Advanced Electromagnetics:
21^{st} Century Electromagnetics

Coupled-Mode Theory

Lecture Outline

- Electromagnetic modes
- Coupled-mode theory
- Codirectional coupling
- Contradirectional coupling
- Non-directional coupling
- Phase matching with gratings
- Mode-matching vs. coupled-wave models

Electromagnetic Modes

What are modes?

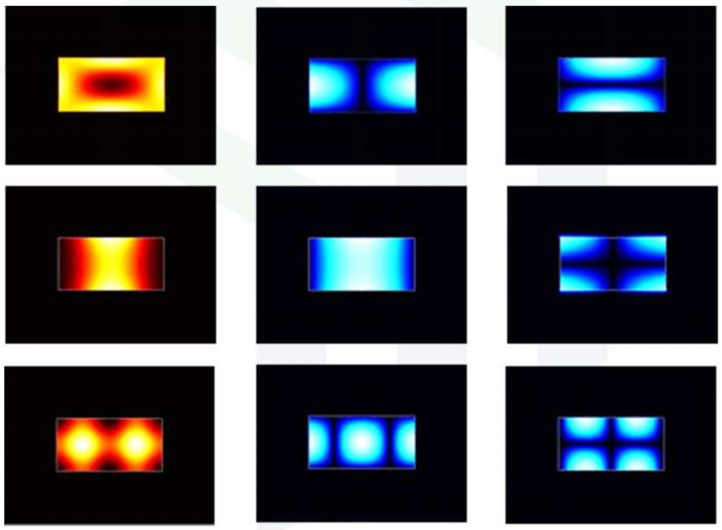
Modes can mean many different things depending on the context it is being used.

- Different discrete eigen-modes in a waveguide
- Different polarizations
- Different directions
- Etc…

Generalized Definition:

An electromagnetic mode is electromagnetic power that exists independent and different from other electromagnetic power.

Modes in a Waveguide



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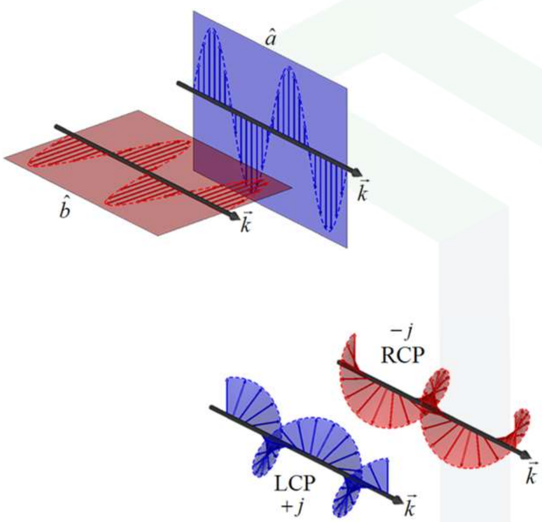
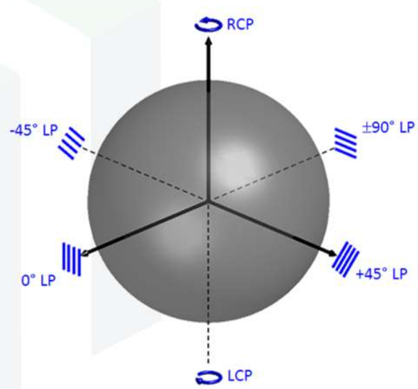
Waves in Free Space





Poincare Sphere

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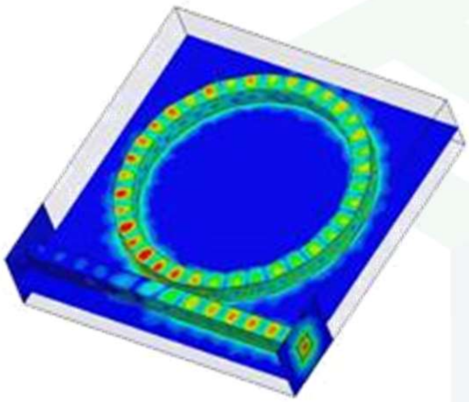
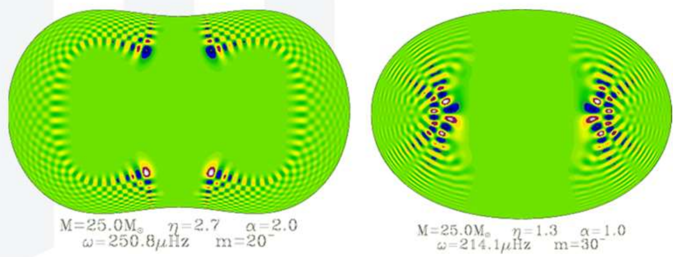
Resonant Modes





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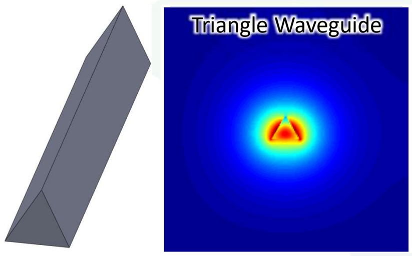
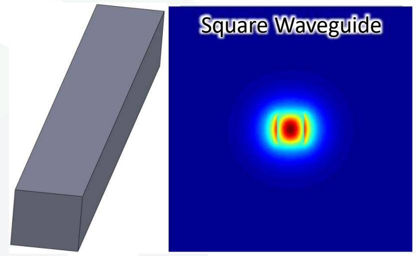
Coupled-Mode Theory

