Advanced Electromagnetics:
21st Century Electromagnetics

Guided-Mode Resonance

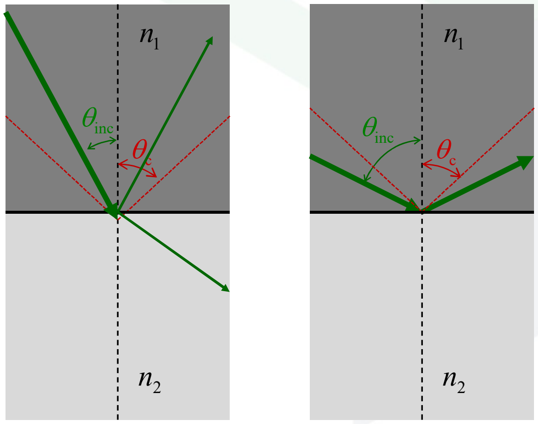
Lecture Outline

- Physics of Guided-Mode Resonance (GMR)
- GMR Filters
- Design of GMR Filters
- Applications

Physics of Guided-Mode Resonance

The Critical Angle and Total Internal Reflection

When an electromagnetic wave is incident on a material with a lower refractive index, it is totally reflected when the angle of incidence is greater than the critical angle.



$$\theta_{\text{c}} = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

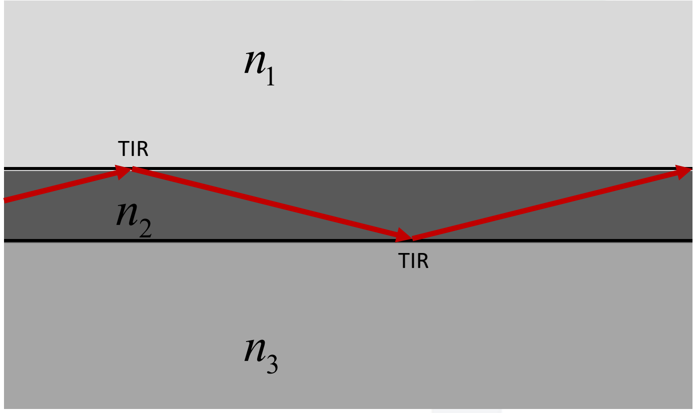
Example
What is the critical angle for fused silica (glass)?

The refractive index at optical waveguides is around 1.5.

$$\theta_{\text{c}} = \sin^{-1}\left(\frac{1.0}{1.5}\right) = 41.81°$$

The Slab Waveguide

If a slab of material is placed between two materials with lower refractive index, a slab waveguide is created.

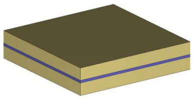


Conditions

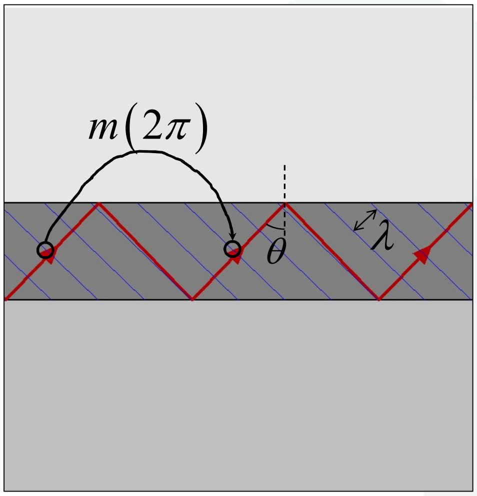
$n_2 > n_1$

and

$n_2 > n_3$



Ray Tracing Analysis



$$\beta = k_0 n_{\text{eff}} = k_0 n \sin\theta$$

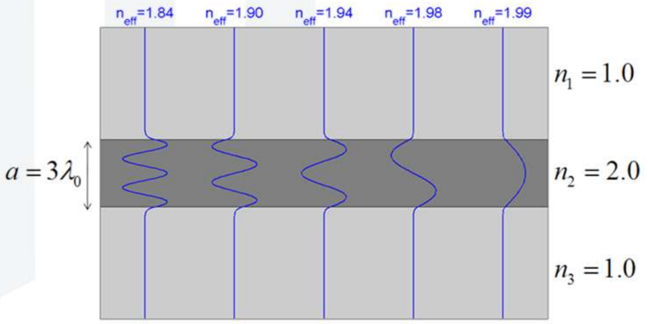
The round trip phase of a ray must be an integer multiple of 2π.
Only certain angles are allowed to propagate in the waveguide.

Rigorous Analysis

A rigorous analysis of slab waveguides involves Maxwell's equations.

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \qquad \nabla \times \vec{H} = j\omega\varepsilon\vec{E}$$

The geometry and mode solutions for a typical slab waveguide are



Diffraction from Gratings

The angles of the diffracted modes are related to the wavelength and grating through the grating equation.

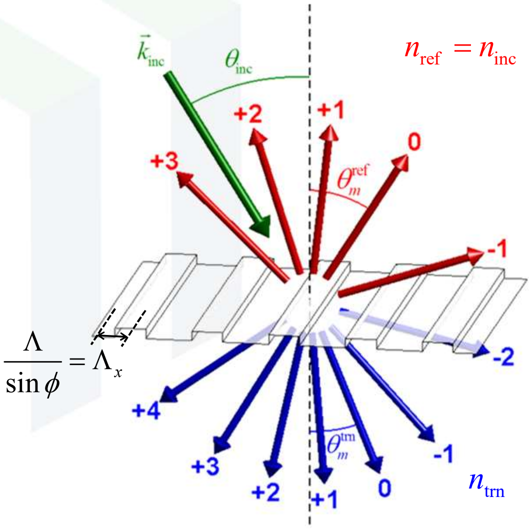
The grating equation only predicts the directions of the modes, not how much power is in them.

