Computational Science:
Introduction to Finite-Difference Time-Domain

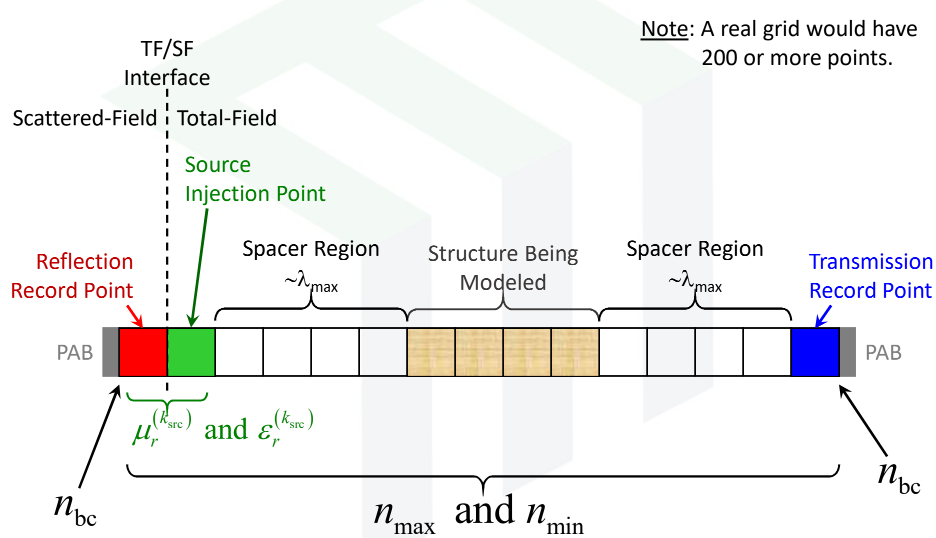
Examples of One-Dimensional FDTD

Lecture Outline

- Review
  - FDTD Algorithm
  - Code walkthrough
- Simple Electromagnetic Structures
- Two Examples

Review of Lecture #8

Typical FDTD Grid Layout



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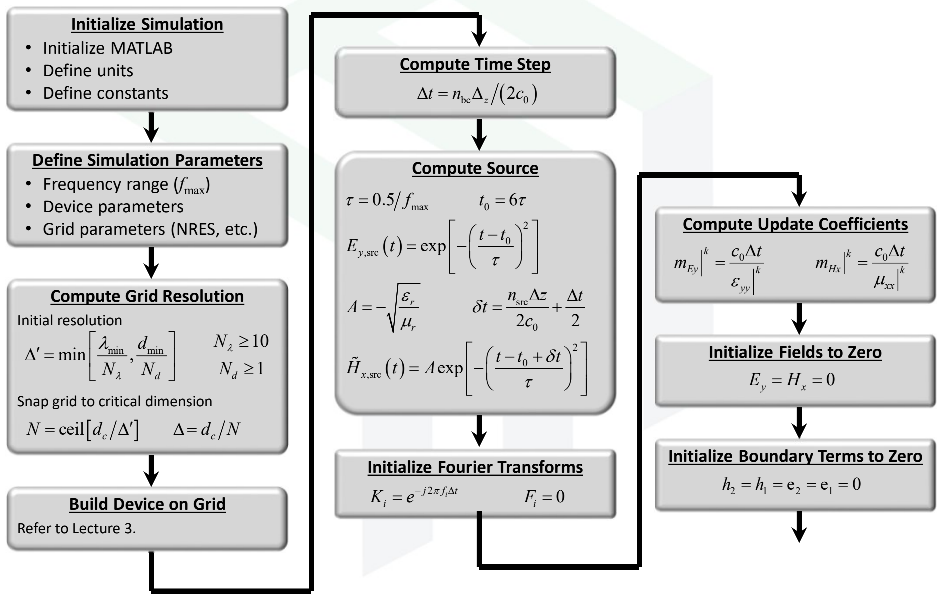
Initializing the FDTD Simulation