Slide 8

Modes in Two Different Waveguides





$$\vec{E}_1 = \vec{E}_{0,1}\left(x,y\right)e^{-j\beta_1 z} \qquad \vec{E}_2 = \vec{E}_{0,2}\left(x,y\right)e^{-j\beta_2 z}$$

$$\vec{H}_1 = \vec{H}_{0,1}\left(x,y\right)e^{-j\beta_1 z} \qquad \vec{H}_2 = \vec{H}_{0,2}\left(x,y\right)e^{-j\beta_2 z}$$

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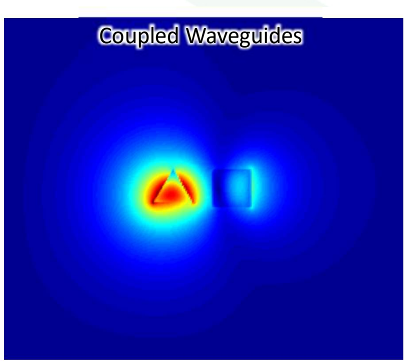
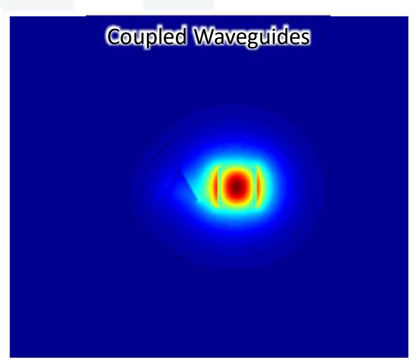
Supermodes

When two waveguides are in close proximity, they become coupled.

The pair forms *supermodes*.





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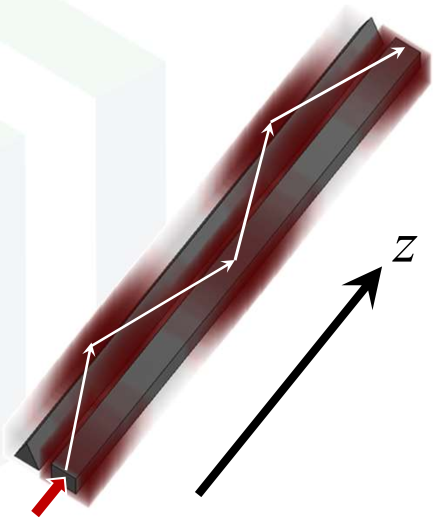
Visualization of Coupled-Modes

When two waveguides are in close proximity, they become coupled and exchange power as a function of z.

Very often, this leads to a periodic exchange of power between the waveguides.

Waveguide arrays are more complicated to analyze, but involve the same concepts.



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Perturbation Analysis

Assumption – To simplify the analysis, it will be assumed that the supermodes can be represented as a weighted sum of the individual guided modes. This implies that the modes do not change at all with the introduction of the second guide. In reality, the modes are deformed slightly, but are still coupled.

$$\vec{E} = A(z)\vec{E}_1 + B(z)\vec{E}_2$$
$$\vec{H} = A(z)\vec{H}_1 + B(z)\vec{H}_2$$

$A(z) \equiv$ amplitude of 1^{st} mode

$B(z) \equiv$ amplitude of 2^{nd} mode

When two waveguides are in close proximity, they become coupled and exchange power as a function of z.

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Assumed Solution in Perturbation Analysis

Start with the following solution.

$$\vec{E} = A(z)\vec{E}_1 + B(z)\vec{E}_2$$

$$\vec{H} = A(z)\vec{H}_1 + B(z)\vec{H}_2$$

Ignoring the magnetic response (i.e. $\mu_r = 1$)

$$\nabla \times \vec{E} = -j\omega\mu_0\vec{H}$$

$$\nabla \times \vec{H} = j\omega\varepsilon_0\varepsilon_r\vec{E}$$

Substitute these into Maxwell's curl equations to obtain

$$\left(\hat{z}\times\vec{E}_1\right)\frac{dA}{dz} + \left(\hat{z}\times\vec{E}_2\right)\frac{dB}{dz} = 0$$

$$\left(\hat{z}\times\vec{H}_1\right)\frac{dA}{dz} - j\omega\varepsilon_0\left(\varepsilon_r - \varepsilon_{r1}\right)A\vec{E}_1 + \left(\hat{z}\times\vec{H}_2\right)\frac{dB}{dz} - j\omega\varepsilon_0\left(\varepsilon_r - \varepsilon_{r2}\right)B\vec{E}_2 = 0$$

To get to this point, the following vector identity was used.

$$\nabla\times\left(A\vec{E}\right) = A\nabla\times\vec{E} + \nabla A\times\vec{E} = A\nabla\times\vec{E} + \frac{dA}{dz}\left(\hat{z}\times\vec{E}\right)$$

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Derivation of the Generalized Coupled-Mode Equations

The following equations were derived that enforce Maxwell's equations.

$$\left(\hat{z}\times\vec{E}_1\right)\frac{dA}{dz} + \left(\hat{z}\times\vec{E}_2\right)\frac{dB}{dz} = 0 \qquad \text{Eq. (1)}$$

$$\left(\hat{z}\times\vec{H}_1\right)\frac{dA}{dz} - j\omega\varepsilon_0\left(\varepsilon_r - \varepsilon_1\right)A\vec{E}_1 + \left(\hat{z}\times\vec{H}_2\right)\frac{dB}{dz} - j\omega\varepsilon_0\left(\varepsilon_r - \varepsilon_2\right)B\vec{E}_2 = 0 \qquad \text{Eq. (2)}$$

The generalized coupled-mode equations are derived by substituting the above expressions into the following integral equations.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\vec{E}_1^*\bullet\left(\text{Eq. 2}\right) - \vec{H}_1^*\bullet\left(\text{Eq. 1}\right)\right]dxdy = 0$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\vec{E}_2^*\bullet\left(\text{Eq. 2}\right) - \vec{H}_2^*\bullet\left(\text{Eq. 1}\right)\right]dxdy = 0$$

