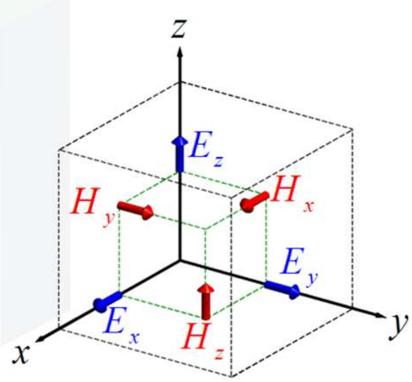
Computational Science:
Introduction to Finite-Difference Time-Domain

Introduction

Lecture Outline

- What is FDTD?
- Theory of FDTD
- How FDTD will be implemented in this class.



What is FDTD?

What is FDTD?

Finite-difference time-domain (FDTD) solves the electromagnetic wave equation in the time domain using finite-difference approximations. Being a time-domain method, FDTD is more intuitive than other techniques and works by creating a "movie" of the fields flowing through a device.

W. Saj, "FDTD simulations of 2D plasmon waveguide on silver nanorods in hexagonal lattice," Opt. Express **13**, 4818-4827 (2005) .

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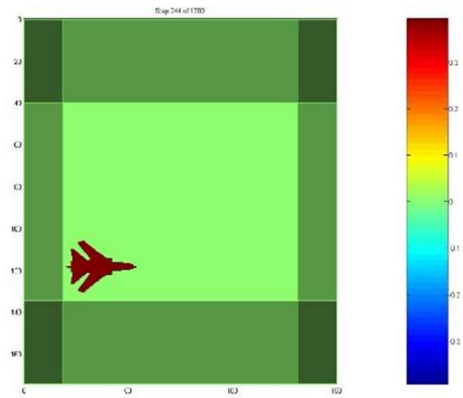
Simulation Example -- RADAR



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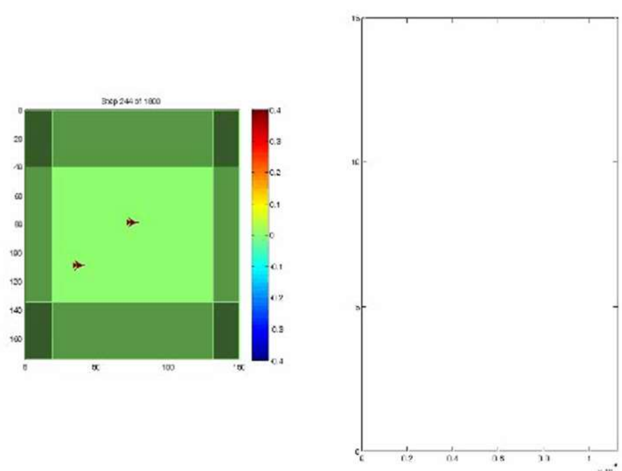
Simulation Example -- RADAR



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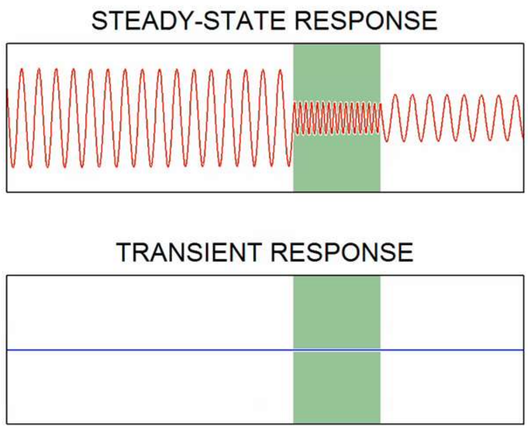
Time-Domain Vs. Frequency-Domain



Frequency-Domain Solution

Here we only see snapshot of the field after a “long” time has passed.

We cannot directly observe the transient response.

Time-Domain Solution

Here we can watch the fields evolve over time.

We can observe the transient response.

