Computational Science: Introduction to Finite-Difference Time-Domain

Examples of One-Dimensional FDTD

Lecture Outline

• Review
  • FDTD Algorithm
  • Code walkthrough
• Simple Electromagnetic Structures
• Two Examples
Review of Lecture #8

Typical FDTD Grid Layout

Note: A real grid would have 200 or more points.
Initializing the FDTD Simulation

- Initialize Simulation
  - Initialize MATLAB
  - Define units
  - Define constants

Define Simulation Parameters
- Frequency range ($f_{max}$)
- Device parameters
- Grid parameters (NRES, etc.)

Compute Grid Resolution
Initial resolution
$$\Delta' = \min \left[ \frac{d_{inc}}{N_x}, \frac{d_{inc}}{N_y} \right]$$
Snap grid to critical dimension
$$N = \text{ceil} \left[ d_{inc} / \Delta' \right]$$
$$\Delta = d_{inc} / N$$

Build Device on Grid
Refer to Lecture 3.

The Main FDTD Loop

Record $E$ at Boundary
$$E_1 = E_{1c}$$
$$E_{1b} = H_{1b}$$

Loop over time
$$t = t + \Delta t$$

Record $H$ at Boundary
$$H_1 = H_{1c}$$
$$H_{1b} = H_{1b}$$

Update $H$ (Perfectly Absorbing Boundary)
$$H_{k+1} = H_{k+1} + \frac{m_y}{\Delta z} \left[ E_{k+1} - E_{k} \right]$$
$$k > 1$$
$$H_{k+1} = H_{k+1} + \frac{m_y}{\Delta z} \left[ E_{k+1} - E_{k} \right]$$
$$k = 1$$

Handle $H$ Source
$$H_{k+1} = H_{k+1} - \frac{m_y}{\Delta z} \left[ E_{k+1} - E_{k} \right]$$

Update $E$ (Perfectly Absorbing Boundary)
$$E_{k+1} = E_{k+1} - \frac{m_y}{\Delta z} \left[ H_{k+1} - H_{k} \right]$$
$$k > 1$$
$$E_{k+1} = E_{k+1} - \frac{m_y}{\Delta z} \left[ H_{k+1} - H_{k} \right]$$
$$k = 1$$

Handle $E$ Source
$$E_{k+1} = E_{k+1} - \frac{m_y}{\Delta z} \left[ H_{k+1} - H_{k} \right]$$

Update Fourier Transforms
$$E_k^{+} = E_k^{+} + \Delta \left( \frac{\epsilon}{\mu} \right) E_k^{+}$$
$$E_k^{-} = E_k^{-} + \Delta \left( \frac{\epsilon}{\mu} \right) E_k^{-}$$

Visualize Simulation
- Superimpose fields on materials
- Show reflectance, transmittance and conservation
- Update only after some number of iterations
The Main FDTD Loop (Pseudo Code)

```plaintext
% MAIN FDTD LOOP
for T = 1 : STEPS
    % Record H-Field at Boundary
    H2(1) = Hx(1)
    % Update H from E
    for nz = 1 : Nz
        Update Hx(nz)
    end
    % H Source
    Correct Hx(nz_src-1)
    % Record E-Field Boundary
    E2(1) = Ey(Nz)
    % Update E from H
    for nz = 1 : Nz
        Update Ey(nz)
    end
    % E Source
    Correct Ey(nz_src)
    % Update Fourier Transforms
    for nf = 1 : NFREQ
        Integrate REF(nf), TRN(nf), and SRC(nf)
    end
    % Visualize
    Plot fields, materials, and response
end
```

Post Processing

- Compute Response
  - $R(f) = \frac{F_E(f)}{\text{FFT}[E_n(t)]}$
  - $T(f) = \frac{F_E(f)}{\text{FFT}[E_n(t)]}$
  - $C(f) = R(f) + T(f)$

- Visualize Results
  - Superimpose fields on materials
  - Show reflectance, transmittance and conservation
  - Show response on linear and dB scale

Done? no yes

Finished!
Outline of Steps for FDTD Analysis

• Step 1: Define problem
  • What device are you modeling?
  • What is its geometry?
  • What materials is it made of?
  • What do you want to learn about the device?