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Generalized Coupled-Mode Equations

After LOTS of algebra, we get (i.e. it is easily shown that… ☺)

$$\frac{dA}{dz}+c_{12}\frac{dB}{dz}e^{-j(\beta_2-\beta_1)z}+j\chi_1 A+j\kappa_{12}Be^{-j(\beta_2-\beta_1)z}=0$$

$$\frac{dB}{dz}+c_{21}\frac{dA}{dz}e^{-j(\beta_2-\beta_1)z}+j\chi_2 B+j\kappa_{21}Ae^{-j(\beta_2-\beta_1)z}=0$$

These are called the generalized coupled-mode equations. These are solved to describe the coupling between the two waveguides.

Mode Coupling Coefficient

$$\kappa_{pq}=\frac{\omega\varepsilon_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\varepsilon_r-\varepsilon_{r,q}\right)\vec{E}_p^*\bullet\vec{E}_q\,dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{z}\bullet\left(\vec{E}_p^*\times\vec{H}_p+\vec{E}_p\times\vec{H}_p^*\right)dxdy}$$

Butt Coupling Coefficient

$$c_{pq}=\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{z}\bullet\left(\vec{E}_p^*\times\vec{H}_q+\vec{E}_q\times\vec{H}_p^*\right)dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{z}\bullet\left(\vec{E}_p^*\times\vec{H}_p+\vec{E}_p\times\vec{H}_p^*\right)dxdy}$$

Change in Propagation Constant

$$\chi_p=\frac{\omega\varepsilon_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\varepsilon_r-\varepsilon_{r,q}\right)\vec{E}_p^*\bullet\vec{E}_p\,dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{z}\bullet\left(\vec{E}_p^*\times\vec{H}_p+\vec{E}_p\times\vec{H}_p^*\right)dxdy}$$

$$p,q=1\text{ or }2$$

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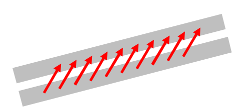
Mode Coupling Coefficient, κ_{pq}

The mode coupling coefficient is calculated according to

$$\kappa_{pq}=\frac{\omega\varepsilon_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\varepsilon_\mathrm{r}-\varepsilon_{\mathrm{r},q}\right)\vec{E}_p^*\bullet\vec{E}_q\,dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{z}\bullet\left(\vec{E}_p^*\times\vec{H}_p+\vec{E}_p\times\vec{H}_p^*\right)dxdy}$$



This parameter quantifies how efficiently power "leaks" from waveguide p to waveguide q due to the behavior of the supermode.

ε_r is the dielectric function containing both waveguides.
$\varepsilon_{\mathrm{r},q}$ is dielectric function with only waveguide q.

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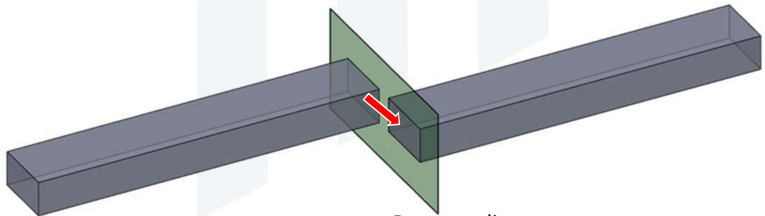
Butt Coupling Coefficient, c_{pq}

The coefficient c_{pq} quantifies the excitation efficiency from one waveguide to the other. It is called the butt coupling coefficient and is calculated according to

$$c_{pq} = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{z} \bullet \left(\vec{E}_p^* \times \vec{H}_q + \vec{E}_q \times \vec{H}_p^* \right) dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{z} \bullet \left(\vec{E}_p^* \times \vec{H}_p + \vec{E}_p \times \vec{H}_p^* \right) dxdy}$$



Butt coupling

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Change in Propagation Constant, χ_p

When the q^{th} waveguide is brought into proximity to p^{th} waveguide, the propagation constant in the p^{th} waveguide changes by χ_p.

$$\chi_p = \frac{\omega\varepsilon_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\varepsilon_{\text{r}} - \varepsilon_{\text{r},q} \right) \vec{E}_p^* \bullet \vec{E}_p dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{z} \bullet \left(\vec{E}_p^* \times \vec{H}_p + \vec{E}_p \times \vec{H}_p^* \right) dxdy}$$

We expect χ_p to be largest when the waveguides are the closest and the fields are perturbed more strongly affecting the propagation constant.

Many analyses just assume $\chi = 0$.

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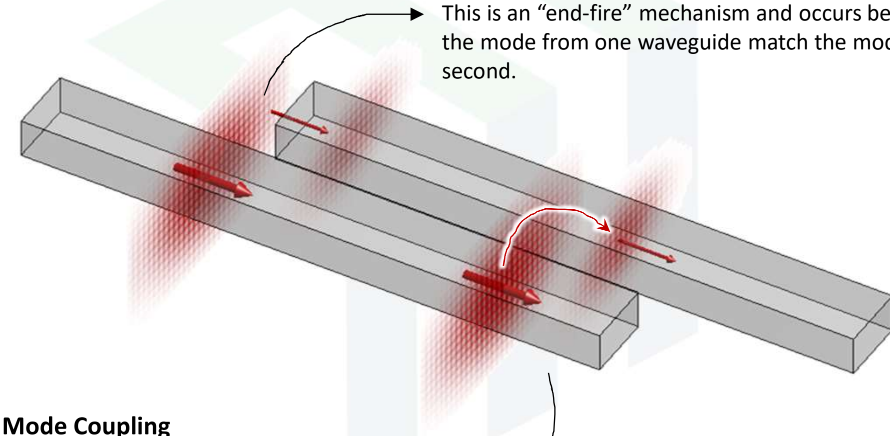
Mode-Coupling Vs. Butt Coupling

Butt Coupling
This is an "end-fire" mechanism and occurs because parts of the mode from one waveguide match the mode from the second.