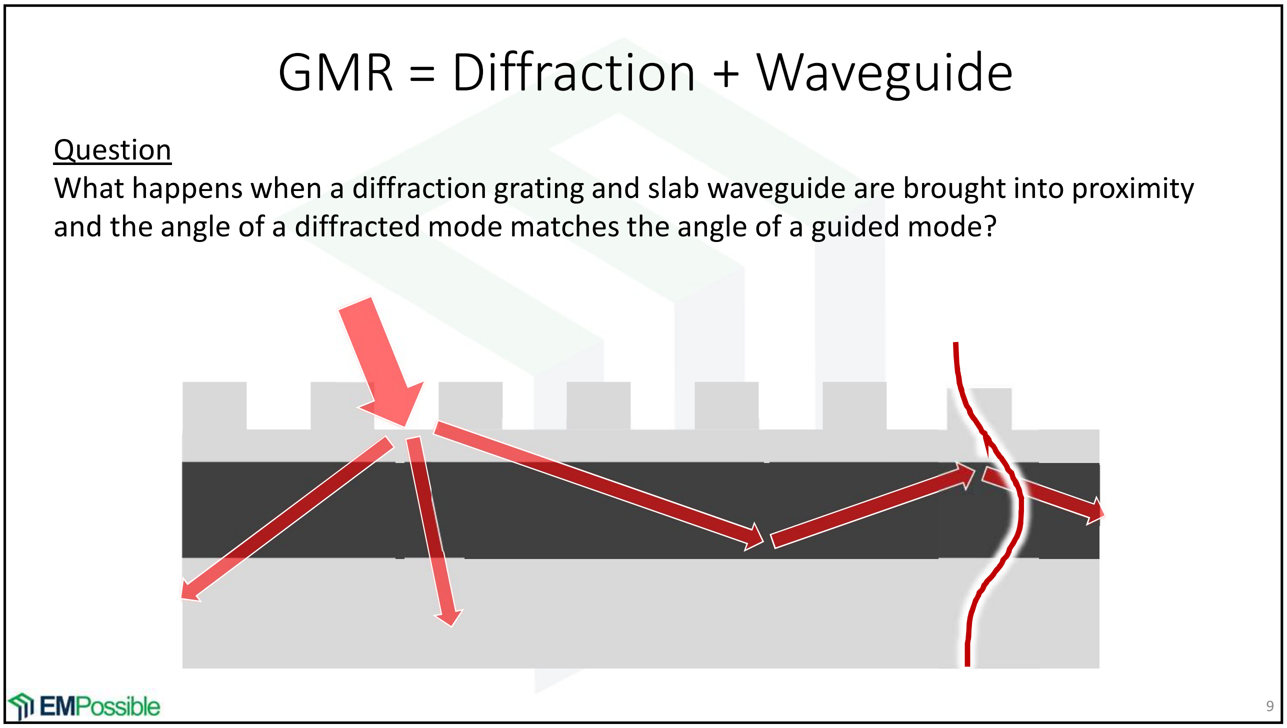
Reflection Region

$$n_{\mathrm{ref}} \sin\left[\theta(m)\right] = n_{\mathrm{inc}} \sin\theta_{\mathrm{inc}} - m\frac{\lambda_0}{\Lambda}\sin\phi$$

Transmission Region

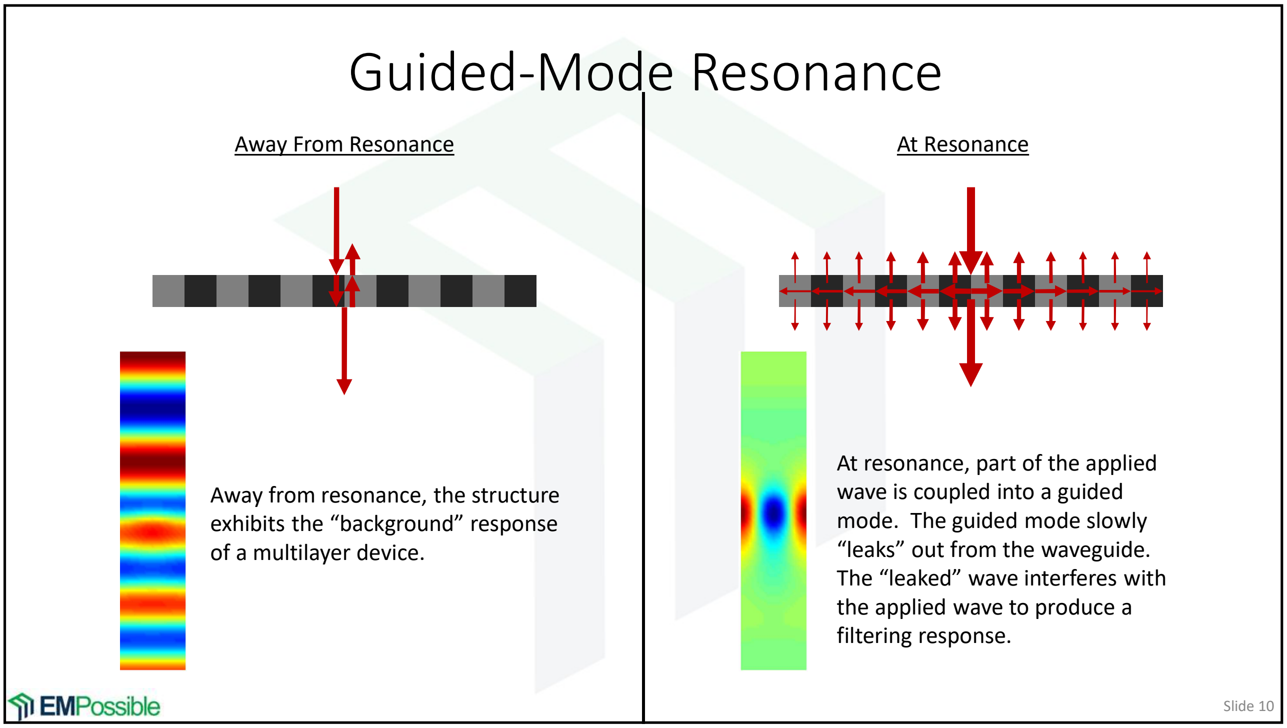
$$n_{\mathrm{trn}} \sin\left[\theta(m)\right] = n_{\mathrm{inc}} \sin\theta_{\mathrm{inc}} - m\frac{\lambda_0}{\Lambda}\sin\phi$$





GMR = Diffraction + Waveguide

Question
What happens when a diffraction grating and slab waveguide are brought into proximity and the angle of a diffracted mode matches the angle of a guided mode?



Guided-Mode Resonance

Away From Resonance

Away from resonance, the structure exhibits the "background" response of a multilayer device.

At Resonance

At resonance, part of the applied wave is coupled into a guided mode. The guided mode slowly "leaks" out from the waveguide. The "leaked" wave interferes with the applied wave to produce a filtering response.