• Step 2: Initialize FDTD
  • Compute grid resolution
  • Assign materials values to points on the grid
  • Compute time step
  • Initialize Fourier transforms

• Step 3: Run FDTD
• Step 4: Analyze the data

Step 1: Define the Problem

What device are you modeling? – A dielectric slab
What is its geometry? – 1 foot thick slab
What materials it is made from? – \( \mu_r = 2.0, \varepsilon_r = 6.0 \) (outside is air)
What do you want to learn? – reflectance and transmittance from 0 to 1 GHz
Step 2: Compute Grid (1 of 2)

Initial Grid Resolution (Wavelength)

\[ N_j = 20 \]

\[ n_{\text{max}} = \sqrt{\mu \varepsilon_r} = \sqrt{(2.0)(6.0)} = 3.46 \]

\[ \lambda_{\text{min}} = \frac{c_0}{f_{\text{max}} n_{\text{max}}} = \frac{299792458}{(1.0 \ \text{GHz})(3.46)} = 8.6543 \ \text{cm} \]

\[ \Delta_j = \frac{\lambda_{\text{min}}}{N_j} = \frac{8.6543 \ \text{cm}}{20} = 0.4327 \ \text{cm} \]

Initial Grid Resolution (Structure)

\[ N_d = 4 \]

\[ \Delta_j = \frac{d}{N_j} = \frac{30.48 \ \text{cm}}{4} = 7.6200 \ \text{cm} \]

Initial Grid Resolution (Overall)

\[ \Delta' = \min[\Delta_j, \Delta_j] = 0.4327 \ \text{cm} \]

Step 2: Compute Grid (2 of 2)

Snap Grid to Critical Dimension(s)

The number of grid cells representing the thickness of the dielectric slab is

\[ N' = \frac{d}{\Delta'} = \frac{30.48 \ \text{cm}}{0.4327 \ \text{cm}} = 70.44 \ \text{cells} \]

It is impossible to represent the thickness of the slab exactly with this grid resolution.

To represent the thickness of the slab exactly, we round \( N' \) up to the nearest integer and then calculate the grid resolution based on this quantity.

\[ N = \text{round } [N'] = 71 \ \text{cells} \]

\[ \Delta' = \frac{d}{N} = \frac{30.48 \ \text{cm}}{71} = 0.4293 \ \text{cm} \]
Step 2: Build Device on the Grid (1 of 2)

Determine Size of Grid

We need to have enough grid cells to fit the device being modeled, some space on either side of the device (10 cells for now), and cells for injecting the source and recording transmitted and reflected fields.

\[ N_z = 71 + 2(10 \text{ cells}) + 3 = 94 \text{ cells} \]

Add Materials to Grid

UR(nz1: nz2) = ur;
ER(nz1: nz2) = er;

Step 2: Build Device on the Grid (2 of 2)

Compute Position of Materials on Grid

\[ n_{z,1} = 2 + 10 + 1 = 13 \]
\[ n_{z,2} = n_{z,1} + \text{round} \left( \frac{d}{\Delta z} \right) - 1 = 13 + 71 - 1 = 83 \]

Add Materials to Grid

UR(nz1: nz2) = ur;
ER(nz1: nz2) = er;
Step 2: Initialize FDTD (1 of 2)

Compute the Time Step
\[
\Delta t = \frac{n_{\Delta z}}{2 c_0} = \frac{(1.0)(0.4293 \text{ cm})}{2(299792458 \frac{\text{cm}}{\text{s}})} = 7.1599 \times 10^{-13} \text{ sec}
\]

Compute Source Parameters
\[
\tau = \frac{1}{2 f_{\text{max}}} = \frac{1}{2(1 \text{ GHz})} = 5.00 \times 10^{-10} \text{ sec}
\]
\[
t_0 = 6\tau = 3.00 \times 10^{-9} \text{ sec}
\]
\[
t_0 = 6\tau \quad \text{Rule of thumb}
\]