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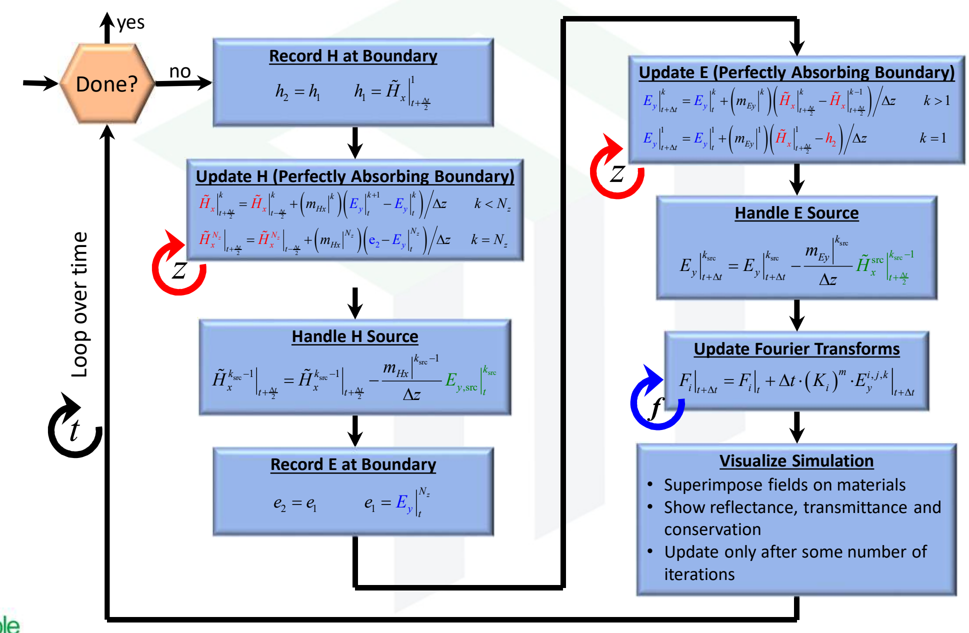
The Main FDTD Loop



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The Main FDTD Loop (Pseudo Code)

```
% MAIN FDTD LOOP
for T = 1 : STEPS

    % Record H-Field at Boundary
    H2←Hx(1)

    % Update H from E
    for nz = 1 : Nz
        Update Hx(nz)
    end

    % H Source
    Correct Hx(nz_src-1)

    % Record E-Field Boundary
    E2←Ey(Nz)

    % Update E from H
    for nz = 1 : Nz
        Update Ey(nz)
    end

    % E Source
    Correct Ey(nz_src)

    % Update Fourier Transforms
    for nf = 1 : NFREQ
        Integrate REF(nf), TRN(nf), and SRC(nf)
    end

    % Visualize
    Plot fields, materials, and response

end
```

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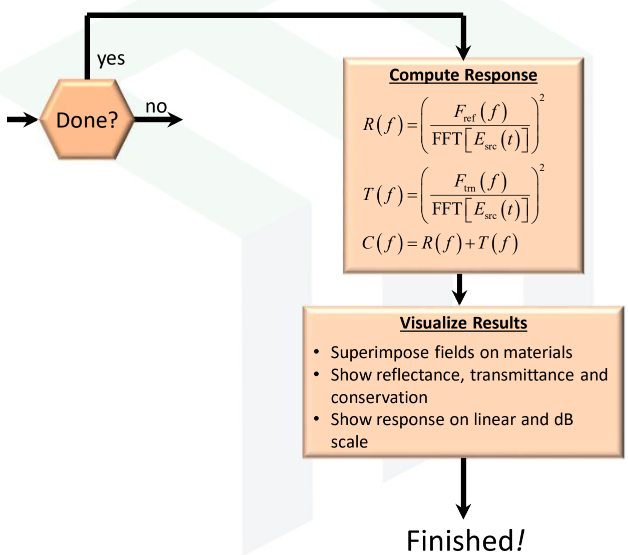
Post Processing



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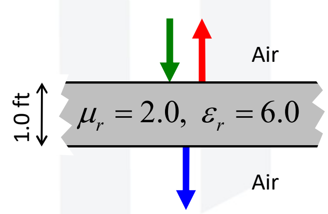
Outline of Steps for FDTD Analysis