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Theory of FDTD

The Math Behind FDTD

$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$	Maxwell's Equations	$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$
$\downarrow$		$\downarrow$
$\frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu} \nabla \times \vec{E}$	Rearrange Terms	$\frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon} \nabla \times \vec{H}$
$\downarrow$		$\downarrow$
$\frac{\vec{H}\left(t+\frac{\Delta t}{2}\right) - \vec{H}\left(t-\frac{\Delta t}{2}\right)}{\Delta t} = -\frac{1}{\mu} \nabla \times \vec{E}(t)$	Finite-Difference Approximation	$\frac{\vec{E}(t+\Delta t) - \vec{E}(t)}{\Delta t} = \frac{1}{\varepsilon} \nabla \times \vec{H}\left(t+\frac{\Delta t}{2}\right)$
$\downarrow$		$\downarrow$
$\vec{H}\left(t+\frac{\Delta t}{2}\right) = \vec{H}\left(t-\frac{\Delta t}{2}\right) - \frac{\Delta t}{\mu} \nabla \times \vec{E}(t)$	Update Equation	$\vec{E}(t+\Delta t) = \vec{E}(t) + \frac{\Delta t}{\varepsilon} \nabla \times \vec{H}\left(t+\frac{\Delta t}{2}\right)$

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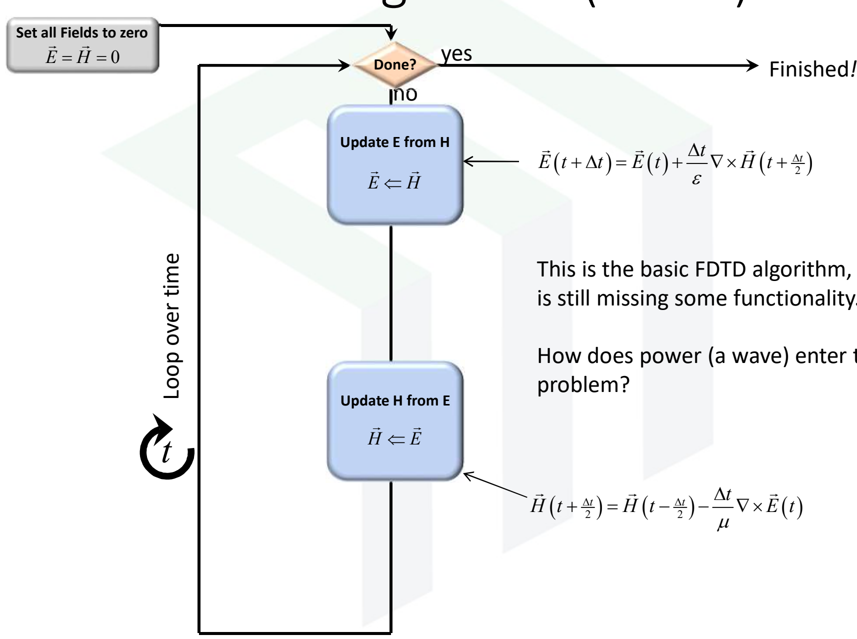
FDTD Algorithm (1 of 6)



This is the basic FDTD algorithm, but it is still missing some functionality.

How does power (a wave) enter the problem?

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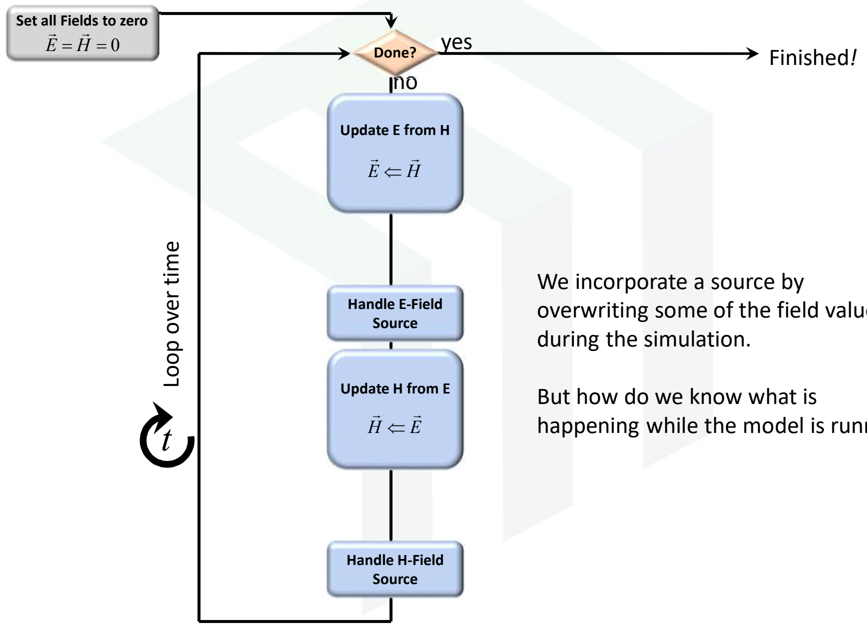
FDTD Algorithm (2 of 6)



We incorporate a source by overwriting some of the field values during the simulation.