Regions of Guided-Mode Resonance (Derivation)

Recall the grating equation

$$n_2 \sin\left[\theta(m)\right] = n_1 \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda}\sin\phi$$

Recall from the ray tracing picture that

$$\beta_m = k_0 n_{\text{eff}} = k_0 n_2 \sin\left[\theta(m)\right]$$

Therefore

$$n_{\text{eff}} = n_1 \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda}\sin\phi$$

Conditions for n_{eff} to represent a guided mode

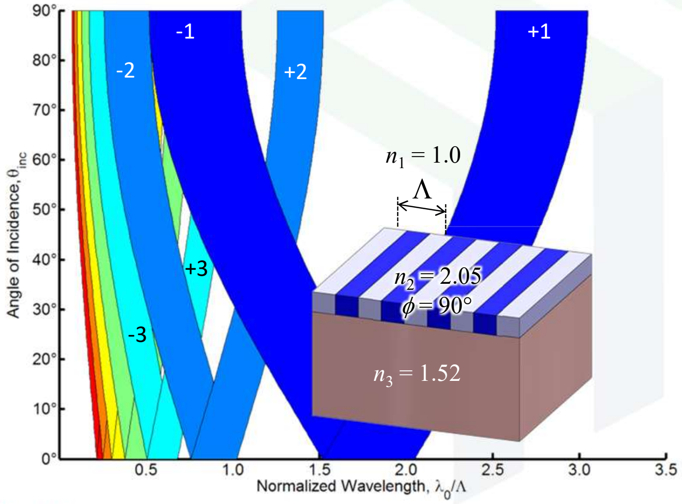
$$\max\left[n_1, n_3\right] \leq n_{\text{eff}} < n_2$$

Combining the above two equations leads to an equation describing the regions of resonance for guided-mode resonance.

$$\max\left[n_1, n_3\right] \leq \left| n_1 \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda}\sin\phi \right| < n_2$$

Regions of Guided-Mode Resonance (Plot)

$$\max\left[n_1, n_3\right] \leq \left| n_1 \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda}\sin\phi \right| < n_2$$



- Estimates ranges for resonant frequencies
- Predicts sensitivity to angle of incidence
- Shows how higher order resonances overlap
- Zero-order modes produces no resonance effects.

Center wavelength at normal incidence:

$$\frac{\lambda_c}{\Lambda} \approx \frac{n_2 + \max\left[n_1, n_3\right]}{2|m|\sin\phi}$$

Bounds:

$$\frac{\lambda_{\min}}{\Lambda} \leq \frac{\lambda_0}{\Lambda} < \frac{\lambda_{\max}}{\Lambda}$$

$$\frac{\lambda_{\min}}{\Lambda} = \begin{cases} \dfrac{n_1 \sin\theta_{\text{inc}} - \max\left[n_1, n_3\right]}{m\sin\phi} & m < 0 \\ \dfrac{n_1 \sin\theta_{\text{inc}} + \max\left[n_1, n_3\right]}{m\sin\phi} & m > 0 \end{cases}$$

$$\frac{\lambda_{\max}}{\Lambda} = \begin{cases} \dfrac{n_1 \sin\theta_{\text{inc}} - n_2}{m\sin\phi} & m < 0 \\ \dfrac{n_1 \sin\theta_{\text{inc}} + n_2}{m\sin\phi} & m > 0 \end{cases}$$

Benefits and Drawbacks

- Benefits
  - All-dielectric for very low loss
  - Extremely strong response from dielectrics
  - Can be made monolithic
  - Potentially better for high power than using metals
- Drawbacks
  - Larger and bulkier than equivalent metallic structures
  - Limited field-of-view and bandwidth compared to metallic devices
  - Response is very sensitive to material properties and structural deformations

Guided-Mode Resonance Filters

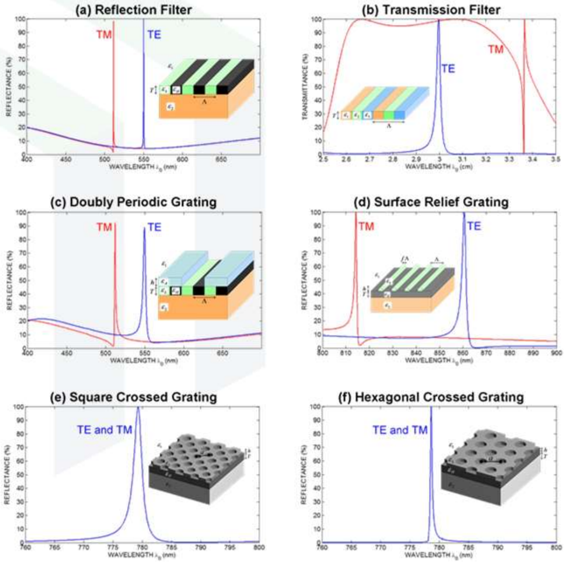
Various GMR Filters

A guided-mode resonance (GMR) filter is both a diffraction grating and slab waveguide.

A resonance occurs when a diffracted mode exactly matches a guided mode.

Away from resonance, the device behaves like an ordinary multilayer structure.

On resonance, the device reverses the background response (roughly speaking).



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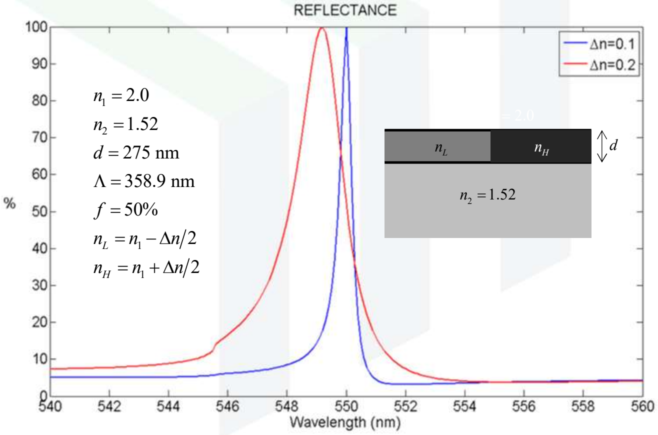
Effect of Index Contrast

Width of the resonance becomes more narrow as index contrast is lowered.
Position of the resonance can change slightly.



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Sensitivity to Angle of Incidence (1 of 2)

Grating Equation

$$n_1 \sin\theta_m = n_{\text{inc}} \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda}\sin\phi$$

Sensitivity to Angle of Incidence

$$\frac{\partial\lambda_0}{\partial\theta_{\text{inc}}} = ?$$

We make the small angle approximation: $\sin\theta_{\text{inc}} \approx \theta_{\text{inc}}$

$$\frac{\partial\lambda_0}{\partial\theta_{\text{inc}}} = \frac{\Lambda n_{\text{inc}}}{m\sin\phi} = \frac{\Lambda_x n_{\text{inc}}}{m}$$

$$\boxed{\frac{\Delta\lambda_{\text{res}}}{\Delta\theta_{\text{inc}}} \cong \frac{\Lambda_x n_{\text{inc}}}{m}}$$