- Step 1: Define problem
  - What device are you modeling?
  - What is its geometry?
  - What materials is it made of?
  - What do you want to learn about the device?
- Step 2: Initialize FDTD
  - Compute grid resolution
  - Assign materials values to points on the grid
  - Compute time step
  - Initialize Fourier transforms
- Step 3: Run FDTD
- Step 4: Analyze the data

Step 1: Define the Problem

What device are you modeling?	–A dielectric slab
What is its geometry?	–1 foot thick slab
What materials it is made from?	–μ_r=2.0, ε_r=6.0 (outside is air)
What do you want to learn?	–reflectance and transmittance from 0 to 1 GHz



Step 2: Compute Grid (1 of 2)

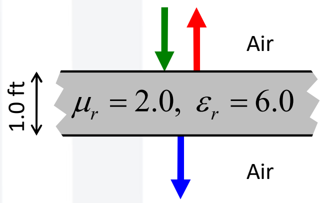
Initial Grid Resolution (Wavelength)

$$N_\lambda = 20$$

$$n_{\max} = \sqrt{\mu_r \varepsilon_r} = \sqrt{(2.0)(6.0)} = 3.46$$

$$\lambda_{\min} = \frac{c_0}{f_{\max} n_{\max}} = \frac{299792458 \ \frac{\text{m}}{\text{s}}}{(1.0\ \text{GHz})(3.46)} = 8.6543\ \text{cm}$$

$$\Delta_\lambda = \frac{\lambda_{\min}}{N_\lambda} = \frac{8.6543\ \text{cm}}{20} = 0.4327\ \text{cm}$$



Initial Grid Resolution (Structure)

$$N_d = 4$$

$$\Delta_d = \frac{d}{N_d} = \frac{30.48\ \text{cm}}{4} = 7.6200\ \text{cm}$$

Initial Grid Resolution (Overall)

$$\Delta z' = \min\left[\Delta_\lambda, \Delta_d\right] = 0.4327\ \text{cm}$$

Step 2: Compute Grid (2 of 2)

Snap Grid to Critical Dimension(s)

The number of grid cells representing the thickness of the dielectric slab is

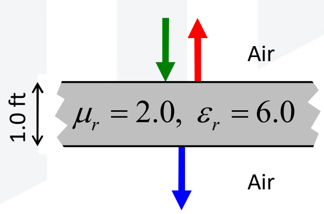
$$N' = \frac{d_c}{\Delta z'} = \frac{30.48\ \text{cm}}{0.4327\ \text{cm}} = 70.44\ \text{cells}$$

It is impossible to represent the thickness of the slab exactly with this grid resolution.

To represent the thickness of the slab exactly, we round N' up to the nearest integer and then calculate the grid resolution based on this quantity.

$$N = \text{round}\uparrow\left[N'\right] = 71\ \text{cells}$$

$$\Delta z = \frac{d_c}{N} = \frac{30.48\ \text{cm}}{71} = 0.4293\ \text{cm}$$



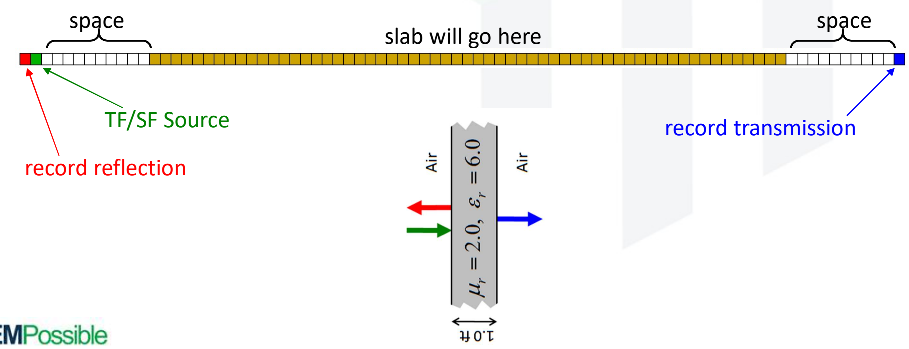
Step 2: Build Device on the Grid (1 of 2)