But how do we know what is happening while the model is running?

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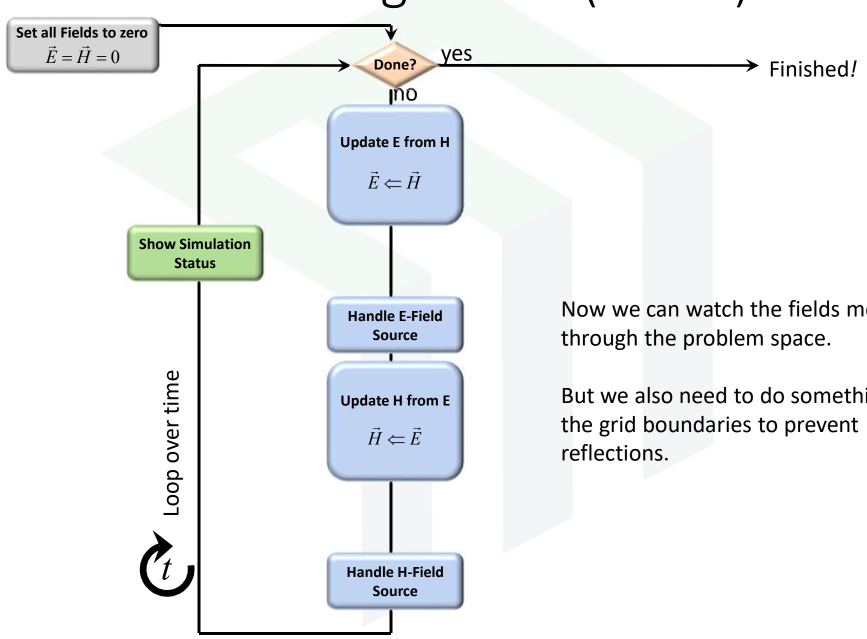
FDTD Algorithm (3 of 6)



Now we can watch the fields move through the problem space.

But we also need to do something at the grid boundaries to prevent reflections.

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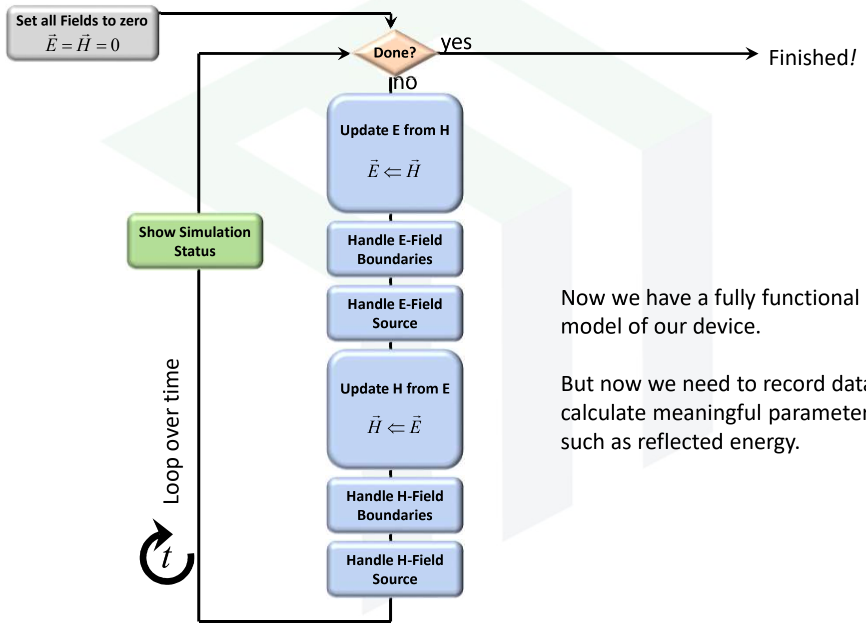
FDTD Algorithm (4 of 6)



Now we have a fully functional FDTD model of our device.

But now we need to record data and calculate meaningful parameters such as reflected energy.

