

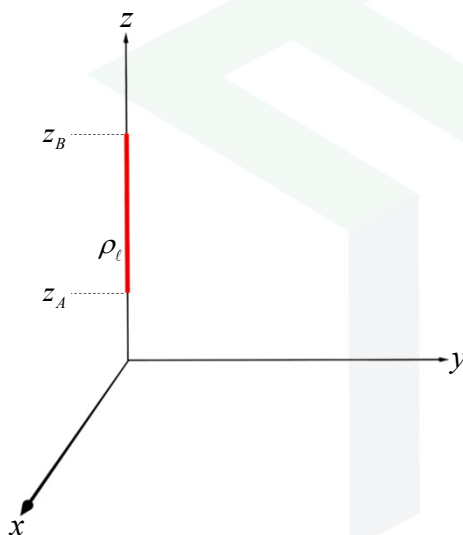


Electromagnetics:  
Electromagnetic Field Theory

Example:  
Uniform Finite Line Charge

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## Total Charge



What is the total charge  $Q_{\text{Total}}$ ?

1. Draw the problem.
2. Choose a coordinate system.

*Cartesian*

3. Write general equation.

$$Q_{\text{Total}} = \int_{\ell} \rho_{\ell} d\ell$$

4. Write expressions for each term.

$$\rho_{\ell} = \rho_{\ell} \quad d\ell = dz$$

5. Choose limits of integration.

$$Q_{\text{Total}} = \int_{z_A}^{z_B} \rho_{\ell} dz$$

6. Solve the integral.

$$Q_{\text{Total}} = \int_{z_A}^{z_B} \rho_{\ell} dz = \rho_{\ell} z \Big|_{z_A}^{z_B} = \rho_{\ell} (z_B - z_A)$$

7. Interpret the result.

$$\boxed{Q_{\text{Total}} = \rho_{\ell} L} \quad \text{for uniform charge density}$$

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## Total Field

What is the total field  $\vec{D}$

1. Draw the problem.
2. Choose a coordinate system.  
*Cartesian*
3. Write general equation.  
$$\vec{D}_{\text{Total}} = \int \frac{\rho_\ell d\ell}{4\pi R^2} \hat{a}_R$$
4. Write expressions for each term.  
$$\rho_\ell = \rho_\ell \quad \hat{a}_R = \frac{\vec{R}}{|\vec{R}|} \quad d\ell = dz'$$
5. Choose limits of integration.  
$$\vec{D}_{\text{Total}} = \frac{\rho_\ell}{4\pi} \int_{z_A}^{z_B} \frac{\vec{R}}{|\vec{R}|^3} dz'$$

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## Total Field

What is the total field  $\vec{D}$

6. Solve the integral.  
$$\vec{D}_{\text{Total}} = \frac{\rho_\ell}{4\pi} \int_{\phi_1}^{\phi_2} \frac{\vec{R}}{|\vec{R}|^3} dz'$$

Rather than integrate over  $z$ , integrate over the angle  $\phi$ .

$z_A \rightarrow \phi_1$   
 $z_B \rightarrow \phi_2$   
 $dz' \rightarrow ?$   
 $\vec{R}/|\vec{R}|^3 \rightarrow ?$

From the figure,  
 $\tan \phi = \frac{z - z'}{\rho} \rightarrow z' = z - \rho \tan \phi$

Differentiate to get  
 $dz' = -\rho \sec^2 \phi d\phi$

$$\vec{D}_{\text{Total}} = \frac{\rho_\ell}{4\pi} \int_{\phi_1}^{\phi_2} ?$$

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### Total Field

What is the total field  $\vec{D}$

6. Solve the integral cont'd...

Write an expression for  $\vec{R}$

$$\begin{aligned} \vec{R} &= \rho \hat{a}_\rho + (z - z') \hat{a}_z \\ &= \rho \hat{a}_\rho + \rho \tan \phi \hat{a}_z \\ &= \frac{\rho}{\cos \phi} (\cos \phi \hat{a}_\rho + \sin \phi \hat{a}_z) \\ &= \rho \sec \phi (\underbrace{\cos \phi \hat{a}_\rho + \sin \phi \hat{a}_z}_{\text{Magnitude is 1}}) \end{aligned}$$