Determine Size of Grid

We need to have enough grid cells to fit the device being modeled, some space on either side of the device (10 cells for now), and cells for injecting the source and recording transmitted and reflected fields.

$$N_z = 71 + 2(10 \text{ cells}) + 3 = 94 \text{ cells}$$

- TRN/REF/SRC cells
- spacer regions
- cells for device

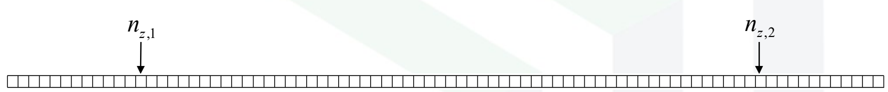


Step 2: Build Device on the Grid (2 of 2)

Compute Position of Materials on Grid

$$n_{z,1} = 2 + 10 + 1 = 13$$

$$n_{z,2} = n_{z,1} + \text{round}[d/\Delta z] - 1 = 13 + 71 - 1 = 83$$



Add Materials to Grid


```
UR(nz1:nz2) = ur;
ER(nz1:nz2) = er;
```

Step 2: Initialize FDTD (1 of 2)

Compute the Time Step

$$\Delta t = \frac{n_{bc}\Delta z}{2c_0} = \frac{(1.0)(0.4293\text{ cm})}{2\left(299792458\ \frac{\text{m}}{\text{s}}\right)} = 7.1599\times10^{-12}\text{ sec}$$

Compute Source Parameters

$$\tau = \frac{1}{2f_{max}} = \frac{1}{2(1\text{ GHz})} = 5.00\times10^{-10}\text{ sec}$$

$$t_0 = 6\tau = 3.00\times10^{-9}\text{ sec}$$

$\boxed{t_0 = 6\tau}$ Rule of thumb

Compute Number of Time Steps

$$t_{prop} = \frac{n_{max}N_z\Delta z}{c_0} = \frac{(3.46)(94)(0.4293\text{ cm})}{\left(299792458\ \frac{\text{m}}{\text{s}}\right)} = 4.6629\times10^{-9}\text{ sec}$$

Time it takes a wave to propagate across the grid.

$$T = 12\tau + 5t_{prop} = 12\left(5\times10^{-10}\text{ s}\right) + 5\left(4.6597\times10^{-9}\text{ s}\right) = 2.9314\times10^{-8}\text{ sec}$$

$$\text{STEPS} = \text{round}\uparrow\left[\frac{T}{\Delta t}\right] = 4095$$

$\boxed{T = 12\tau + 5t_{prop}}$ Rule of thumb

STEPS must be an integer

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Step 2: Initialize FDTD (2 of 2)

Compute the Source Functions for E_y/H_x Mode

$$\delta t = \frac{n_{src}\Delta z}{2c_0} + \frac{\Delta t}{2} = \frac{3\Delta t}{2} = 1.0740\times10^{-11}\text{ sec}$$

$$A = -\sqrt{\frac{\varepsilon_r^{(k_{src})}}{\mu_r^{(k_{src})}}} = -\sqrt{\frac{1.0}{1.0}} = -1$$

$$E_y(t) = \exp\left[-\left(\frac{t-t_0}{\tau}\right)^2\right]$$

$$\tilde{H}_x(t) = A\exp\left[-\left(\frac{t-t_0+\delta t}{\tau}\right)^2\right]$$

```
% COMPUTE GAUSSIAN SOURCE FUNCTIONS
t    = [0:STEPS-1]*dt;                          %time axis
delt = nsrc*dz/(2*c0) + dt/2;                   %total delay between E and H
A    = - sqrt(ersrc/ursrc);                     %amplitude of H field
Esrc = exp(-((t-t0)/tau).^2);                   %E field source
Hsrc = A*exp(-((t-t0+delt)/tau).^2);            %H field source
```

Initialize the Fourier Transforms

```
% INITIALIZE FOURIER TRANSFORMS
NFREQ = 100;
FREQ  = linspace(0,1*gigahertz,NFREQ);
K     = exp(-i*2*pi*dt*FREQ);
REF   = zeros(1,NFREQ);
TRN   = zeros(1,NFREQ);
SRC   = zeros(1,NFREQ);
```