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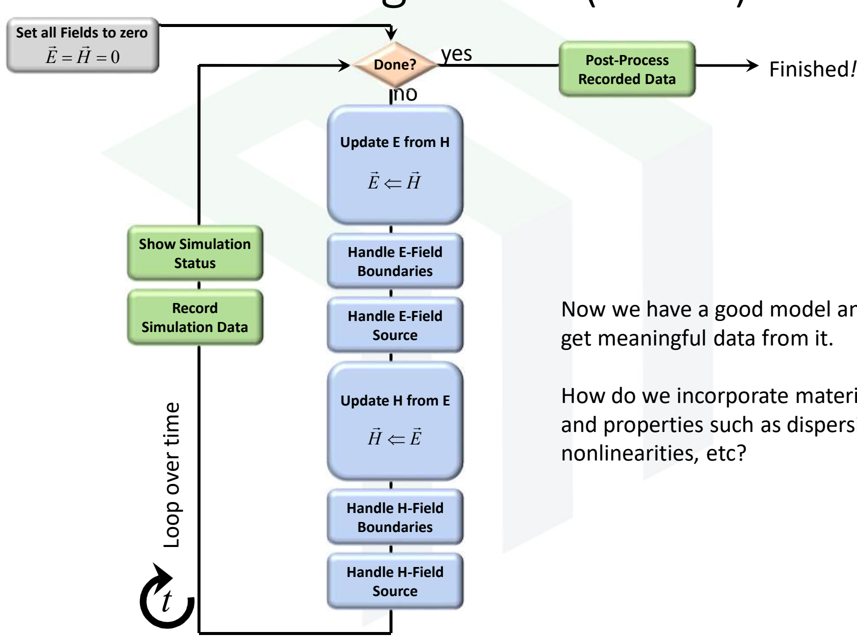
FDTD Algorithm (5 of 6)



Now we have a good model and can get meaningful data from it.

How do we incorporate materials and properties such as dispersion, nonlinearities, etc?

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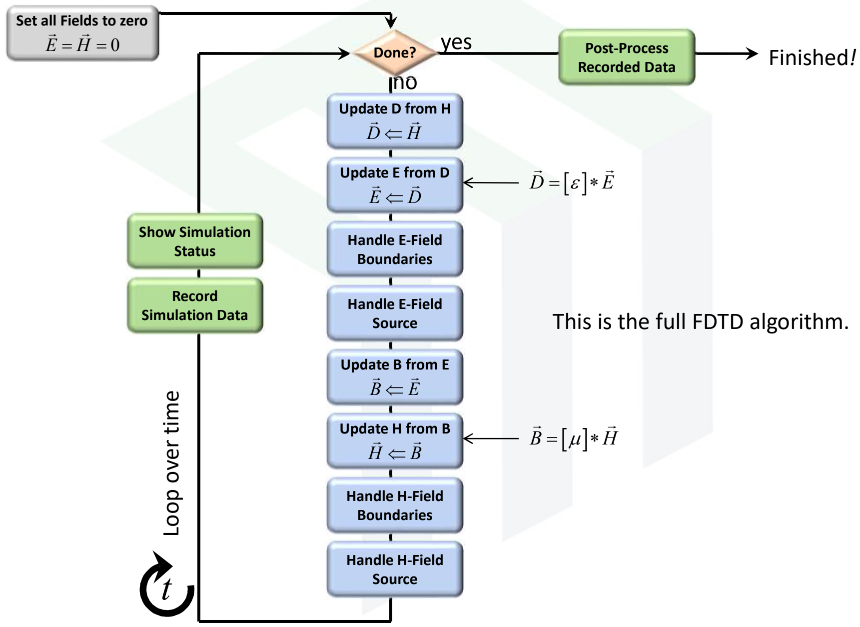
FDTD Algorithm (6 of 6)



This is the full FDTD algorithm.