Mode Coupling
This is a "leakage" mechanism and occurs due to the propagation behavior of the supermode.

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Normalized Power in Eigen-Modes

The total power in waveguide p is

$$P_p = \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\vec{E}_p \times \vec{H}_p^*\right)\bullet \hat{z}\,dxdy$$

The denominator in the prior equations was $4P_p$.

Without loss of generality, the power in the eigen-modes is normalized according to

$$4P_p = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\hat{z}\bullet\left(\vec{E}_p^* \times \vec{H}_p + \vec{E}_p \times \vec{H}_p^*\right)dxdy = 1$$

After normalizing the power, it is then easily shown that… ☺

$$c_{21} = c_{12}^* \qquad \chi_p = \chi_q^*$$

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Power in Supermode

The power in the supermode is

$$P = \frac{1}{2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\vec{E} \times \vec{H}^* \right) \bullet \hat{z} dx dy$$

After some algebra, this becomes

$$P = \frac{1}{4} \left[|A|^2 + |B|^2 + A^* B c_{12} e^{-j2\delta z} + AB^* c_{12}^* e^{j2\delta z} \right]$$

Total power

$$\delta = \frac{\beta_2 - \beta_1}{2}$$

Difference in propagation constants of the two waveguides.

Consequences of Conservation of Power

For waveguides without loss or gain,

$$\frac{dP}{dz} = 0$$

So, the equation for total power is differentiated to get

$$jA^* B \left(\kappa_{21}^* - \kappa_{12} - 2\delta c_{12} \right) e^{-j2\delta z} - jAB^* \left(\kappa_{21} - \kappa_{12}^* - 2\delta c_{12}^* \right) e^{j2\delta z} = 0$$

For this to be satisfied independent of z, it must be that

$$\kappa_{21} = \kappa_{12}^* + 2\delta c_{12}^*$$

Note, $\kappa_{21} = \kappa_{12}^*$ only when:

- $\beta_1 = \beta_2$ (identical waveguides) → $\delta = 0$, or
- Waveguides are sufficiently separated so that $c_{12}^* \cong 0$

If the waveguides are very close or are very different, the $2\delta c_{12}^*$ term cannot be ignored.

Revised Coupled Mode Equations

Our coupled-mode equations can now be written as

$$\frac{dA}{dz} = -j\kappa_a B e^{-j2\delta z} + j\alpha_a A$$
$$\frac{dB}{dz} = -j\kappa_b A e^{j2\delta z} + j\alpha_b B$$

$$\kappa_a = \frac{\kappa_{12} - c_{12}\chi_2}{1 - |c_{12}|^2}$$
$$\kappa_b = \frac{\kappa_{21} - c_{12}^{*}\chi_1}{1 - |c_{12}|^2}$$
$$\alpha_a = \frac{\kappa_{21}c_{21} - \chi_1}{1 - |c_{12}|^2}$$
$$\alpha_b = \frac{\kappa_{12}c_{12}^{*} - \chi_2}{1 - |c_{12}|^2}$$

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Simplified Coupled Mode Equations

Assuming $c_{pq} = \chi_p = 0$ (i.e. modes in the individual waveguides are unperturbed), the coupled-mode equations can be written as

$$\frac{dA}{dz} = -j\kappa_{12} B e^{-j(\beta_2 - \beta_1)z}$$
$$\frac{dB}{dz} = -j\kappa_{21} A e^{+j(\beta_2 - \beta_1)z}$$

These are the equations that most analyses use.

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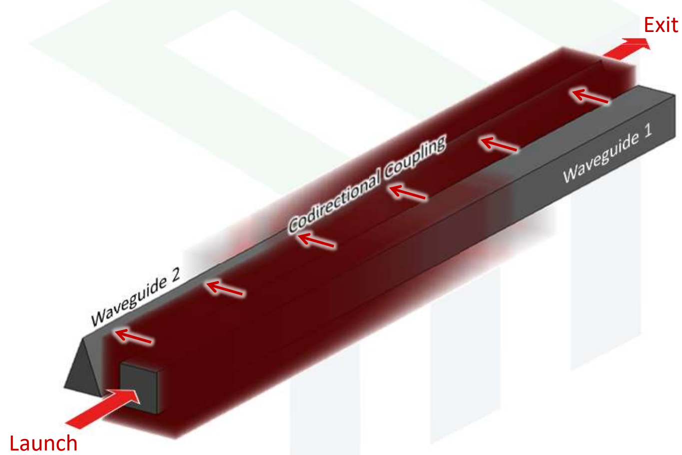
Codirectional Coupling

Picture of Codirectional Coupling



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General Coupled-Mode Solution

In codirectional coupling, both modes are propagating in the same direction and usually have similar propagation constants (i.e. $\beta_1 \approx \beta_2$).

$$\beta_1 > 0 \quad \text{and} \quad \beta_2 > 0$$

Reciprocity requires that $\kappa_{12} = \kappa_{21}^*$. Most often, κ_{pq} is real so when this is the case

$$\kappa = \kappa_{12} = \kappa_{21}$$

The general solution to the simplified coupled-mode equations are now

$$A(z) = \left[a_1 e^{j\psi z} + a_2 e^{-j\psi z} \right] e^{-j\delta z}$$
$$B(z) = \left[b_1 e^{j\psi z} + b_2 e^{-j\psi z} \right] e^{+j\delta z}$$

