Advanced Electromagnetics: 21st Century Electromagnetics

Guided-Mode Resonance

Lecture Outline

• Physics of Guided-Mode Resonance (GMR)
• GMR Filters
• Design of GMR Filters
• Applications
The Critical Angle and Total Internal Reflection

When an electromagnetic wave is incident on a material with a lower refractive index, it is totally reflected when the angle of incidence is greater than the critical angle.

\[ \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \]

**Example**
What is the critical angle for fused silica (glass)?

The refractive index at optical waveguides is around 1.5.

\[ \theta_c = \sin^{-1} \left( \frac{1.0}{1.5} \right) = 41.81^\circ \]
The Slab Waveguide

If a slab of material is placed between two materials with lower refractive index, a slab waveguide is created.

\[ n_1 > n_2 \quad \text{and} \quad n_2 > n_3 \]

Ray Tracing Analysis

\[ \beta = k_0 n_{\text{eff}} = k_0 n \sin \theta \]

The round trip phase of a ray must be an integer multiple of \(2\pi\). Only certain angles are allowed to propagate in the waveguide.
Rigorous Analysis

A rigorous analysis of slab waveguides involves Maxwell’s equations.

\[
\nabla \times \vec{E} = -j\omega \mu \vec{H} \quad \nabla \times \vec{H} = j\omega \varepsilon \vec{E}
\]

The geometry and mode solutions for a typical slab waveguide are

---

Diffraction from Gratings

The angles of the diffracted modes are related to the wavelength and grating through the grating equation.

The grating equation only predicts the directions of the modes, not how much power is in them.

**Reflection Region**

\[
n_{\text{ref}} \sin \left[ \theta(m) \right] = n_{\text{inc}} \sin \theta_{\text{inc}} - m \frac{\lambda}{\Lambda} \sin \phi
\]

**Transmission Region**

\[
n_{\text{trans}} \sin \left[ \theta(m) \right] = n_{\text{inc}} \sin \theta_{\text{inc}} - m \frac{\lambda}{\Lambda} \sin \phi
\]
GMR = Diffraction + Waveguide

Question
What happens when a diffraction grating and slab waveguide are brought into proximity and the angle of a diffracted mode matches the angle of a guided mode?

Guided-Mode Resonance

Away From Resonance

At resonance, part of the applied wave is coupled into a guided mode. The guided mode slowly “leaks” out from the waveguide. The “leaked” wave interferes with the applied wave to produce a filtering response.

Away from resonance, the structure exhibits the “background” response of a multilayer device.
Regions of Guided-Mode Resonance (Derivation)

Recall the grating equation

\[ n_2 \sin \theta(m) = n_1 \sin \theta_{inc} - m \frac{\lambda_0}{\Lambda} \sin \phi \]

Recall from the ray tracing picture that

\[ \beta_n = k_n n_{\text{eff}} = k_n n_2 \sin \theta(m) \]

Therefore

\[ n_{\text{eff}} = n_1 \sin \theta_{inc} - m \frac{\lambda_0}{\Lambda} \sin \phi \]

Conditions for \( n_{\text{eff}} \) to represent a guided mode

\[ \max [n_1, n_2] \leq n_{\text{eff}} < n_2 \]

Combining the above two equations leads to an equation describing the regions of resonance for guided-mode resonance.

\[ \max [n_1, n_3] \leq \left| n_1 \sin \theta_{inc} - m \frac{\lambda_0}{\Lambda} \sin \phi \right| < n_2 \]

Regions of Guided-Mode Resonance (Plot)

- Estimates ranges for resonant frequencies
- Predicts sensitivity to angle of incidence
- Shows how higher order resonances overlap
- Zero-order modes produces no resonance effects.

Center wavelength at normal incidence:

\[ \frac{\lambda_0}{\Lambda} = \frac{n_1 + \max [n_1, n_3]}{2 \sin \phi} \]

Bounds:

\[ \frac{\lambda_{\text{inc}}}{\Lambda} \leq \frac{\lambda_{\text{inc}}}{\Lambda} \]

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Benefits and Drawbacks

• Benefits
  • All-dielectric for very low loss
  • Extremely strong response from dielectrics
  • Can be made monolithic
  • Potentially better for high power than using metals

• Drawbacks
  • Larger and bulkier than equivalent metallic structures
  • Limited field-of-view and bandwidth compared to metallic devices
  • Response is very sensitive to material properties and structural deformations

Guided-Mode Resonance Filters
Various GMR Filters

A guided-mode resonance (GMR) filter is both a diffraction grating and slab waveguide.

A resonance occurs when a diffracted mode exactly matches a guided mode.

Away from resonance, the device behaves like an ordinary multilayer structure.

On resonance, the device reverses the background response (roughly speaking).

