c_0}\cdot n = 2\pi\frac{c_0}{n\lambda}\cdot\frac{1}{c_0}\cdot n = \frac{2\pi}{\lambda}$$

$$\boxed{k = \frac{2\pi}{\lambda}}$$

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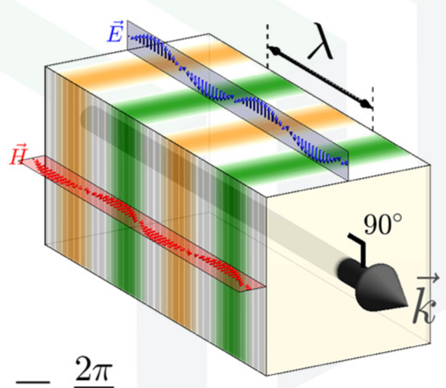
Wave Vector $\vec{k}$

The wave vector $\vec{k}$ conveys two pieces of information at the same time: (1) Magnitude conveys the wavelength λ inside the medium, and (2) direction conveys the direction of the wave and is perpendicular to the wave fronts.



$$|\vec{k}| = \frac{2\pi}{\lambda}$$

$$\vec{k} = k_x\hat{a}_x + k_y\hat{a}_y + k_z\hat{a}_z$$

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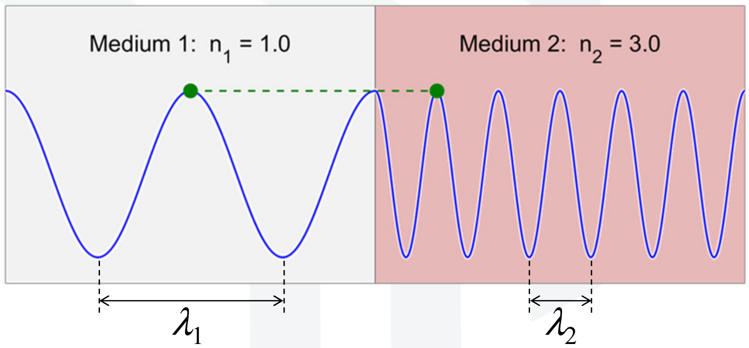
Magnitude $\left|\vec{k}\right|$ Conveys Wavelength λ

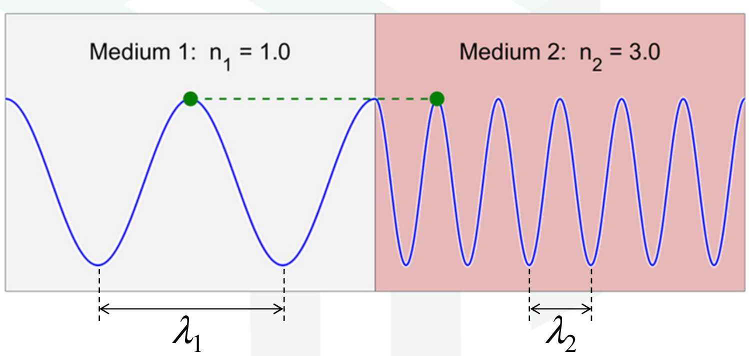
Most fundamentally, the magnitude of the wave vector conveys the wavelength of the wave inside of the medium.



$$\left|\vec{k}_1\right| = \frac{2\pi}{\lambda_1} \qquad \left|\vec{k}_2\right| = \frac{2\pi}{\lambda_2}$$

Magnitude $\left|\vec{k}\right|$ Can Also Convey Refractive Index n

When the frequency ω of a wave is known, the magnitude of the wave vector $\left|\vec{k}\right|$ conveys refractive index n.



$$\left|\vec{k}_1\right| = \frac{2\pi n_1}{\lambda_0} = k_0 n_1 \qquad \left|\vec{k}_2\right| = \frac{2\pi n_2}{\lambda_0} = k_0 n_2 \qquad k_0 = \frac{2\pi}{\lambda_0}$$

Material Impedance, η (1 of 3)

Impedance η is defined as the relationship between the amplitudes of $\vec{E}$ and $\vec{H}$.

$$\eta = E_0 / H_0$$

Recall the relationship between $\vec{E}$ and $\vec{H}$.