Initial conditions…

$$a_1 + a_2 = A(0)$$
$$b_1 + b_2 = B(0)$$

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Solution with Boundary Conditions

The final solution for $A(z)$ and $B(z)$ are then

$$A(z) = \left\{ \left[\cos(\psi z) + \frac{j\delta}{\psi}\sin(\psi z) \right] A(0) - \frac{j\kappa}{\psi}\sin(\psi z) B(0) \right\} e^{-j\delta z}$$

$$B(z) = \left\{ -\frac{j\kappa}{\psi}\sin(\psi z) A(0) + \left[\cos(\psi z) + \frac{j\delta}{\psi}\sin(\psi z) \right] B(0) \right\} e^{j\delta z}$$

$$\psi = \sqrt{\kappa^2 + \delta^2}$$

Note, when perturbation of the modes in the waveguide is minimal, $\delta \approx 0$ and $\psi \approx \kappa$.

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Typical Solution in Terms of Power

In most cases, power is injected into only one waveguide. If this is the first waveguide,

$$A(0) = A_0 \qquad B(0) = 0$$

Our equations for $A(z)$ and $B(z)$ reduce to

$$A(z) = A_0\left[\cos(\psi z) + \frac{j\delta}{\psi}\sin(\psi z)\right]e^{-j\delta z} \qquad B(z) = -A_0\frac{j\kappa}{\psi}\sin(\psi z)e^{j\delta z}$$

It is often more meaningful to write similar expressions in terms of the power in each waveguide as a function of z.

$$\tilde{P}_a(z) = \frac{|A(z)|^2}{|A_0|^2} = 1 - F\sin^2(\psi z)$$

$$\tilde{P}_b(z) = \frac{|B(z)|^2}{|A_0|^2} = F\sin^2(\psi z)$$

Maximum power-coupling efficiency…

$$F = \left(\frac{\kappa}{\psi}\right)^2 = \frac{1}{1+(\delta/\kappa)^2}$$

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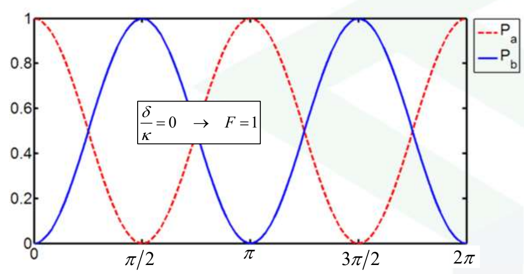
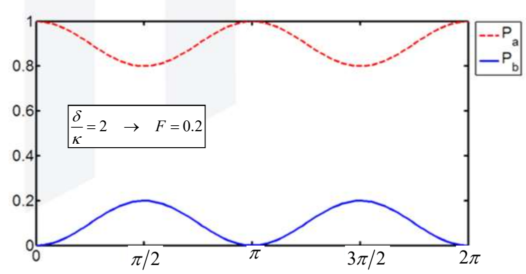
Typical Response of Codirectional Couplers



Maximums occur at

$$z_m = \frac{\pi}{2\psi}(2m+1) \qquad m = 0,1,2,\ldots$$



Coupling Length

The length over which maximum power is transferred to the second waveguide is called the coupling length.

$$L_c = \frac{\pi}{2\psi} = \frac{\pi}{2\sqrt{\kappa^2+\delta^2}}$$

When β_1=β_2 (i.e. δ= 0), then $L_c = \pi/2\kappa$

$$\boxed{L_c = \frac{\pi}{2\kappa}}$$

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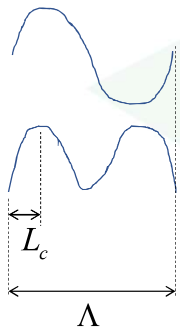
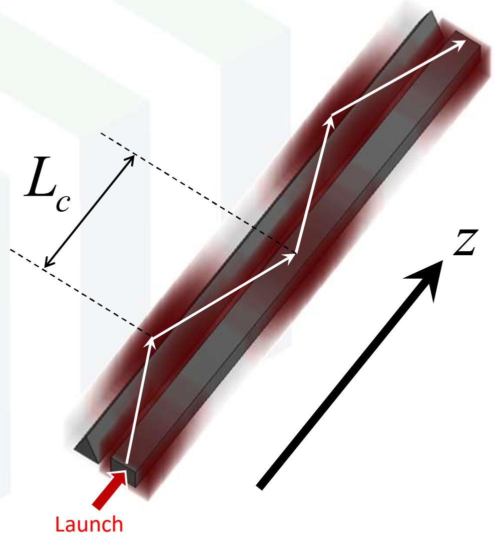
Visualization of the Terms



$\kappa \approx \psi = \frac{2\pi}{\Lambda}$

$L_c = \frac{\Lambda}{4}$

$\kappa \approx \frac{\pi}{2L_c}$



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Contradirectional Coupling (Bragg Grating)

Slide 32

Contradirectional Coupling

In contradirectional coupling, the coupled modes are propagating in opposite directions.

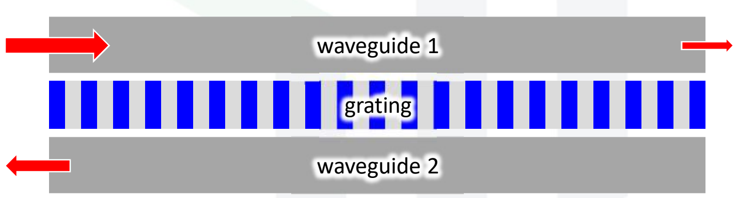
Let the second mode be the backward propagation mode.

$$\beta_1 > 0 \quad \text{and} \quad \beta_2 < 0$$

Reciprocity for contradirection modes requires that $\kappa_{12} = -\kappa_{21}^*$.