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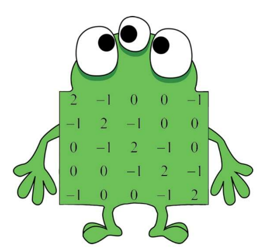
Benefits of FDTD

- Excellent for large scale simulations
  - It is said that numerical complexity scales linearly with problem size Typically, methods scale exponentially
  - Easily parallelized
- Excellent for broadband and/or transient simulations
- Accurate, robust, and mature method
  - Sources of error are well understood and it is a proven method in many fields
  - Lots of literature available
- Naturally handles nonlinear behavior
  - Directly handles nonlinearities due to nonlinear materials or incorporation of circuit elements
- Great for learning electromagnetics
  - Field animations and direct simulation of Maxwell's equations make FDTD a great learning tool
- **When all else fails, FDTD does not!**



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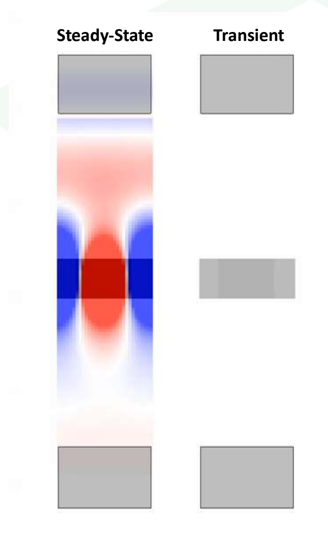
Drawbacks of FDTD

- Tedious to incorporate dispersion
- Tedious to incorporate PML
- Structured grid does not efficiently represent curved surfaces
- Slow for small devices
- Very inefficient for highly resonant devices

Time-Domain Simulation of Highly Resonant Devices



For More Information on FDTD

- http://en.wikipedia.org/wiki/Finite-difference_time-domain_method
- http://www.fdtd.org/
- For a simpler introduction
  - Dennis M. Sullivan, *Electromagnetic Simulation Using the FDTD Method*, IEEE Press.
- For more advanced topics
  - Allen Taflove, et al, *Advances in FDTD Computational Electrodynamics*, Artech House.

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How FDTD Will Be Implemented in This Class

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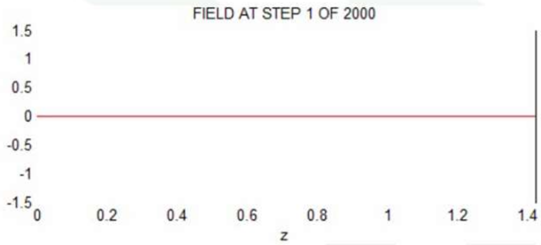
Step 1 – Basic 1D-FDTD Algorithm

- Basic update equations



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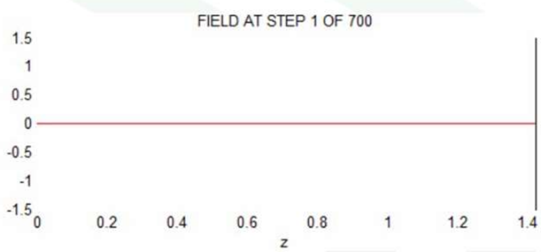
Step 2 – Add Simple Soft Source

- Basic update equations
- Add a soft source



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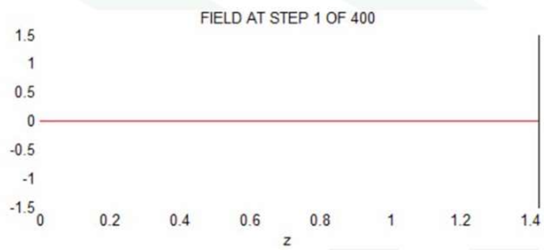
Step 3 – Add Absorbing Boundary



- Basic update equations
- Add a soft source
- Add perfect boundary condition

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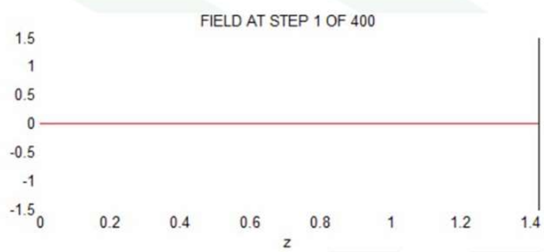
Step 4 – Add TF/SF



- Basic update equations
- Add a soft source
- Add perfect boundary condition
- Incorporate TF/SF "one-way" source

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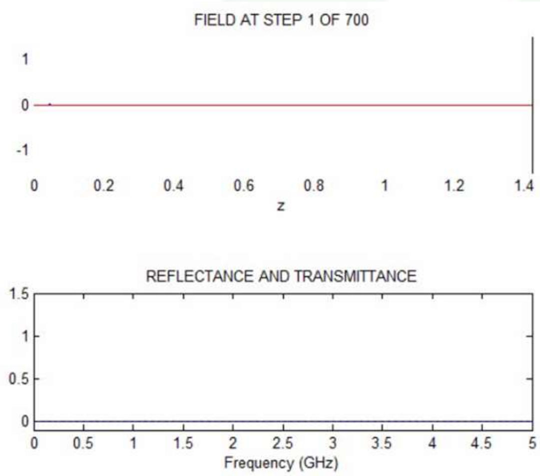
Step 5 – Move Source & Add T/R