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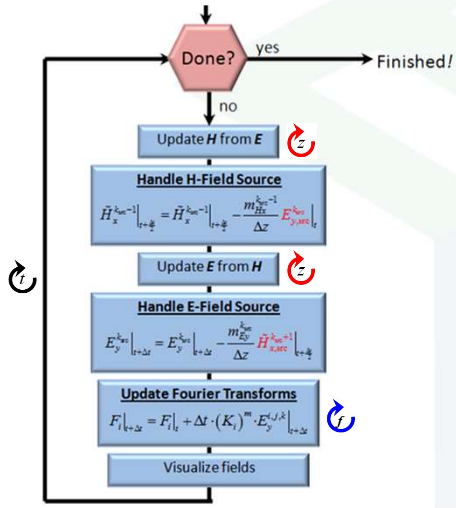
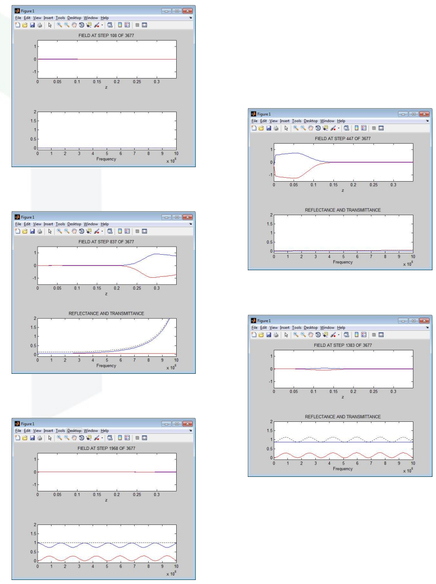
Step 3: Run FDTD (3 of 3)





Step 4: Analyze the Data

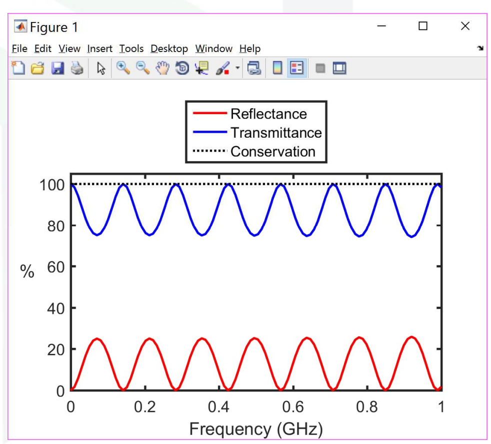
Normalize the Data to the Source Spectrum

$$R(f)=\left(\frac{F_{\text{ref}}(f)}{\text{FFT}\left[E_{\text{src}}(t)\right]}\right)^2$$

$$T(f)=\left(\frac{F_{\text{trn}}(f)}{\text{FFT}\left[E_{\text{src}}(t)\right]}\right)^2$$

$$C(f)=R(f)+T(f)$$

```
% COMPUTE REFLECTANCE
% AND TRANSMITTANCE
REF = abs(REF./SRC).^2;
TRN = abs(TRN./SRC).^2;
CON = REF + TRN;
```



Simple Electromagnetic Structures

Reflection and Transmission at an Interface

Reflection and Transmission Coefficients

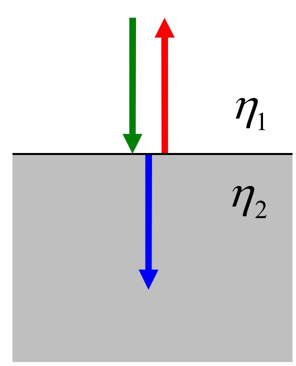
At normal incidence, the field amplitude of waves reflected from, or transmitted through, an interface are related to the incident wave through the reflection and transmission coefficients.

$$r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \qquad t = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Reflectance and Transmittance

The reflectance and transmittance quantify the fraction of power that is reflected from, or transmitted through, an interface.

$$R = |r|^2 \qquad T = |t|^2$$



Useful Special Cases

For ε_r=9.0 and μ_r=1.0, R=25% and T=75%.
For ε_r=1.0 and μ_r=9.0, R=25% and T=75%.
For ε_r=9.0 and μ_r=9.0, R=0% and T=100%.