Conditions for Contradirectional Coupling

Contradirectional coupling cannot occur by simply bringing two waveguides in proximity. Typically a grating is used to couple the counter propagating modes.



$$\kappa_{12}(z) = \kappa_G e^{-j\frac{2\pi}{\Lambda}z}$$

The mode coupling coefficient is now a periodic function.

Contradirectional Coupled-Mode Equations

The coupled-mode equations are now written as

$$\frac{dA}{dz} = -j\kappa_G B e^{-j\left(\beta_2-\beta_1+\frac{2\pi}{\Lambda}\right)z}$$

$$\frac{dB}{dz} = -j\kappa_G A e^{+j\left(\beta_2-\beta_1+\frac{2\pi}{\Lambda}\right)z}$$

$$\kappa_{12} = -\kappa_{21}^{*} = \kappa_G e^{-j\frac{2\pi}{\Lambda}z}$$

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Phase Matching Conditions

Introduce the following phase matching condition of the grating.

$$\varphi = \frac{\beta_1 - \beta_2 - \frac{2\pi}{\Lambda}}{2}$$

Three are three cases

Case 1: $|\varphi| > \kappa_G$ — Pass band. Forward output. Temporary and confined peak in reflected mode.

Case 2: $|\varphi| = \kappa_G$ — Band edge.

Case 3: $|\varphi| < \kappa_G$ — Stop band. Reflected output. Band of reflection.

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Case 1: $|\varphi| > \kappa_G$ (Pass Band)

The mode amplitudes are:

$$A(z) = A_0 \frac{\rho\cos\left[\rho(z-L)\right] - j\varphi\sin\left[\rho(z-L)\right]}{\rho\cos(\rho L) + j\varphi\sin(\rho L)} e^{j\varphi z}$$

$$B(z) = A_0 \frac{j\kappa_G \sin\left[\rho(z-L)\right]}{\rho\cos(\rho L) + j\varphi\sin(\rho L)} e^{-j\varphi z}$$

$$\rho = \sqrt{\varphi^2 - \kappa_G^2}$$

The normalized forward and backward power

$$P_f(z) = \frac{|A(z)|^2}{|A_0|^2} = \frac{\rho^2 + \kappa_G^2 \sin^2\left[\rho(z-L)\right]}{\rho^2 + \kappa_G^2 \sin^2(\rho L)}$$

$$P_b(z) = \frac{|B(z)|^2}{|A_0|^2} = \frac{\kappa_G^2 \sin^2\left[\rho(z-L)\right]}{\rho^2 + \kappa_G^2 \sin^2(\rho L)}$$

L is the distance over which the periodic perturbation exists.

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Case 2: $|\varphi| = \kappa_G$ (Band Edge)

The mode amplitudes are:

$$A(z) = A_0 \frac{1 - j\varphi(z-L)}{1 + j\varphi L} e^{j\varphi z}$$

$$B(z) = A_0 \frac{j\kappa_G(z-L)}{1 + j\varphi L} e^{-j\varphi z}$$

The normalized forward and backward power

$$P_f(z) = \frac{|A(z)|^2}{|A_0|^2} = \frac{1 + \kappa_G^2 (z-L)^2}{1 + \kappa_G^2 L^2}$$

$$P_b(z) = \frac{|B(z)|^2}{|A_0|^2} = \frac{\kappa_G^2 (z-L)^2}{1 + \kappa_G^2 L^2}$$

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Case 3: $|\varphi| < \kappa_G$ (Stop Band)

The mode amplitudes are:

$$A(z) = A_0 \frac{\alpha\cosh\left[\alpha(z-L)\right] - j\alpha\sinh\left[\alpha(z-L)\right]}{\alpha\cosh(\alpha L) + j\alpha\sinh(\alpha L)} e^{j\alpha z}$$

$$B(z) = A_0 \frac{j\kappa_G \sinh\left[\alpha(z-L)\right]}{\alpha\cosh(\alpha L) + j\alpha\sinh(\alpha L)} e^{-j\alpha z}$$

$$\alpha = \sqrt{\kappa_G^2 - \varphi^2}$$

The normalized forward and backward power

$$P_f(z) = \frac{|A(z)|^2}{|A_0|^2} = \frac{\alpha^2 + \kappa_G^2 \sinh^2\left[\alpha(z-L)\right]}{\alpha^2 + \kappa_G^2 \sinh^2(\alpha L)}$$

$$P_b(z) = \frac{|B(z)|^2}{|A_0|^2} = \frac{\kappa_G^2 \sinh^2\left[\alpha(z-L)\right]}{\alpha^2 + \kappa_G^2 \sinh^2(\alpha L)}$$

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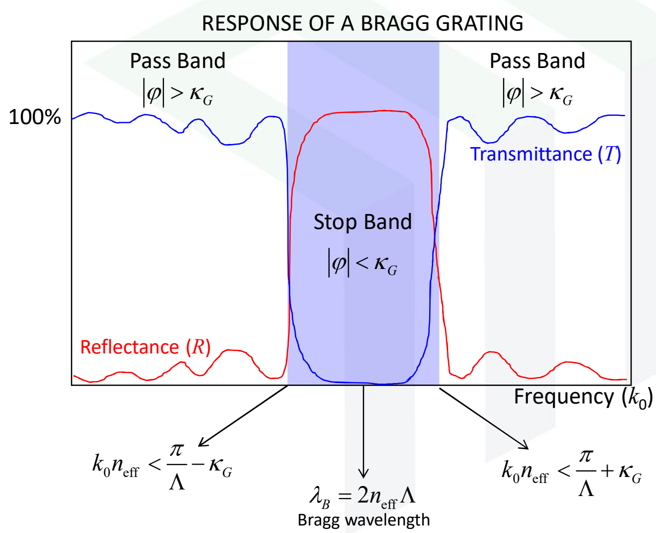
Slide 39