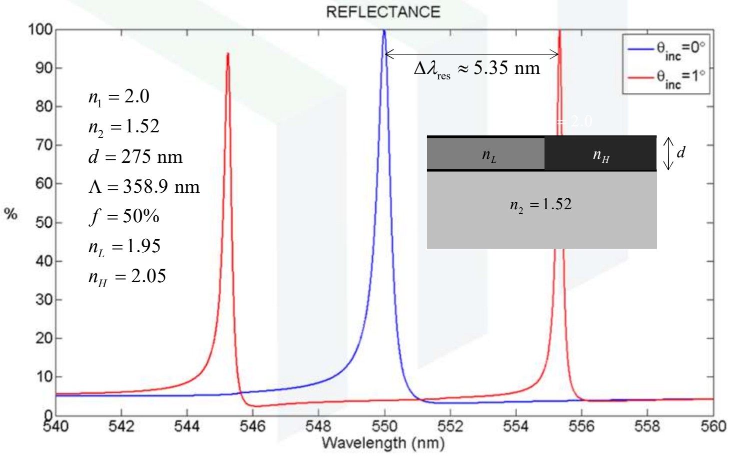
Example

$$\frac{\partial\lambda_{\text{res}}}{\partial\theta_{\text{inc}}} = \frac{\Lambda_x n_{\text{inc}}}{m} = \frac{(358.9\text{ nm})(1.0)}{1} = 358.9\ \tfrac{\text{nm}}{\text{rad}}$$

$$= \left(358.9\ \tfrac{\text{nm}}{\text{rad}}\right)\left(\frac{\pi\text{ rad}}{180^\circ}\right) = 6.3\ \tfrac{\text{nm}}{\text{deg}}$$

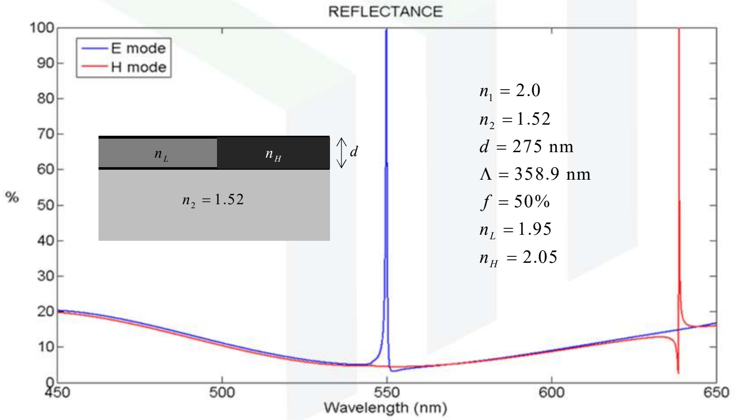
Sensitivity to Angle of Incidence (2 of 2)

Deviating from normal incidence splits the resonance. Increasing angle of incidence shifts the position of the resonance and alters background response.

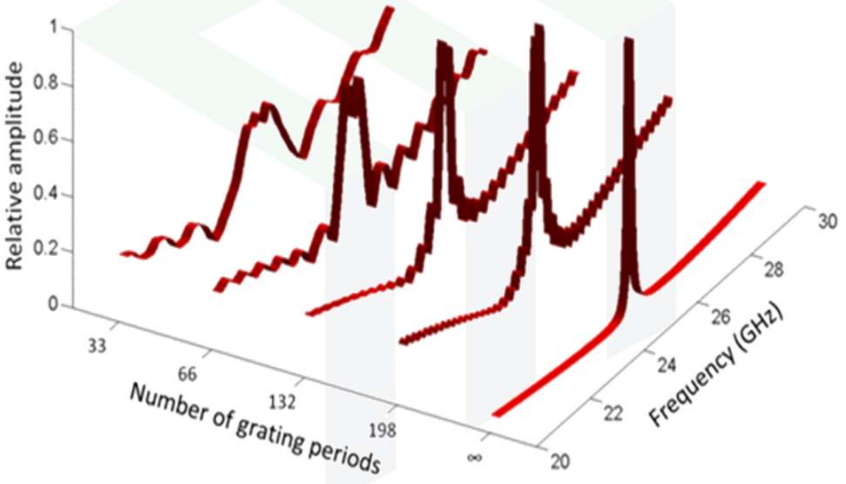


Sensitivity to Polarization

Polarization can have a dramatic effect on the response of a GMR.
See Lecture on subwavelength gratings.



Effect of Having a Finite Number of Periods



Design of GMR Filters

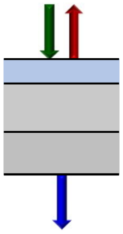
A Simple Design Procedure

Step 1: Design a multilayer structure that provides the desired background response.

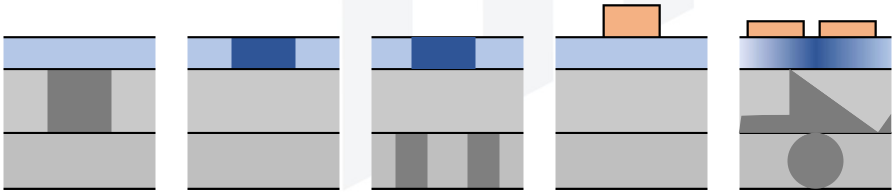
- For low background reflection, think anti-reflection coatings.

  $n_{\text{ar}} = \sqrt{n_1 n_2} \qquad L = \lambda_0 / (4 n_{\text{ar}})$

- For low background transmission, think Bragg gratings.
- This part of the design can also be performed using any number of optimization algorithms. There may be other constraints which you need to consider when choosing layer thicknesses.

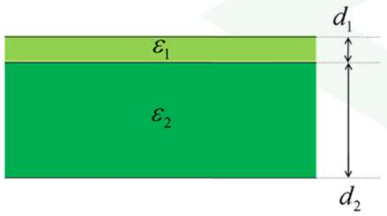


Step 2: Incorporate a grating (or gratings).
Set duty cycle to realize effective material properties.
Set grating period to place resonance at desired frequency.



Design Example #1:
Monolithic GMR Filter (1 of 2)

Step 1: Design a multilayer structure with minimal background reflection at 1.5 GHz.