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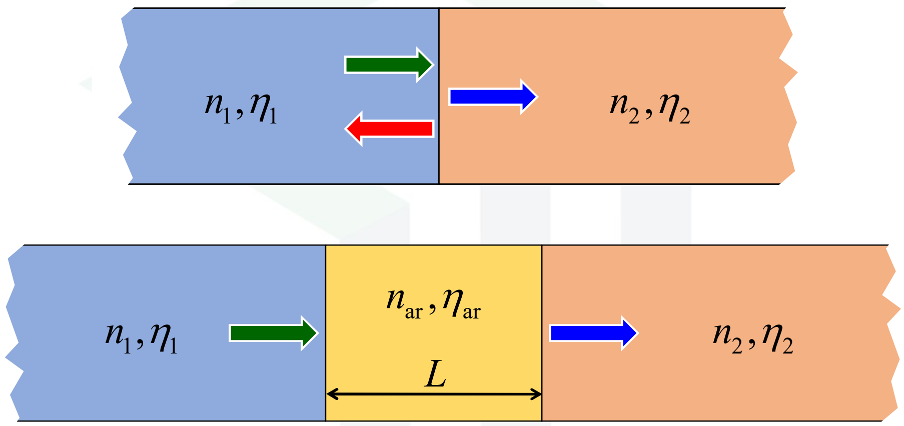
Anti-Reflection Layer



General Case

$$\eta_{ar} = \sqrt{\eta_1 \eta_2} \qquad L = \frac{\lambda_0}{4 n_{ar}}$$

No magnetic response

$$n_{ar} = \sqrt{n_1 n_2} \qquad L = \frac{\lambda_0}{4 n_{ar}}$$

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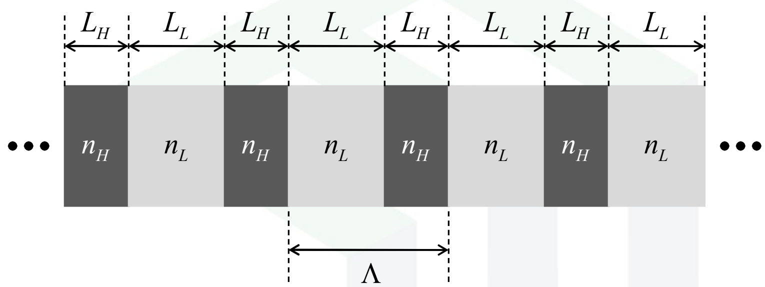
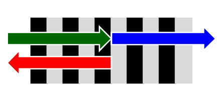
Bragg Gratings

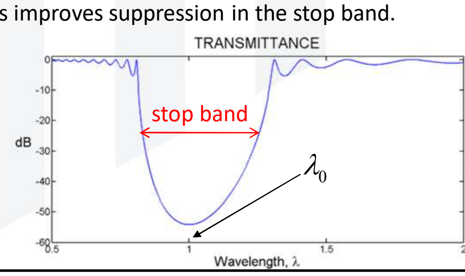


$$L_L = \frac{\lambda_0}{4 n_L}$$

$$L_H = \frac{\lambda_0}{4 n_H}$$

A Bragg grating is typically composed of alternating layers of high and low refractive index. Each layer is λ/4 thick. Higher index contrast provides wider stop band. More layers improves suppression in the stop band.





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Example #1:
The Invisible Slab

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Design Problem

A radome is being designed to protect an antenna operating at
2.4 GHz. For mechanical reasons, it must be constructed from 1 ft thick plastic with dielectric constant 12. How could you modify the design to maximize transmission through the radome? Simulate the design using 1D FDTD.

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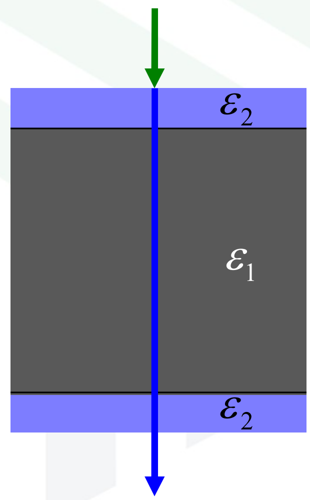
A Solution

Add anti-reflection layers to both sides of the radome.



