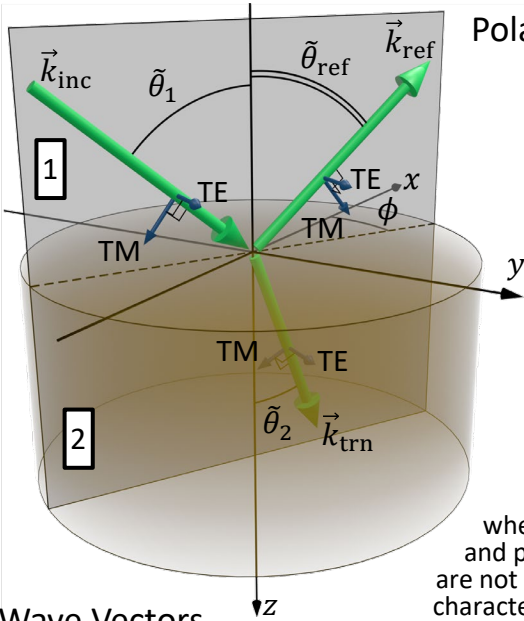


# Scattering at An Interface

## Geometry of an Interface



Polarization Directions

$$\hat{a}_{TE} = \frac{\hat{a}_z \times \vec{k}_{inc}}{|\hat{a}_z \times \vec{k}_{inc}|}$$

$$\hat{a}_{TM} = \frac{\vec{k}_{inc} \times \hat{a}_{TE}}{|\vec{k}_{inc} \times \hat{a}_{TE}|}$$

For normal incidence, the directions of TE and TM cannot be defined.

### Complex Angles $\tilde{\theta}$

When loss or gain is considered, waves can become inhomogeneous where planes of equal phase and planes of equal amplitude are not aligned. These waves are characterized by complex angles.

Wave Vectors

$$\vec{k}_{inc} = k_0 \tilde{n}_1 (\sin \tilde{\theta}_1 \cos \phi \hat{a}_x + \sin \tilde{\theta}_1 \sin \phi \hat{a}_y + \cos \tilde{\theta}_1 \hat{a}_z)$$

$$\vec{k}_{ref} = k_0 \tilde{n}_1 (\sin \tilde{\theta}_1 \cos \phi \hat{a}_x + \sin \tilde{\theta}_1 \sin \phi \hat{a}_y - \cos \tilde{\theta}_1 \hat{a}_z)$$

$$\vec{k}_{trn} = k_0 \tilde{n}_2 (\sin \tilde{\theta}_2 \cos \phi \hat{a}_x + \sin \tilde{\theta}_2 \sin \phi \hat{a}_y + \cos \tilde{\theta}_2 \hat{a}_z)$$

## Snell's Laws

Snell's laws are only valid for plane waves and materials without loss or gain.

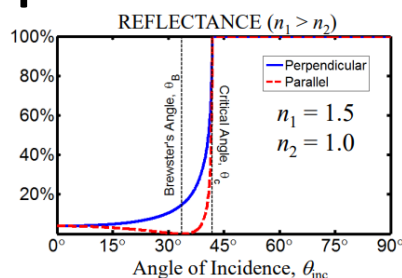
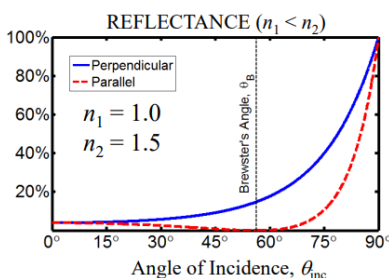
Reflection

$$\tilde{\theta}_{ref} = \tilde{\theta}_1$$

Refraction

$$\tilde{n}_1 \sin \tilde{\theta}_1 = \tilde{n}_2 \sin \tilde{\theta}_2$$

## Examples



## Fresnel Equations

TE Polarization

$$r_{TE} = \frac{\tilde{\eta}_2 \cos \tilde{\theta}_1 - \tilde{\eta}_1 \cos \tilde{\theta}_2}{\tilde{\eta}_2 \cos \tilde{\theta}_1 + \tilde{\eta}_1 \cos \tilde{\theta}_2}$$

$$t_{TE} = \frac{2\tilde{\eta}_2 \cos \tilde{\theta}_1}{\tilde{\eta}_2 \cos \tilde{\theta}_1 + \tilde{\eta}_1 \cos \tilde{\theta}_2}$$

$$1 + r_{TE} = t_{TE}$$

If  $\text{Re} \left[ \frac{\tilde{\eta}_2 \cos \tilde{\theta}_1}{\tilde{\eta}_1 \cos \tilde{\theta}_2} \right] < 0$ , then change sign of  $\tilde{\eta}_1$  or  $\tilde{\eta}_2$ , but not both.