- Basic update equations
- Add a soft source
- Add perfect boundary condition
- Incorporate TF/SF "one-way" source
- Move position of source
- Calculate transmittance and reflectance

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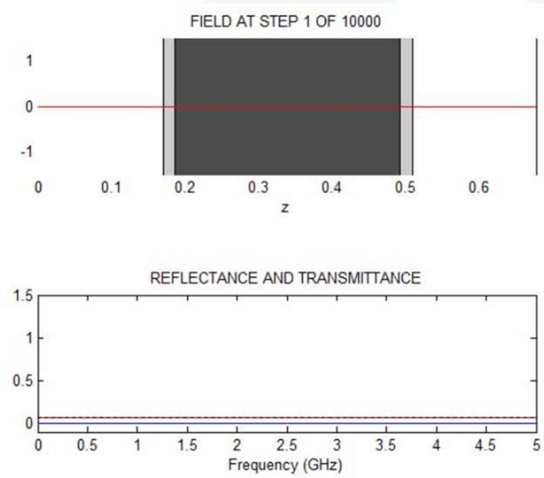
Step 6 – Add Device (Complete 1D FDTD)



- Basic update equations
- Add a soft source
- Add perfect boundary condition
- Incorporate TF/SF "one-way" source
- Move position of source
- Calculate transmittance and reflectance
- Add a real device

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Step 7 – Basic 2D-FDTD Update Equations

The basic update equations are implemented along with simple Dirichlet boundary conditions.

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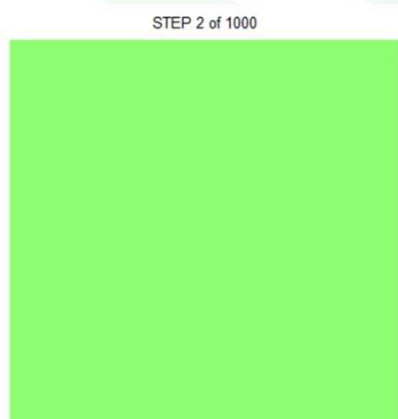
Step 8 – Incorporate Periodic Boundaries

Periodic boundary conditions are incorporated so that a wave leaving the grid reenters the grid at the other side.



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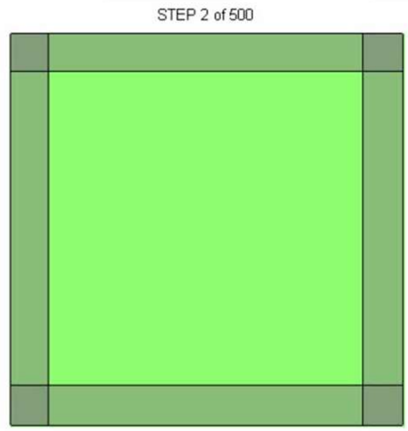
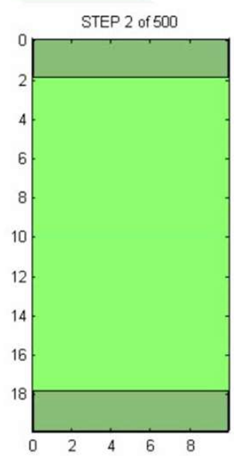
Step 9 – Incorporate a PML

The perfectly matched layer absorbing boundary condition is incorporated to absorb outgoing waves.



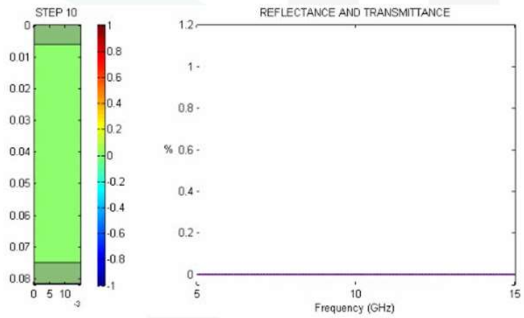
Step 10 – Total-Field/Scattered-Field

We use periodic boundaries for the horizontal axis and the PML for the vertical axis. When then implement TF/SF at the center vertically.