Given

$\varepsilon_2 = 2.35$

Design Constraints

$1.0 \le \varepsilon_1 \le \varepsilon_2$

$d_1 + d_2 < 3.0"$

Design After Optimization

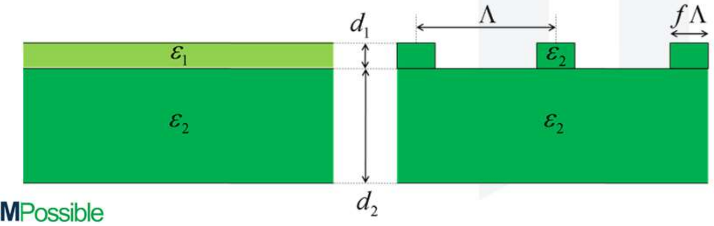
$\varepsilon_1 = 1.1$

$\varepsilon_1 = 2.35$

$d_1 = 0.787"$

$d_2 = 2.20"$

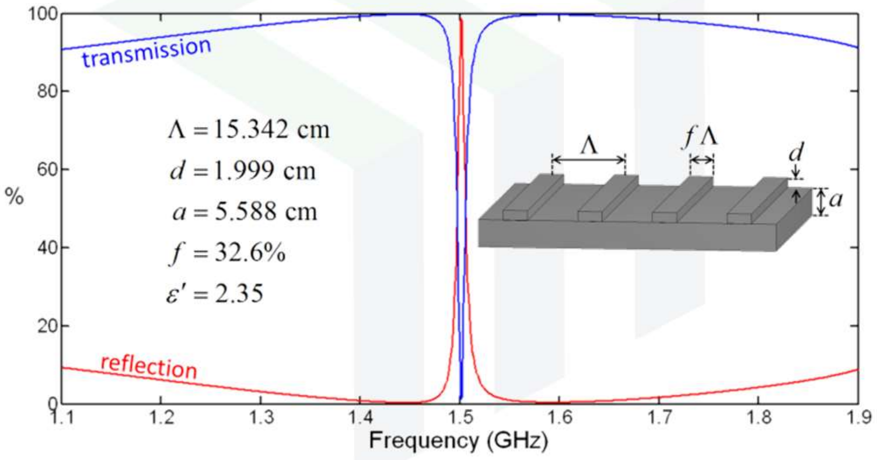
Step 2: Incorporate a grating to place resonance at 1.5 GHz.



For E mode,
$f = 32.6\%$

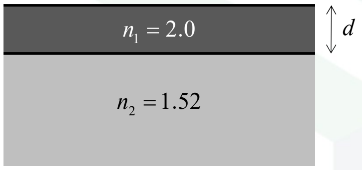
For 1.5 GHz,
$\Lambda = 6.04"$

Design Example #1:
Monolithic GMR Filter (2 of 2)



Design Example #2:
GMR Filter on a Substrate

Step 1: Design a multilayer structure with minimal background reflection at λ_0=550 nm.



Given

$n_1 = 2.0$

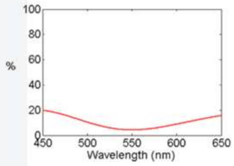
$n_2 = 1.52$

Design Constraints

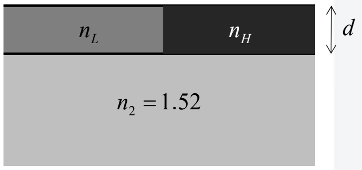
$0.1\lambda_0 \le d \le \lambda_0$

Design After Optimization

$d = 0.5\lambda_0$



Step 2: Incorporate a grating to place resonance at 1.5 GHz.



Let,

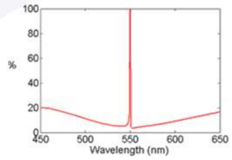
$f = 50\%$

$n_L = n_1 - \Delta n/2$

$n_H = n_1 + \Delta n/2$

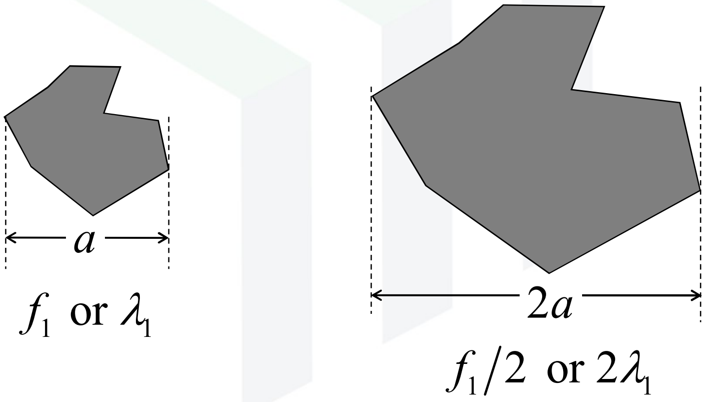
For λ_0=550 nm.

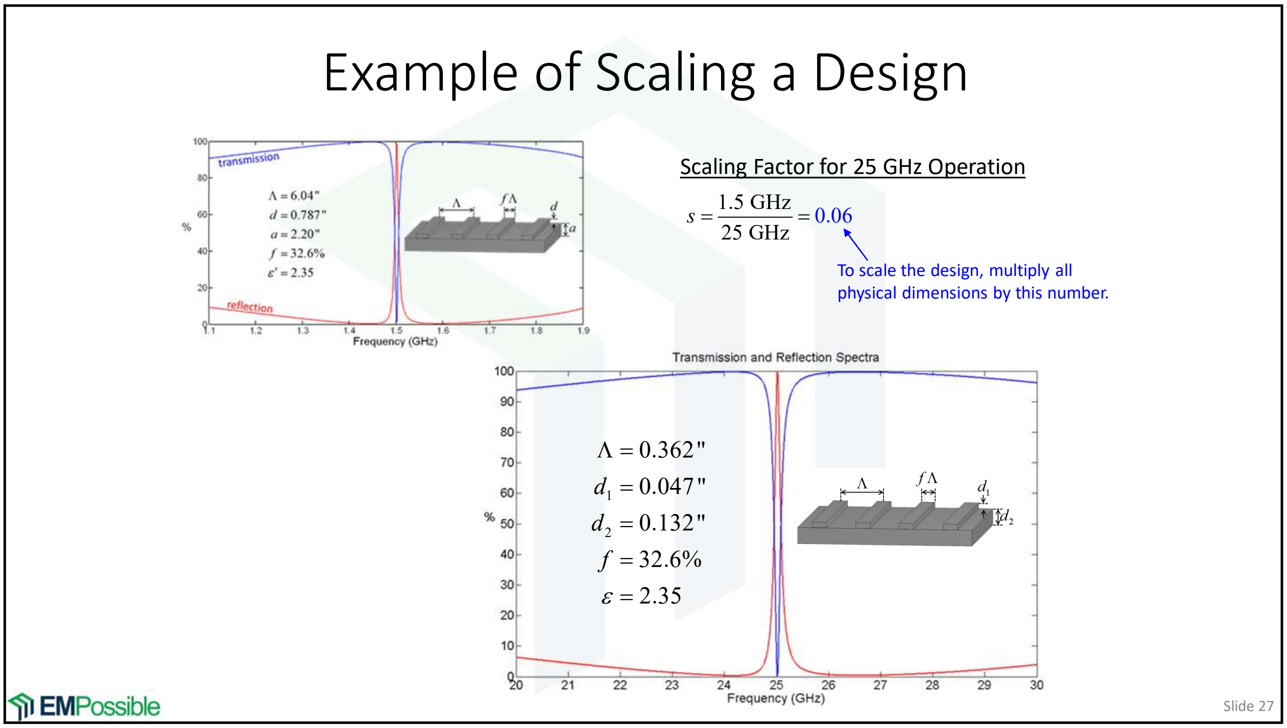
$\Lambda = 358.9$ nm



Scalability

Maxwell's equations have no fundamental length scale so designs can be made to operate at different frequencies just by scaling the dimensions.