TM Polarization

$$r_{TM} = \frac{\tilde{\eta}_2 \cos \tilde{\theta}_2 - \tilde{\eta}_1 \cos \tilde{\theta}_1}{\tilde{\eta}_2 \cos \tilde{\theta}_2 + \tilde{\eta}_1 \cos \tilde{\theta}_1}$$

$$t_{TM} = \frac{2\tilde{\eta}_2 \cos \tilde{\theta}_1}{\tilde{\eta}_2 \cos \tilde{\theta}_2 + \tilde{\eta}_1 \cos \tilde{\theta}_1}$$

$$1 + r_{TM} = t_{TM} \frac{\cos \tilde{\theta}_2}{\cos \tilde{\theta}_1}$$

If  $\text{Re} \left[ \frac{\tilde{\eta}_2 \cos \tilde{\theta}_2}{\tilde{\eta}_1 \cos \tilde{\theta}_1} \right] < 0$ , then change sign of  $\tilde{\eta}_1$  or  $\tilde{\eta}_2$ , but not both.

Normal Incidence

$$r = \frac{\tilde{\eta}_2 - \tilde{\eta}_1}{\tilde{\eta}_2 + \tilde{\eta}_1}$$

$$t = \frac{2\tilde{\eta}_2}{\tilde{\eta}_2 + \tilde{\eta}_1}$$

$$1 + r = t$$

These equations handle complex  $\tilde{\mu}_r$  and  $\tilde{\epsilon}_r$ .

## Reflectance & Transmittance

$$R_{TE} = |r_{TE}|^2$$

$$T_{TE} = |t_{TE}|^2 \text{Re} \left[ \frac{\tilde{\eta}_1^* \cos \tilde{\theta}_2}{\tilde{\eta}_2^* \cos \tilde{\theta}_1} \right]$$

$$1 = R_{TE} + T_{TE}$$

$$R_{TM} = |r_{TM}|^2$$

$$T_{TM} = |t_{TM}|^2 \text{Re} \left[ \frac{\tilde{\eta}_1^* \cos \tilde{\theta}_2}{\tilde{\eta}_2^* \cos \tilde{\theta}_1} \right]$$

$$1 = R_{TM} + T_{TM}$$

$$R = |r|^2$$

$$T = |t|^2 \text{Re} \left[ \frac{\tilde{\eta}_1^*}{\tilde{\eta}_2^*} \right]$$

$$1 = R + T$$

## Brewster's Angles

Transmission is 100% when angle of incidence equals the Brewster's angle.

$$\sin^2 \tilde{\theta}_B \Big|_{TE} = \frac{1 - \tilde{\mu}_{r1} \tilde{\epsilon}_{r2} / \tilde{\mu}_{r2} \tilde{\epsilon}_{r1}}{1 - (\tilde{\mu}_{r1} / \tilde{\mu}_{r2})^2}$$

No Brewster's angle for TE polarization of there is no change in permeability

$$\sin^2 \tilde{\theta}_B \Big|_{TM} = \frac{1 - \tilde{\mu}_{r2} \tilde{\epsilon}_{r1} / \tilde{\mu}_{r1} \tilde{\epsilon}_{r2}}{1 - (\tilde{\epsilon}_{r1} / \tilde{\epsilon}_{r2})^2}$$

$$\tan \tilde{\theta}_B \Big|_{TM} = \sqrt{\frac{\tilde{\epsilon}_{r2}}{\tilde{\epsilon}_{r1}}} = \frac{\tilde{n}_2}{\tilde{n}_1} \quad \text{No change in permeability}$$

## Critical Angle

$$\tilde{\theta}_c = \sin^{-1}(\tilde{n}_2 / \tilde{n}_1)$$

Critical angle only exists when  $\tilde{n}_1 > \tilde{n}_2$ .

Reflection is 100% for all angles of incidence greater than the critical angle.