



Advanced Electromagnetics:
21st Century Electromagnetics

Metamaterials

Lecture Outline

- Introduction to Metamaterials
- Resonant Metamaterials
 - Left-handed metamaterials
 - Refractive index less than one
 - Biisotropic and bianisotropic metamaterials
- Nonresonant Metamaterials
 - Anisotropic metamaterials
 - Hyperbolic metamaterials

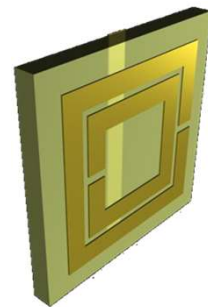


Introduction to Metamaterials

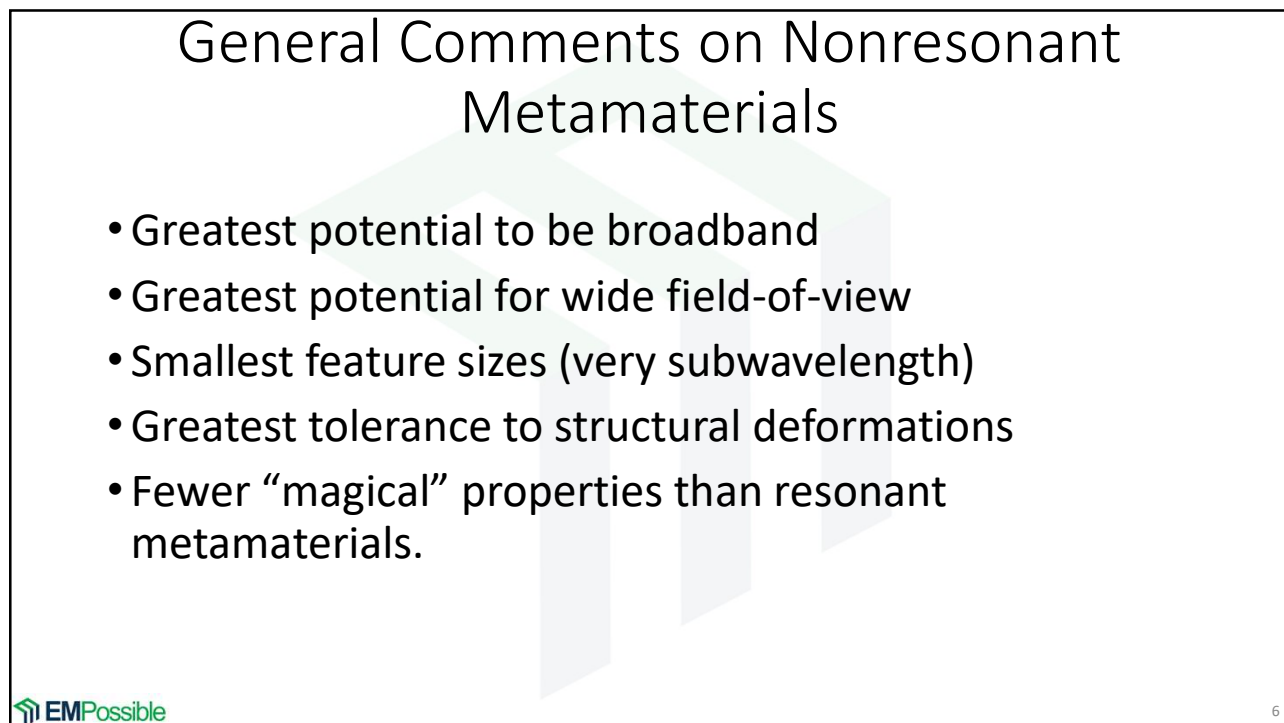
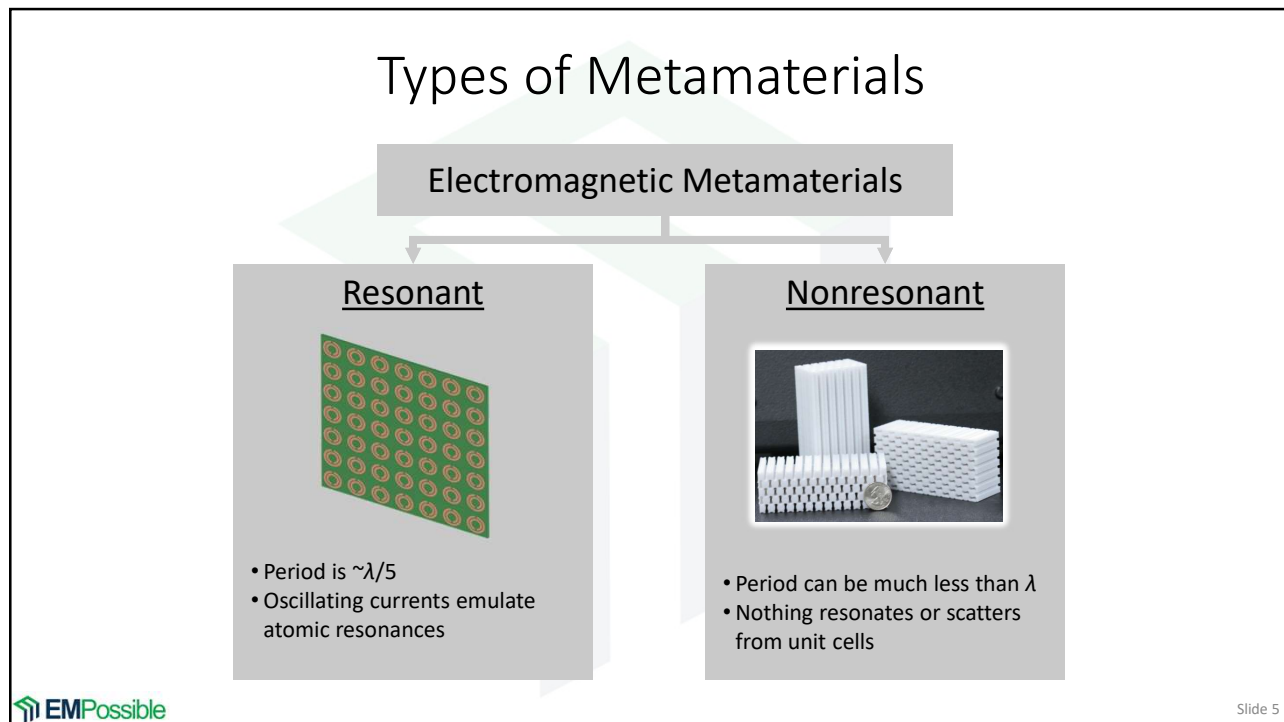
Slide 3