The Design

To match the slab material to air on both sides, the dielectric constant and thickness of the anti-reflection layers should be

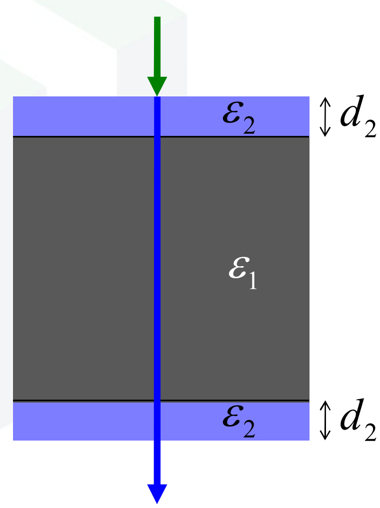
$$\varepsilon_2 = \sqrt{\varepsilon_1 \varepsilon_{\text{air}}} = \sqrt{(12)(1)} = \boxed{3.46}$$

$$n_2 = \sqrt{\varepsilon_2} = \sqrt{3.46} = 1.86$$

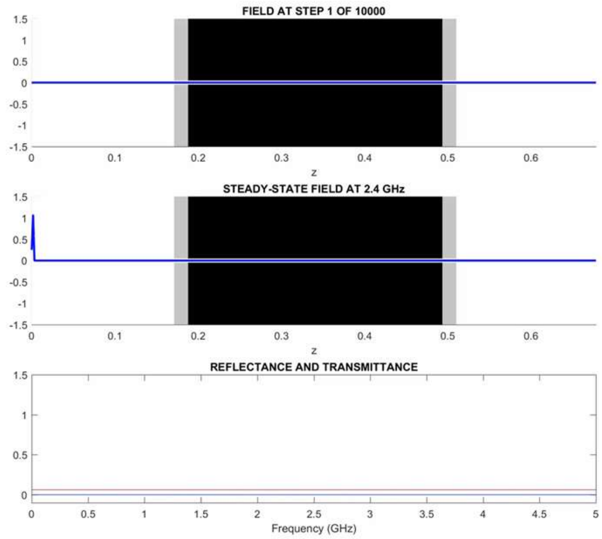
$$\lambda_0 = \frac{c_0}{f_0} = \frac{299792458\ \frac{\text{m}}{\text{s}}}{2.4\times10^9\ \text{Hz}}$$

$$= \frac{299792458\ \frac{\text{m}}{\text{s}}}{2.4\times10^9\ \text{Hz}} = 12.49\ \text{cm}$$

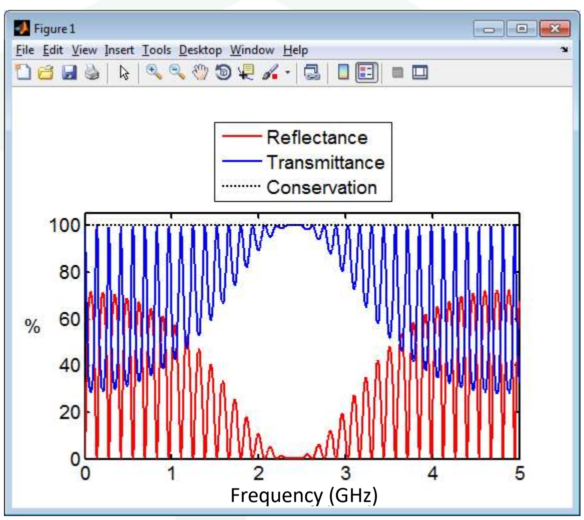
$$d_2 = \frac{\lambda_0}{4n_2} = \frac{12.49\ \text{cm}}{4(1.86)} = \boxed{1.6779\ \text{cm}}$$