Example of Scaling a Design

Scaling Factor for 25 GHz Operation

$$s = \frac{1.5\text{ GHz}}{25\text{ GHz}} = 0.06$$

To scale the design, multiply all physical dimensions by this number.

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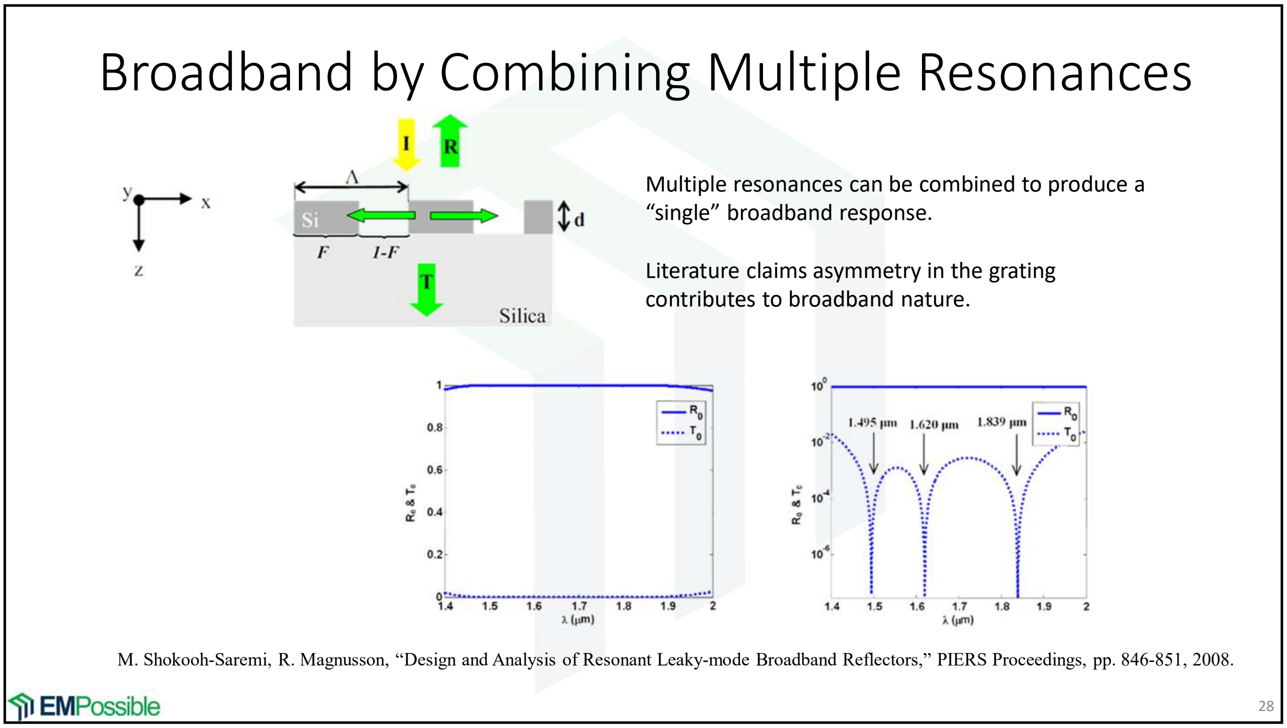

Broadband by Combining Multiple Resonances

Multiple resonances can be combined to produce a "single" broadband response.

Literature claims asymmetry in the grating contributes to broadband nature.

M. Shokooh-Saremi, R. Magnusson, "Design and Analysis of Resonant Leaky-mode Broadband Reflectors," PIERS Proceedings, pp. 846-851, 2008.

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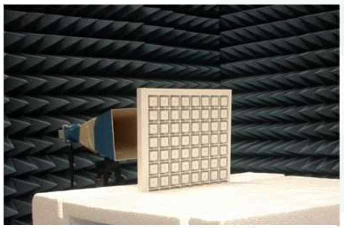
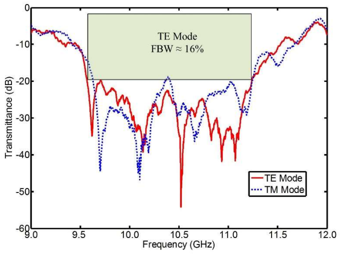
Broadband Microwave GMRFs

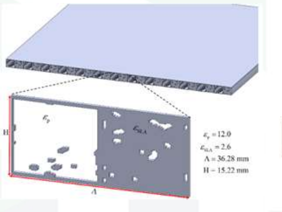
CNC Machined GMRF

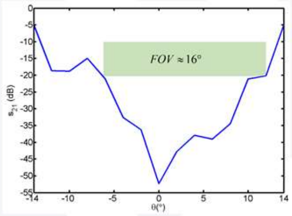


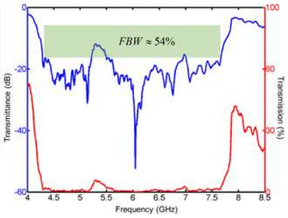


J. H. Barton, C. R. Garcia, E. A. Berry, R. G. May, D. T. Gray, R. C. Rumpf, "All-Dielectric Frequency Selective Surface for High Power Microwaves," IEEE Transactions on Antennas and Propagation, 2014.

3D Printed GMRF





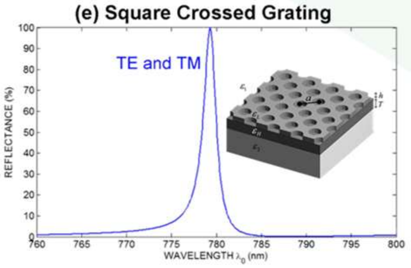


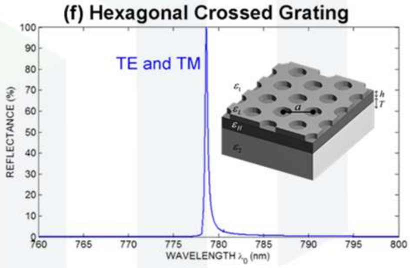
J. H. Barton, C. R. Garcia, E. A. Berry, R. Salas, R. C. Rumpf, "3D Printed All–Dielectric Frequency Selective Surface with Large Bandwidth and Field-of-View," IEEE Trans. Antennas and Propagation, Vol. 63, No. 3, pp. 1032-1039, 2015.