Typical Bragg Response



$$R = \frac{|B(0)|^2}{|A_0|^2}$$

$$T = \frac{|A(L)|^2}{|A_0|^2}$$

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Slide 40

Non-Directional Coupling

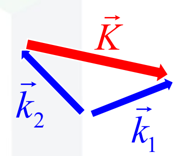
Slide 41

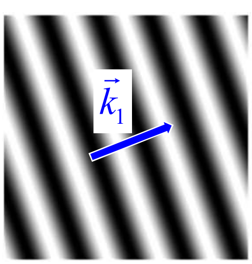
Non-Directional Coupling

It turns out that we can couple waves travelling in different directions. This is called non-directional coupling.

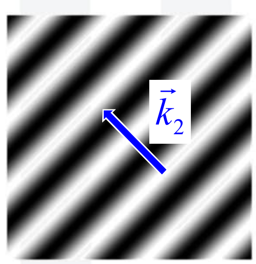
$$\vec{K} = \pm\left(\vec{k}_1 - \vec{k}_2\right)$$

Grating vectors in opposite directions describe the same grating.

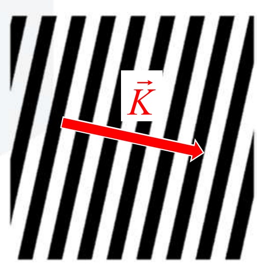




Wave 1



Wave 2



Grating that would couple wave 1 and wave 2

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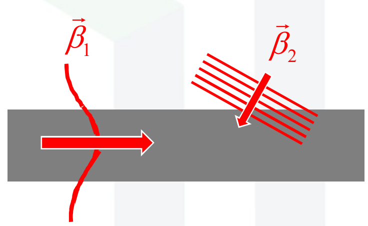
42

Phase Matching with Gratings

Generalized Framework

How can two completely different modes be force to couple so that they can exchange power efficiently? Ordinarily, this will not happen.



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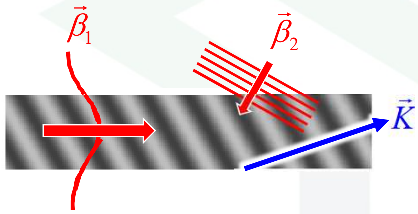
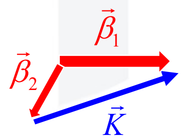
Phase Matching

Any two modes can be coupled using a grating.



The phase matching condition to couple power between two modes is

$$\vec{K} = \pm\left(\vec{\beta}_1 - \vec{\beta}_2\right)$$



Grating Coupler Regimes

Short period gratings
Bragg gratings
Contradirectional coupling

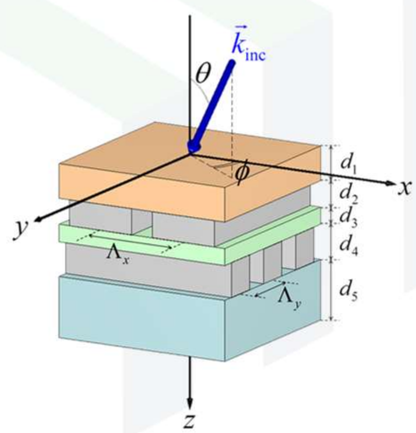
"Medium" period gratings
Non-directional coupling

Long period gratings
Codirectional coupling

Mode-Matching Vs. Coupled-Wave

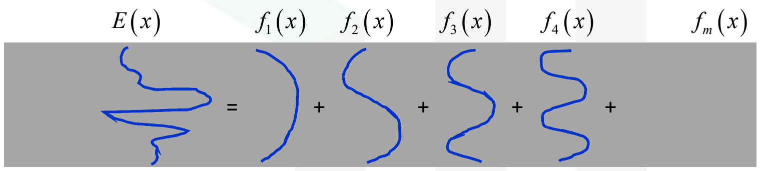
Frameworks to Model Propagation

Both mode-matching and coupled-mode frameworks view devices as consisting of a series of segments that are uniform in the z-direction.



Mode-Matching Framework (1 of 3)

Mode matching views the field in a segment as being the sum of a set of orthogonal basis functions (eigen-modes).

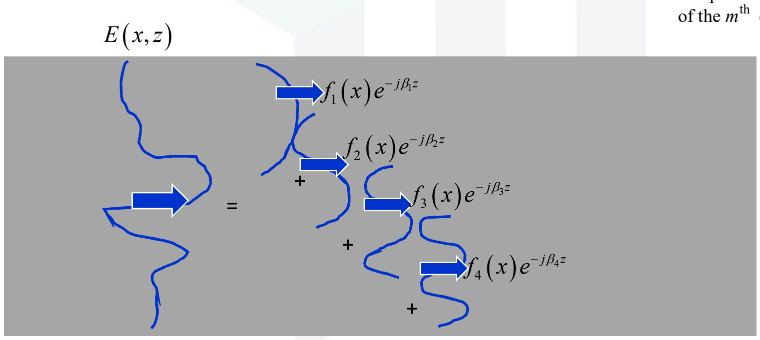


$$E(x) = \sum_m a_m f_m(x)$$

Mode-Matching Framework (2 of 3)