Magnitude is

$$|\vec{R}|^3 = (\rho \sec \phi)^3$$

Plug everything back into integral.

$$\vec{D}_{\text{Total}} = \frac{\rho_\ell}{4\pi} \int_{\phi_1}^{\phi_2} \frac{\rho \sec \phi (\cos \phi \hat{a}_\rho + \sin \phi \hat{a}_z)}{(\rho \sec \phi)^3} (-\rho \sec^2 \phi d\phi)$$

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### Total Field

What is the total field  $\vec{D}$

6. Solve the integral cont'd...

Simplify integral.

$$\begin{aligned} \vec{D}_{\text{Total}} &= \frac{\rho_\ell}{4\pi} \int_{\phi_1}^{\phi_2} \frac{\cancel{\rho} \sec \phi (\cos \phi \hat{a}_\rho + \sin \phi \hat{a}_z)}{(\cancel{\rho} \sec \phi)^3} (-\cancel{\rho} \sec^2 \phi d\phi) \\ &= -\frac{\rho_\ell}{4\pi \rho} \int_{\phi_1}^{\phi_2} (\cos \phi \hat{a}_\rho + \sin \phi \hat{a}_z) d\phi \end{aligned}$$

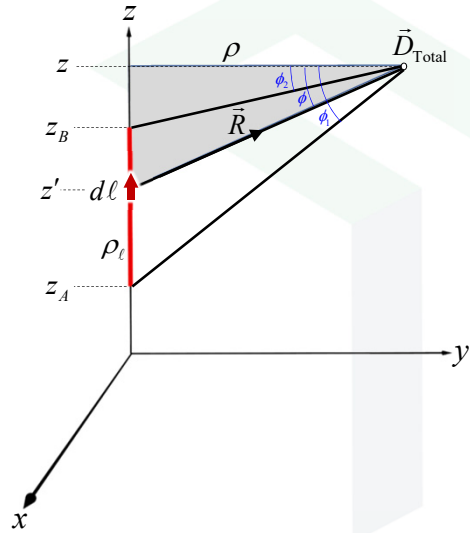
Perform integration.

$$\begin{aligned} \vec{D}_{\text{Total}} &= -\frac{\rho_\ell}{4\pi \rho} (\sin \phi \hat{a}_\rho - \cos \phi \hat{a}_z) \Big|_{\phi_1}^{\phi_2} \\ &= -\frac{\rho_\ell}{4\pi \rho} \left[ (\sin \phi_2 \hat{a}_\rho - \cos \phi_2 \hat{a}_z) - (\sin \phi_1 \hat{a}_\rho - \cos \phi_1 \hat{a}_z) \right] \\ &= -\frac{\rho_\ell}{4\pi \rho} \left[ (\sin \phi_2 - \sin \phi_1) \hat{a}_\rho + (\cos \phi_1 - \cos \phi_2) \hat{a}_z \right] \\ &= \frac{\rho_\ell}{4\pi \rho} \left[ (\sin \phi_1 - \sin \phi_2) \hat{a}_\rho + (\cos \phi_2 - \cos \phi_1) \hat{a}_z \right] \end{aligned}$$

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## Total Field



What is the total field  $\vec{D}$

7. Interpret the result.

The final answer...

$$\vec{D}_{Total} = \frac{\rho_L}{4\pi\rho} [(\sin\phi_1 - \sin\phi_2)\hat{a}_\rho + (\cos\phi_2 - \cos\phi_1)\hat{a}_z]$$

$D$  decays as  $1/\rho$ .