Polarization Independent GMR Devices

GMR devices can be made polarization-independent in several ways.

1. Special cases can be found where both polarizations exhibit a resonance at the same frequency.
2. Crossed gratings with rotational symmetry are polarization independent at normal incidence.

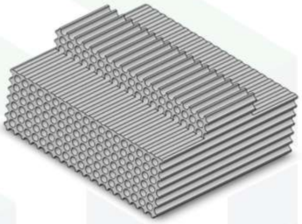


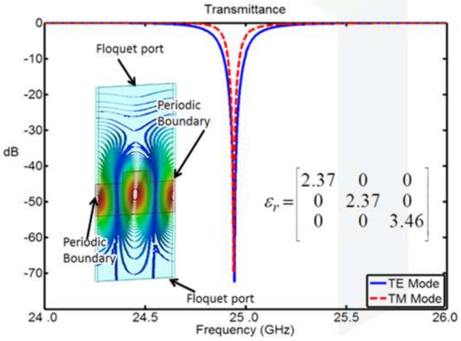


3. Anisotropy can be incorporated to compensate for the birefringence produced by gratings.
4. More?

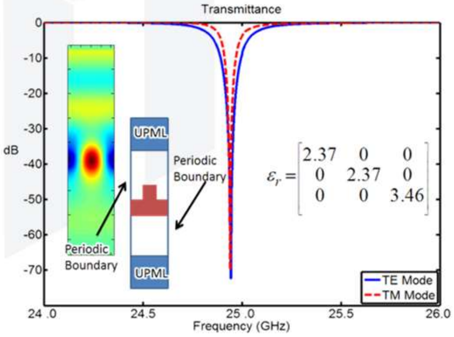
Anisotropy for Polarization Independence

R. C. Rumpf, C. R. Garcia, E. A. Berry, J. H. Barton, "Finite-Difference Frequency-Domain Algorithm for Modeling Electromagnetic Scattering from General Anisotropic Objects," PIERS B, Vol. 61, pp. 55-67, 2014.





$$\varepsilon_r = \begin{bmatrix} 2.37 & 0 & 0 \\ 0 & 2.37 & 0 \\ 0 & 0 & 3.46 \end{bmatrix}$$



$$\varepsilon_r = \begin{bmatrix} 2.37 & 0 & 0 \\ 0 & 2.37 & 0 \\ 0 & 0 & 3.46 \end{bmatrix}$$

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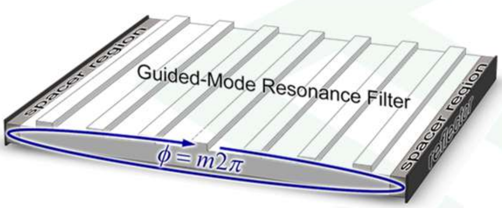
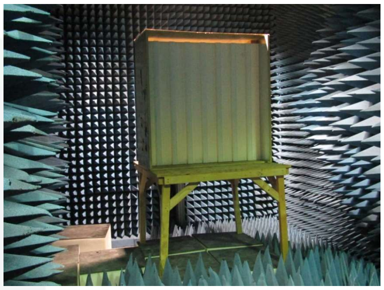
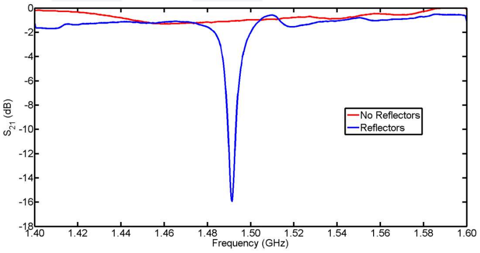
GMR Devices with Few Periods





GMR device is made “effectively” infinite length by incorporating reflectors at the ends of the device.



Jay H. Barton, R. C. Rumpf, R. W. Smith, "All-Dielectric Frequency Selective Surfaces with Few Periods," PIERS B, Vol. 41, pp. 269-283, 2012.

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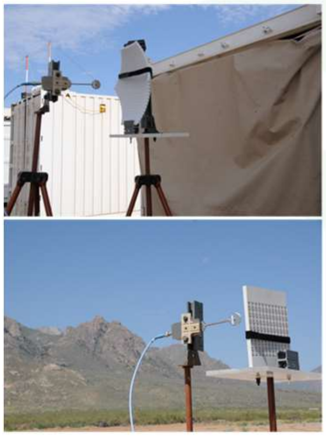
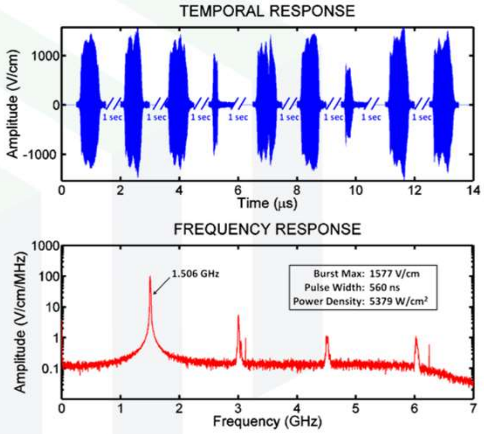
Applications

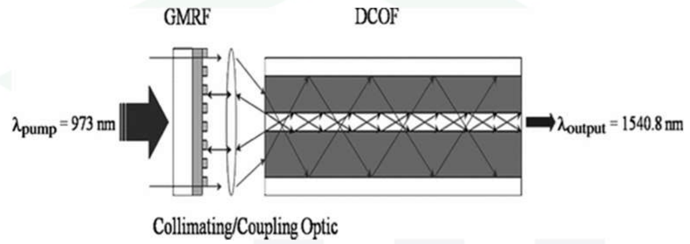
High Power Microwave Frequency Selective Surfaces

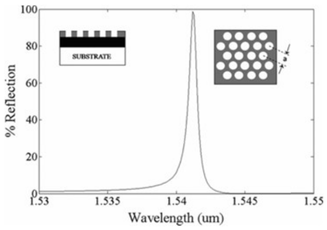


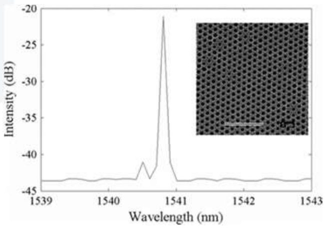


J. H. Barton, C. R. Garcia, E. A. Berry, R. G. May, D. T. Gray, R. C. Rumpf, "All-Dielectric Frequency Selective Surface for High Power Microwaves," IEEE Transactions on Antennas and Propagation, 2014.