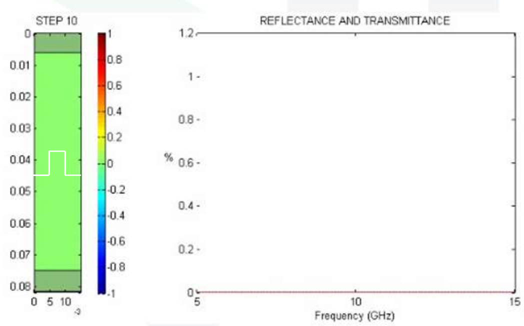
Step 11 – Calculate TRN, REF, and CON

We move the TF/SF interface to a unit cell or two outside the top PML. We include code to calculate Fourier transforms and then transmittance, reflectance, and conservation of energy.

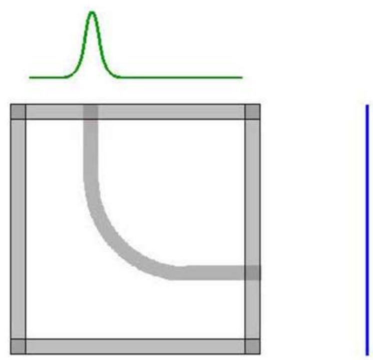


Step 12 – Add Device (Complete 2D FDTD)

We build a device on the grid that has a known solution. We run the simulation and duplicate the known results to benchmark the code.

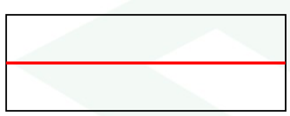


Step 13 – Waveguide Analysis

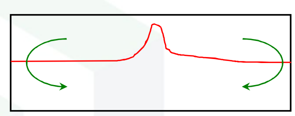


Summary of Code Development Sequence for 1D FDTD

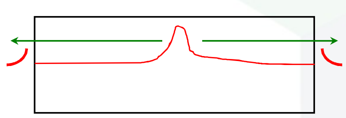
Step 1 – Implement basic FDTD algorithm



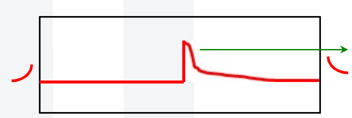
Step 2 – Add the source



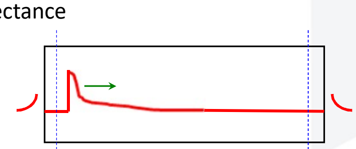
Step 3 – Add absorbing boundary



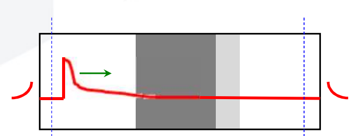
Step 4 – Add “one-way” source



Step 5 – Calculate transmittance and reflectance

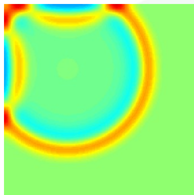


Step 6 – Add a device

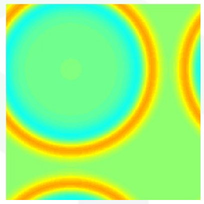


Summary of Code Development Sequence for 2D FDTD

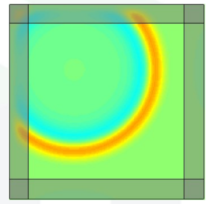
Step 7 – Basic Update + Dirichlet



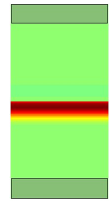
Step 8 – Basic Update + Periodic BC



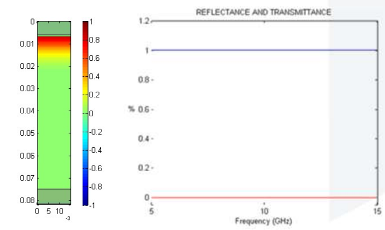
Step 9 – Add PML



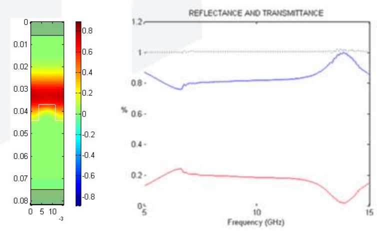
Step 10 – TF/SF



Step 11 – Calculate Response



Step 12 – Add a Device and Benchmark



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