$$\vec{E}(\vec{r}) = \vec{P}e^{-j\vec{k}\bullet\vec{r}} \quad \text{and} \quad \vec{H} = \frac{1}{\omega\mu}\left(\vec{k}\times\vec{P}\right)e^{-j\vec{k}\bullet\vec{r}}$$

An expression for impedance η can be derived by collecting all of the amplitude terms together in the expression for $\vec{H}$.

$$\vec{H} = \frac{1}{\omega\mu}\left[\left(k\hat{k}\right)\times\left(E_0\hat{P}\right)\right]e^{-j\vec{k}\bullet\vec{r}} = \frac{E_0 k}{\omega\mu}\left(\hat{k}\times\hat{P}\right)e^{-j\vec{k}\bullet\vec{r}}$$

This term is the amplitude of $\vec{H}$

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Material Impedance, η (2 of 3)

From the last slide, the amplitude of $\vec{H}$ is

$$\vec{H} = \frac{E_0 k}{\omega\mu}\left(\hat{k}\times\hat{P}\right)e^{-j\vec{k}\bullet\vec{r}} \quad \rightarrow \quad H_0 = \frac{E_0 k}{\omega\mu}$$

Expressions have been obtained for both E_0 and H_0. It follows that impedance is

$$\eta = \frac{E_0}{H_0} = \frac{E_0}{E_0 k/\omega\mu} = \frac{\omega\mu}{k}$$

Since $k = \omega\sqrt{\mu\varepsilon}$, the final expression for impedance η is

$$\eta = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \frac{\mu}{\sqrt{\mu\varepsilon}} \quad \rightarrow \quad \boxed{\eta = \sqrt{\frac{\mu}{\varepsilon}}}$$

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Material Impedance, η (3 of 3)

The expressions for the electric and magnetic field components of a wave can now be written to be in terms of impedance.

$$\vec{E}(\vec{r}) = \vec{P}e^{-j\vec{k}\bullet\vec{r}} \quad \text{and} \quad \vec{H} = \frac{\hat{k}\times\vec{P}}{\eta}e^{-j\vec{k}\bullet\vec{r}}$$

where

$$\eta = \frac{E_0}{H_0} = \sqrt{\frac{\mu}{\varepsilon}}$$

Vacuum Impedance

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.73011346177\ \Omega$$

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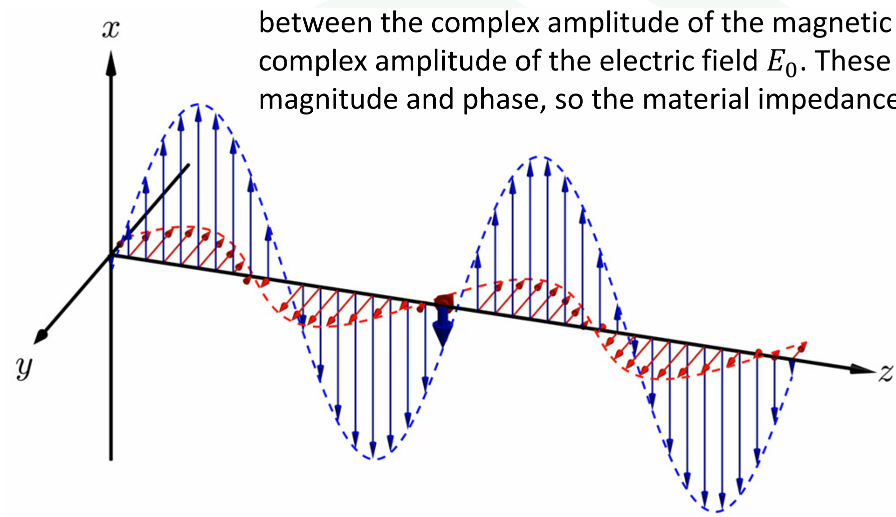
What is Material Impedance η?

The material impedance quantifies the magnitude and phase relationship between the complex amplitude of the magnetic field H_0 and the complex amplitude of the electric field E_0. These can differ in both magnitude and phase, so the material impedance is a complex number.



$$\eta = \frac{E_0}{H_0}$$

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