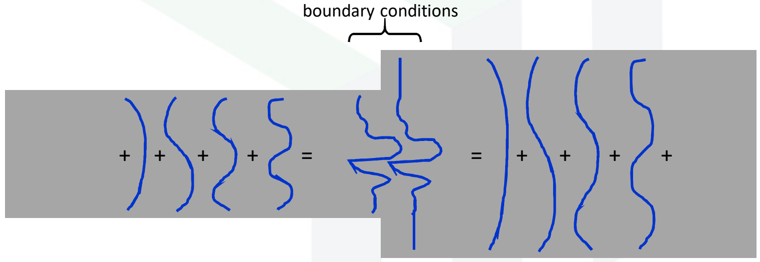
The modes within a segment accumulate phase differently as they propagate, but they do not interact and they propagate independently.

$$E(x,z) = \sum_m a_m \underbrace{f_m(x) e^{-j\beta_m z}}_{\substack{\text{complete description} \\ \text{of the } m^{\text{th}} \text{ eigen-mode}}}$$



Mode-Matching Framework (3 of 3)

At an interface, the power redistributes itself among the eigen-modes in the next segment.



$$E_1\left(x, z_0\right) = E_2\left(x, z_0\right)$$

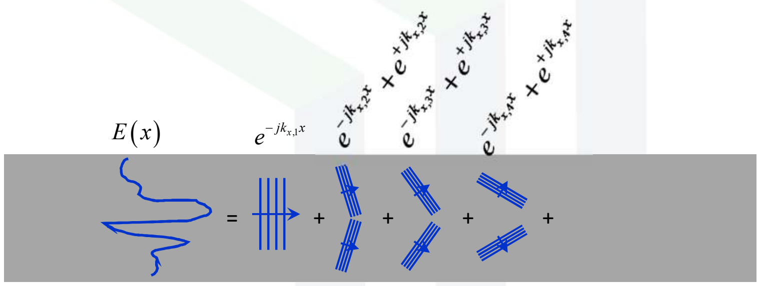
$$\sum_m a_{1,m} f_{1,m}\left(x\right) e^{-j\beta_{1,m} z_0} = \sum_m a_{2,m} f_{2,m}\left(x\right) e^{-j\beta_{2,m} z_0}$$

Conclusions About Mode-Matching

- The mode-matching framework applies to more than waveguides
  - Metamaterials, gratings, electromagnetic band gap materials, frequency selective surfaces, transmission lines, guided-mode resonance filters, photonic crystals, and more.
- Modes do not interact and they propagate independently with their own propagation constant.
- Power among the modes “scrambles” at an interface.
- The overall field is the sum of the eigen-modes

Coupled-Wave Framework (1 of 3)

Coupled-wave views the field in a segment as being the sum of a set of plane wave basis functions.

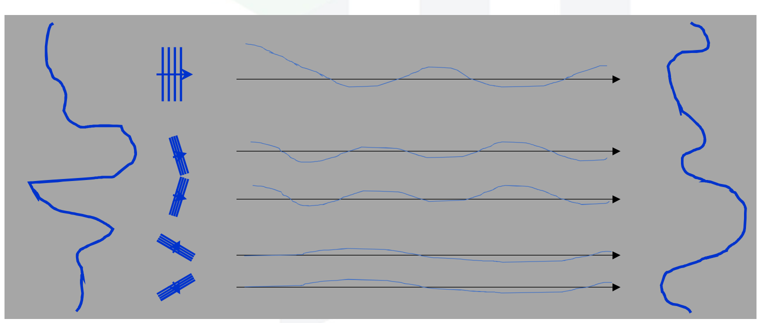


$$E(x)=\sum_m a_m e^{-jk_{x,m}x}$$

Coupled-Wave Framework (2 of 3)

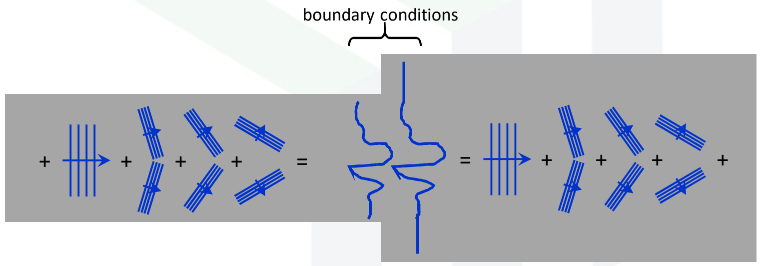
The waves within a segment are coupled. So, in addition to accumulating phase as they propagate, they also interact by exchanging power (coupled). The mode coefficients are therefore a function of z.

$$E(x,z)=\sum_m a_m(z)e^{-jk_{x,m}x}$$



Coupled-Wave Framework (3 of 3)

At an interface, the amplitudes of the plane waves on either side remain the same to enforce boundary conditions. This is because the same basis functions are being used on both sides..



$$E_1(x,z_0) = E_2(x,z_0)$$

$$\sum_m a_{1,m}(z)e^{-jk_{x,m}x} = \sum_m a_{2,m}(z)e^{-jk_{x,m}x}$$

EMPossible

Conclusions about Coupled-Wave

- The coupled-mode framework applies to more than waveguides
  - Metamaterials, gratings, electromagnetic band gap materials, frequency selective surfaces, transmission lines, guided-mode resonance filters, photonic crystals, and more.
- Modes can interact. In addition to accumulating phase as they propagate, modes can exchange power.
- Nothing interesting happens at an interface as the amplitudes of the modes remain constant across the interface (ignoring reflections)
- The overall field is the sum of the basis functions

EMPossible

How Are These Two Models Reconciled?

Plane waves do not exist in inhomogeneous materials.

If they are forced to exist, they exist in “sets” and the plane waves exchange power as they propagate.

In this sense, think of the modes as the set of plane waves that propagate independently of other sets of plane waves.

This transforms the coupled-mode framework to the mode-matching frame work.