Narrow-Line Feedback Elements for Lasers





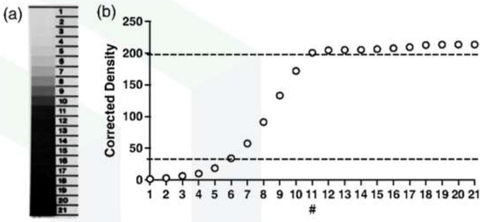


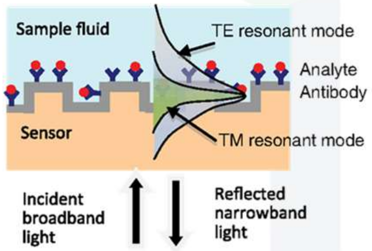
Alok Mehta, Raymond C. Rumpf, Zachary A. Roth, and Eric G. Johnson, "Guided Mode Resonance Filter as an External Feedback Element in a Double-Cladding Optical Fiber Laser," IEEE Photonics Tech Letters, VOL. 19, NO. 24, pp. 2030-2032 (2007).

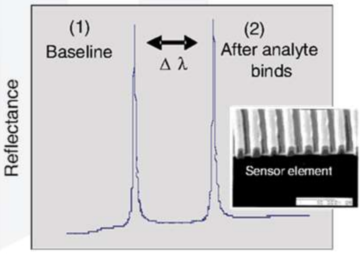
GMRs as Biosensors

The extreme sensitivity of GMRs make them ideally suited for detecting small changes in dimensions and refractive index.

They are becoming more popular in biosensing for this reason.







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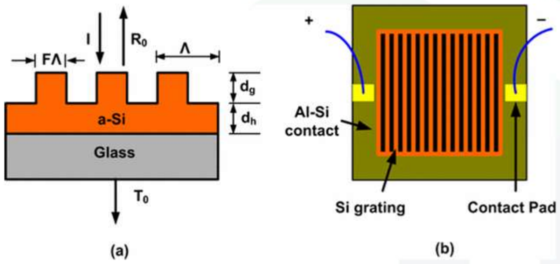
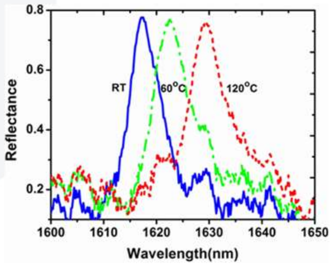
Tunable Optical Filters



High sensitivity of GMR devices is exploited to make a tunable filter.



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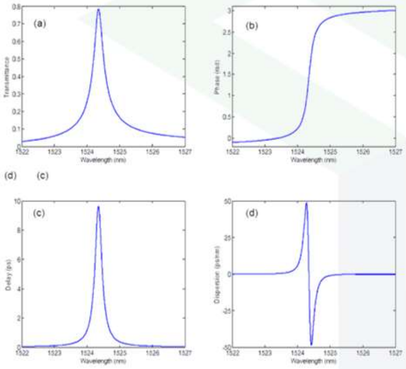
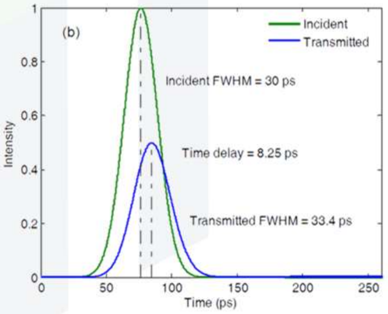
EMPossible

Dispersion Engineering and Pulse Shaping

Response of a Typical GMR





Xin Wang, "Dispersion Engineering with Leaky-Mode Resonance Structures," MS Thesis, University of Texas at Arlington, 2010.

EMPossible

Polarization Beam Splitter

Incident polarization

Reflected and
transmitted polarizations

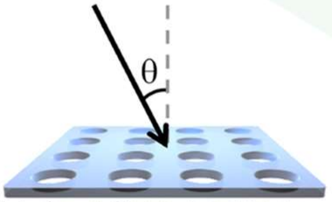


Fig. 7. (Color online) Illustration showing light incident on a PCS at an oblique angle. The azimuthal orientation of the incidence plane is irrelevant.

O. Kilic, et al, "Analysis of guided-resonance-based polarization beam splitting in photonic crystal slabs," J. Opt. Soc. Am. A, Vol. 25, No. 11, pp. 2680-2692, 2008.

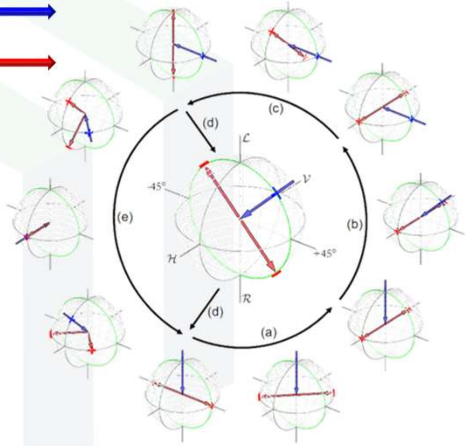


Fig. 6. (Color online) Illustration of the PCS device on the Poincaré sphere. The arrow with its head just at the center of the sphere denotes the incident polarization, while the two arrows pointing away from the center show the reflected and transmitted polarizations. The circle passing through $\mathcal{P}+45°$, $\mathcal{L}$, $\mathcal{P}-45°$, and $\mathcal{R}$ shows the polarizations aligned to the PBS axes, as in Eq. (13). Note that for the case when the incident polarization is aligned to the PBS, the arrows denoting the transmission and reflection polarizations are always antiparallel to each other and orthogonal to the arrow depicting the incidence polarization, showing that the incident polarization is separated into its two orthogonal constituents. The paths show what happens when some of the parameters are changed gradually, summarizing the operation of this type of PBS. (a) β changes from 0 to $\frac{1}{2}$ to 1 for the incidence polarization fixed at $\mathcal{L}$. (b) Incidence polarization changes from $\mathcal{L}$ to elliptical to $\mathcal{P}+45°$, for β fixed at 1. (c) β changes from 1 to $\frac{1}{2}$ to 0 for the incidence polarization fixed at $\mathcal{P}+45°$. (d) Incidence polarization changes from $\mathcal{P}+45°$ to elliptical to $\mathcal{L}$ for β fixed at 0. (e) The incident polarization is misaligned, rotating between the $\mathcal{P}+45°$ and $\mathcal{H}$ polarizations, and then between $\mathcal{H}$ and $\mathcal{L}$ polarizations.