Effect of Index Contrast

Width of the resonance becomes more narrow as index contrast is lowered. Position of the resonance can change slightly.

\[ \begin{align*}
    n_1 &= 2.0 \\
    n_2 &= 1.52 \\
    d &= 275 \text{ nm} \\
    \Lambda &= 358.9 \text{ nm} \\
    f &= 50\% \\
    n_1 &= n_1 - \Delta n/2 \\
    n_2 &= n_1 + \Delta n/2 \\
\end{align*} \]
Sensitivity to Angle of Incidence (1 of 2)

Grating Equation

\[ n_1 \sin \theta_n = n_\text{inc} \sin \theta_\text{inc} - m \frac{\lambda}{\Lambda} \sin \phi \]

Sensitivity to Angle of Incidence

\[ \frac{\partial \lambda}{\partial \theta_\text{inc}} = \frac{\Lambda}{m \sin \phi} \]

We make the small angle approximation: \( \sin \theta_\text{inc} \approx \theta_\text{inc} \)

\[ \frac{\partial \lambda}{\partial \theta_\text{inc}} = \frac{\Lambda n_\text{inc}}{m \sin \phi} = \frac{\Lambda n_\text{inc}}{m} \]

Example

\[ \frac{\partial \lambda}{\partial \theta_\text{inc}} = \frac{\Lambda n_\text{inc}}{m} = \frac{(358.9 \text{ nm})(1.0)}{1} = 358.9 \text{ mm} \]

\[ = (358.9 \text{ mm}) \left( \frac{\pi \text{ rad}}{180^\circ} \right) = 6.3 \text{ mm} \]

\[ \Delta \lambda, \Delta \theta_\text{inc} \approx \frac{\Lambda}{m} \]

Sensitivity to Angle of Incidence (2 of 2)

Deviating from normal incidence splits the resonance. Increasing angle of incidence shifts the position of the resonance and alters background response.
Sensitivity to Polarization

Polarization can have a dramatic effect on the response of a GMR. See Lecture on subwavelength gratings.

Effect of Having a Finite Number of Periods
A Simple Design Procedure

**Step 1:** Design a multilayer structure that provides the desired background response.
- For low background reflection, think anti-reflection coatings.
  \[ n_a = \sqrt{n_1 n_2} \quad L = \frac{\lambda}{4n_a} \]
- For low background transmission, think Bragg gratings.
  - This part of the design can also be performed using any number of optimization algorithms.
  - There may be other constraints which you need to consider when choosing layer thicknesses.

**Step 2:** Incorporate a grating (or gratings).
- Set duty cycle to realize effective material properties.
- Set grating period to place resonance at desired frequency.
Design Example #1: Monolithic GMR Filter (1 of 2)

Step 1: Design a multilayer structure with minimal background reflection at 1.5 GHz.

Given
\[ \varepsilon_1 = 2.35 \]

Design Constraints
\[ 1.0 \leq \varepsilon_1 \leq \varepsilon_2 \]
\[ d_1 + d_2 < 3.0'' \]

Design After Optimization
\[ \varepsilon_1 = 1.1 \]
\[ \varepsilon_2 = 2.35 \]
\[ d_1 = 0.787'' \]
\[ d_2 = 2.20'' \]

Step 2: Incorporate a grating to place resonance at 1.5 GHz.

For E mode, \( f = 32.6\% \)

For 1.5 GHz, \( \Lambda = 6.04'' \)

Design Example #1: Monolithic GMR Filter (2 of 2)

\[ \Lambda = 15.342 \text{ cm} \]
\[ d = 1.999 \text{ cm} \]
\[ a = 5.588 \text{ cm} \]
\[ f = 32.6\% \]
\[ \varepsilon'' = 2.35 \]
**Design Example #2: GMR Filter on a Substrate**

**Step 1:** Design a multilayer structure with minimal background reflection at $\lambda_0 = 550$ nm.

- Given
  - $n_1 = 2.0$
  - $n_2 = 1.52$
- Design Constraints
  - $0.1 \lambda_0 \leq d \leq \lambda_0$
- Design After Optimization
  - $d = 0.5 \lambda_0$

**Step 2:** Incorporate a grating to place resonance at 1.5 GHz.

Let,
- $f = 50\%$
- $n_g = n_i - \Delta n/2$
- $n_H = n_i + \Delta n/2$

For $\lambda_0 = 550$ nm.
- $\Lambda = 358.9$ nm

**Scalability**

Maxwell’s equations have no fundamental length scale so designs can be made to operate at different frequencies just by scaling the dimensions.

- $f_1$ or $\lambda_1$
- $2a$
- $f_1/2$ or $2\lambda_1$
Example of Scaling a Design

Scaling Factor for 25 GHz Operation

\[
\frac{S}{25 \text{ GHz}} = 0.06
\]

To scale the design, multiply all physical dimensions by this number.

\[
A = 0.362''
\]
\[
d_1 = 0.047''
\]
\[
d_2 = 0.132''
\]
\[
f = 32.6\%
\]
\[
\epsilon = 2.35
\]

Broadband by Combining Multiple Resonances

Multiple resonances can be combined to produce a “single” broadband response.

Literature claims asymmetry in the grating contributes to broadband nature.

GMR devices can be made polarization-independent in several ways.

1. Special cases can be found where both polarizations exhibit a resonance at the same frequency.
2. Crossed gratings with rotational symmetry are polarization independent at normal incidence.
3. Anisotropy can be incorporated to compensate for the birefringence produced by gratings.
4. More?
Anisotropy for Polarization Independence


GMR Devices with Few Periods

GMR device is made "effectively" infinite length by incorporating reflectors at the ends of the device.

Applications

High Power Microwave Frequency Selective Surfaces

Narrow-Line Feedback Elements for Lasers


GMRs as Biosensors

The extreme sensitivity of GMRs make them ideally suited for detecting small changes in dimensions and refractive index.

They are becoming more popular in biosensing for this reason.

High sensitivity of GMR devices is exploited to make a tunable filter.


Response of a Typical GMR