Compute Number of Time Steps
\[
T_{\text{prop}} = \frac{n_{\Delta x} \Delta x}{c_0} = \frac{(3.46)(94)(0.4293 \text{ cm})}{299792458 \frac{\text{cm}}{\text{s}}} = 4.6629 \times 10^{-9} \text{ sec}
\]
\[
T = 12\tau + 5T_{\text{prop}} = 12\left(5 \times 10^{-10} \text{ s}\right) + 5(4.6597 \times 10^{-9} \text{ s}) = 2.9314 \times 10^{-8} \text{ sec}
\]
\[
\text{STEPS} = \text{round} \left( \frac{T}{\Delta t} \right) = 4095
\]
\[
\frac{T}{\Delta t} = 12\tau + 5T_{\text{prop}} \quad \text{Rule of thumb}
\]
\[
\text{STEPS} \text{ must be an integer}
\]

Step 2: Initialize FDTD (2 of 2)

Compute the Source Functions for \(E_y/H_x\) Mode
\[
\delta t = \frac{n_{\Delta y} \Delta y}{2 c_0} + \frac{3\Delta t}{2} = 1.0740 \times 10^{-11} \text{ sec}
\]
\[
A = \sqrt{\left(\frac{\epsilon_{(\omega=1)}}{\mu_{(\omega=1)}}\right)} = -\frac{1.0}{1.0} = -1
\]
\[
E_{y}(t) = \exp \left[ -\frac{(t-t_0)^2}{\tau} \right]
\]
\[
\tilde{H}_x(t) = A \exp \left[ -\frac{(t-t_0 + \delta t)^2}{\tau} \right]
\]

\% COMPUTE GAUSSIAN SOURCE FUNCTIONS
\[
t = (0:\text{STEPS}=1) \times \Delta t; \quad \text{\% time axis}
\]
\[
delt = nsrc \times dz/(2 \times c0) + dt/2; \quad \text{\% total delay between E and H}
\]
\[
A = - \text{sqrt}(\text{ersrc} / \text{ursrc}); \quad \text{\% amplitude of H field}
\]
\[
Esrc = \exp(-((t-t0)/tau).^2); \quad \text{\% E field source}
\]
\[
Hsrc = A*\exp(-(t-t0+delt)/tau).^2; \quad \text{\% H field source}
\]

Initialize the Fourier Transforms
\% INITIALIZE FOURIER TRANSFORMS
\[
\text{NFREQ} = 100;
\]
\[
\text{FREQ} = \text{linspace}(0,1*\text{gigahertz},\text{NFREQ});
\]
\[
\text{K} = \exp(-1i*2\pi*\text{dt}*\text{FREQ});
\]
\[
\text{REF} = \text{zeros}(1,\text{NFREQ});
\]
\[
\text{TRN} = \text{zeros}(1,\text{NFREQ});
\]
\[
\text{SRC} = \text{zeros}(1,\text{NFREQ});
\]
Step 3: Run FDTD (3 of 3)

Step 4: Analyze the Data

Normalize the Data to the Source Spectrum

\[
R(f) = \left( \frac{F_{\text{ref}}(f)}{\text{FFT}[E_{\text{src}}(t)]} \right)^2
\]

\[
T(f) = \left( \frac{F_{\text{trn}}(f)}{\text{FFT}[E_{\text{src}}(t)]} \right)^2
\]

\[
C(f) = R(f) + T(f)
\]

% COMPUTE REFLECTANCE
% AND TRANSMITTANCE
REF = abs(REF./SRC).^2;
TRN = abs(TRN./SRC).^2;
CON = REF + TRN;
Reflection and Transmission at an Interface

Reflection and Transmission Coefficients
At normal incidence, the field amplitude of waves reflected from, or transmitted through, an interface are related to the incident wave through the reflection and transmission coefficients.