What are Metamaterials?

- No universally accepted definition
- Themes of a definition
 - Engineered composites
 - Properties are derived from their physical structure, not their chemistry
 - Exhibit properties not observed in nature.
 - Exhibit properties not observed in their constituent materials.
- A composite material that is purposely engineered to provide material properties that are not otherwise attainable with ordinary materials.



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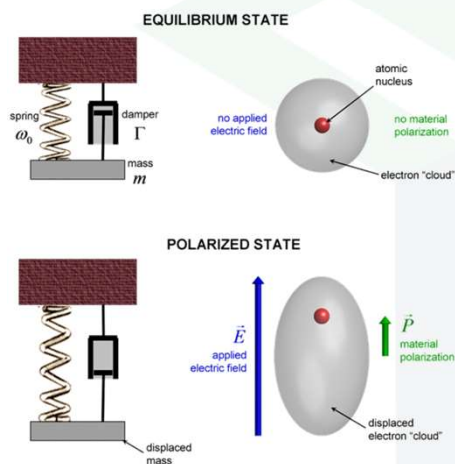


Resonant Metamaterials

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Dielectric Response of Ordinary Materials

Lorentz Oscillator Model



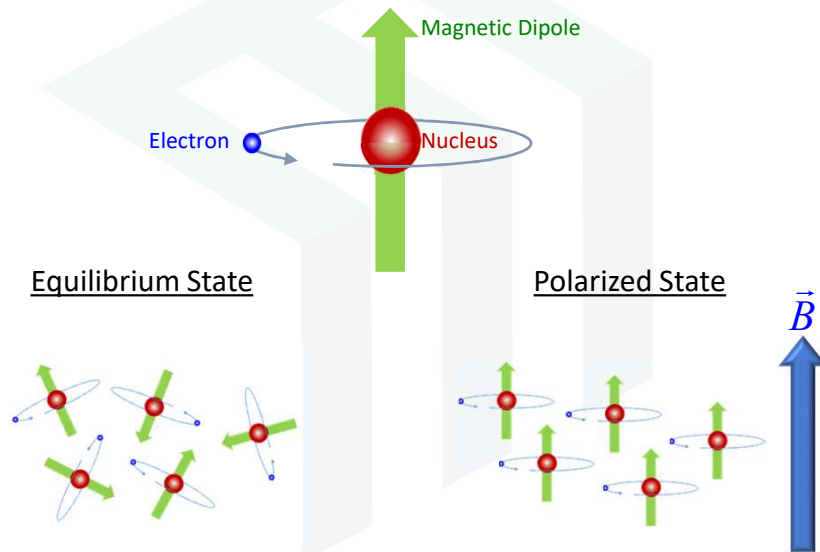
Mechanisms Producing Electric Polarization

Mechanism	No applied field	Applied field
Dipole or orientational polarization		
Ionic or molecular polarization		
Electronic polarization		

C. Balanis, *Advanced Engineering Electromagnetics*, (Wiley, New York, 1989).

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Magnetic Response of Ordinary Materials



Lorentz Oscillator Model for Dielectrics

Governing Equation

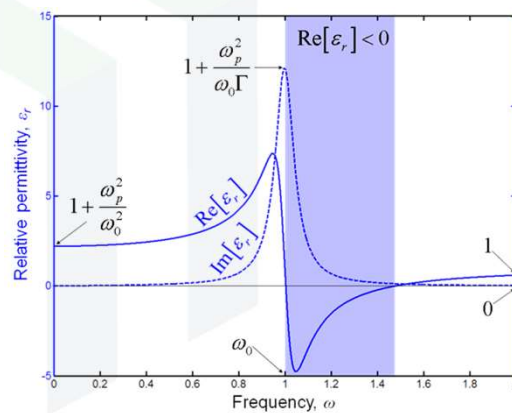
$$m \frac{\partial^2 \vec{r}}{\partial t^2} + m\Gamma \frac{\partial \vec{r}}{\partial t} + m\omega_0^2 \vec{r} = -q\vec{E}$$

inertia loss restoring force applied electric force

Resulting Dielectric Function

$$\epsilon_r = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - j\omega\Gamma}$$

$$\omega_p^2 = \frac{Nq^2}{\epsilon_0 m}$$



Drude Model for Metals

Governing Equation

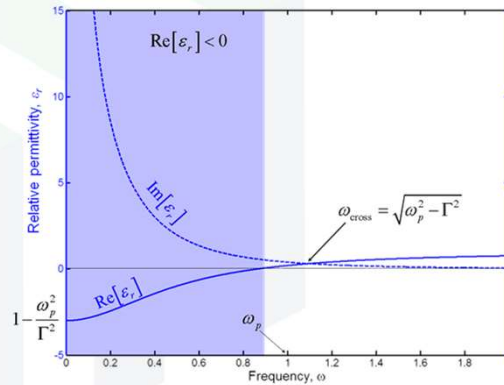
$$m \frac{\partial^2 \vec{r}}{\partial t^2} + m\Gamma \frac{\partial \vec{r}}{\partial t} + \cancel{m\omega_0^2 \vec{r}} = -q\vec{E}$$

Electrons are not bound so restoring force is zero.

Resulting Dielectric Function

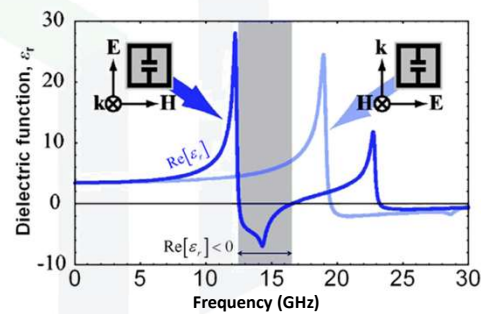
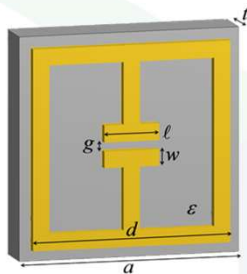
$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2 + j\omega\Gamma}$$