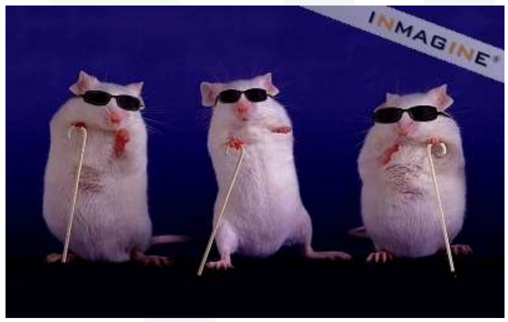
FDTD Simulation



Final Transmittance/Reflectance/Conservation



Example #2: The Blinded Missile



Design Problem

A heat-seeking missile is vulnerable to jamming from high power lasers operating at λ_0=980 nm. Design a multilayer cover that would prevent this energy from reaching the infrared camera. The design should provide at least 30 dB of suppression at 980 nm. Simulate the design using 1D FDTD. The only materials available to you are SiO_2 (n_{SiO2} = 1.5) and SiN (n_{SiN} = 2.0).

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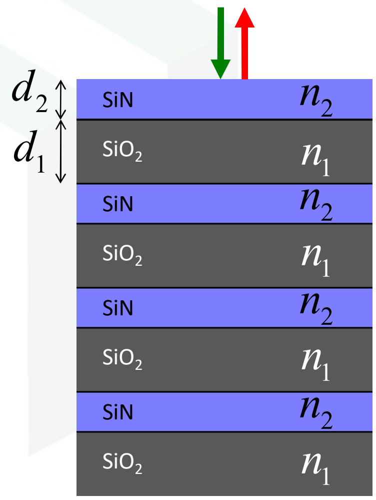
A Solution

Use a Bragg grating with alternating layers of SiO_2 and SiN.

$$n_{\mathrm{SiO_2}} = 1.5$$
$$n_{\mathrm{SiN}} = 2.0$$



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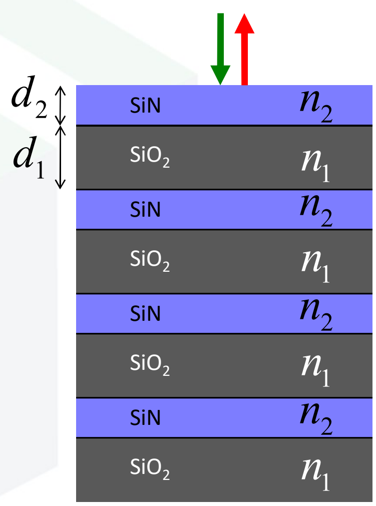
The Design

$$d_1 = \frac{\lambda_0}{4n_1} = \frac{980\ \mathrm{nm}}{4(1.5)} = 163\ \mathrm{nm}$$
$$d_2 = \frac{\lambda_0}{4n_2} = \frac{980\ \mathrm{nm}}{4(2.0)} = 122\ \mathrm{nm}$$

But how many layers?



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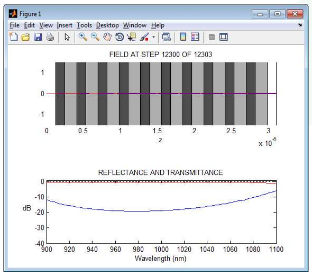
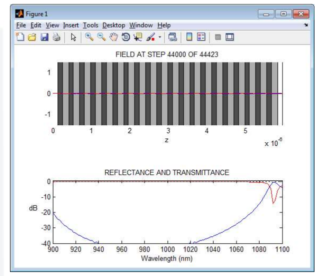
Number of Layers for 30 dB Suppression

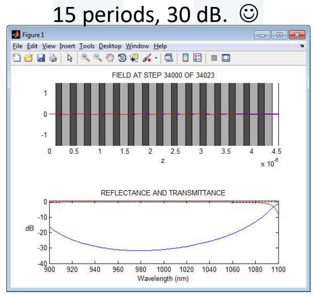
10 periods, barely 20 dB ☹



20 periods, 45 dB! Yikes!!



15 periods, 30 dB. ☺



In practice, you may want to include a few extra layers as a safety margin.

Manufacturing inaccuracies often degrade performance.

FDTD Simulation Results