\[ r = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad t = \frac{2\eta_2}{\eta_2 + \eta_1} \]

Reflectance and Transmittance
The reflectance and transmittance quantify the fraction of power that is reflected from, or transmitted through, an interface.

\[ R = |r|^2 \quad T = |t|^2 \]

Useful Special Cases
For \( \varepsilon_r = 9.0 \) and \( \mu_r = 1.0 \), \( R = 25\% \) and \( T = 75\% \).
For \( \varepsilon_r = 1.0 \) and \( \mu_r = 9.0 \), \( R = 25\% \) and \( T = 75\% \).
For \( \varepsilon_r = 9.0 \) and \( \mu_r = 9.0 \), \( R = 0\% \) and \( T = 100\% \).
**Anti-Reflection Layer**

![Diagram of Anti-Reflection Layer](image)

General Case:

\[ n_{at} = \sqrt{n_1 n_2} \]

\[ L = \frac{\lambda_0}{4 n_{at}} \]

No magnetic response:

\[ n_{at} = \frac{\lambda_0}{4 n_{at}} \]

**Bragg Gratings**

![Diagram of Bragg Gratings](image)

A Bragg grating is typically composed of alternating layers of high and low refractive index. Each layer is \( \lambda/4 \) thick. Higher index contrast provides wider stop band. More layers improves suppression in the stop band.

\[ L_L = \frac{\lambda_0}{4 n_L} \]

\[ L_H = \frac{\lambda_0}{4 n_H} \]
Example #1: The Invisible Slab

A radome is being designed to protect an antenna operating at 2.4 GHz. For mechanical reasons, it must be constructed from 1 ft thick plastic with dielectric constant 12. How could you modify the design to maximize transmission through the radome? Simulate the design using 1D FDTD.
A Solution

Add anti-reflection layers to both sides of the radome.

The Design

To match the slab material to air on both sides, the dielectric constant and thickness of the anti-reflection layers should be

\[ \varepsilon_2 = \sqrt{\varepsilon_1 \varepsilon_{\text{air}}} = \sqrt{(12)(1)} = 3.46 \]

\[ n_2 = \sqrt{\varepsilon_2} = \sqrt{3.46} = 1.86 \]

\[ \lambda_0 = \frac{c_0}{f_0} = \frac{299792458 \text{ m/s}}{2.4 \times 10^9 \text{ Hz}} = 12.49 \text{ cm} \]

\[ d_2 = \frac{\lambda_0}{4n_2} = \frac{12.49 \text{ cm}}{4(1.86)} = 1.6779 \text{ cm} \]
FDTD Simulation

Final Transmittance/Reflectance/Conservation
Example #2: The Blinded Missile

Design Problem

A heat-seeking missile is vulnerable to jamming from high power lasers operating at $\lambda_0=980$ nm. Design a multilayer cover that would prevent this energy from reaching the infrared camera. The design should provide at least 30 dB of suppression at 980 nm. Simulate the design using 1D FDTD. The only materials available to you are SiO$_2$ ($n_{\text{SiO}_2} = 1.5$) and SiN ($n_{\text{SiN}} = 2.0$).
A Solution

Use a Bragg grating with alternating layers of SiO₂ and SiN.

\[ n_{\text{SiO}_2} = 1.5 \]
\[ n_{\text{SiN}} = 2.0 \]

The Design

\[ d_1 = \frac{\lambda}{4n_1} = \frac{980 \text{ nm}}{4(1.5)} = 163 \text{ nm} \]
\[ d_2 = \frac{\lambda}{4n_2} = \frac{980 \text{ nm}}{4(2.0)} = 122 \text{ nm} \]

But how many layers?
Number of Layers for 30 dB Suppression

10 periods, barely 20 dB 😊

20 periods, 45 dB! Yikes!!

15 periods, 30 dB. 😊

In practice, you may want to include a few extra layers as a safety margin.

Manufacturing inaccuracies often degrade performance.

FDTD Simulation Results