$$\omega_p^2 = \frac{Nq^2}{\epsilon_0 m}$$



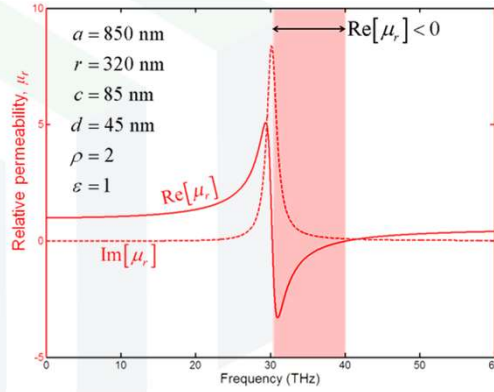
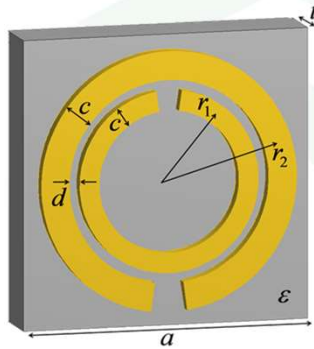
Artificial Permittivity, ϵ

$a = 3.333$ mm
 $d = 3.0$ mm
 $\ell = 1.0$ mm
 $w = 0.25$ mm
 $g = 0.25$ mm
 $t = 0.203$ mm
 $\epsilon = 4.3$



D. Schurig, J. J. Mock, D. R. Smith, "Electric-field-coupled resonators for negative permittivity metamaterials," Appl. Phys. Lett. **88**, 041109 (2006).

Artificial Permeability, μ

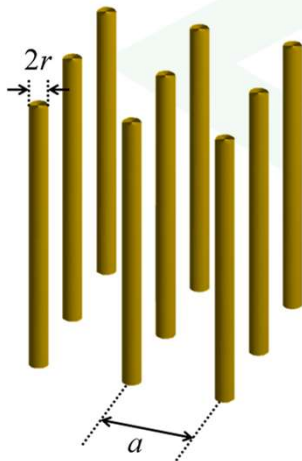


J. B. Pendry, A. J. Holden, D. J. Robbins, W. J. Stewart, "Magnetism from Conductors and Enhanced Nonlinear Phenomena," IEEE Trans. Microwave Theory and Techniques 47(11), 2075–2084 (1999).



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Artificial Plasma Frequency



$$N_{\text{eff}} = \frac{\pi r^2 N}{a^2}$$

$$m_{\text{eff}} = \frac{\mu_0 \pi r^2 N_{\text{eff}} q^2}{2\pi} \ln\left(\frac{a}{2r}\right)$$

$$\omega_{\text{p,eff}}^2 = \frac{2\pi c^2}{a^2 \ln(a/2r)}$$

Quantities previously related to the atoms and molecules can now be engineered through structural dimensions.

J. B. Pendry, A. J. Holden, W. J. Stewart, I Youngs, "Extremely Low Frequency Plasmons in Metallic Mesostructures," Phys. Rev. Lett. 76, 4773-4776 (1996).



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Categorizing Metamaterials Based on Sign of Effective Properties

Double Positive (DP)

$$+\varepsilon, +\mu$$

- Ordinary effective media
- Right-handed media (RHM)

Single Negative (SN)

$$-\varepsilon, +\mu \quad \text{or} \quad +\varepsilon, -\mu$$

- Can be left-handed or right-handed
- Artificial metals and magnetic conductors

Double Negative (DN)

$$-\varepsilon, -\mu$$

- Left-handed media (LHM)
- Negative refractive index (NRI)
- Negative thickness devices

$$n < 1.0$$

$$|n| = \left| \sqrt{\mu\varepsilon} \right| < 1.0$$

- Faster than light propagation
- Invisibility and cloaking
- Angle insensitive devices

Left-Handed Metamaterials

What Happens When Both μ and ε are Negative?

What happens when both μ and ε are negative?

$$\begin{aligned} \nabla \times \vec{E} &= -j\omega(-\tilde{\mu})\vec{H} \\ \nabla \times \vec{H} &= j\omega(-\tilde{\varepsilon})\vec{E} \end{aligned} \quad \longrightarrow \quad \begin{aligned} \nabla \times \vec{E} &= -j\omega\tilde{\mu}(-\vec{H}) \\ \nabla \times (-\vec{H}) &= j\omega\tilde{\varepsilon}\vec{E} \end{aligned}$$

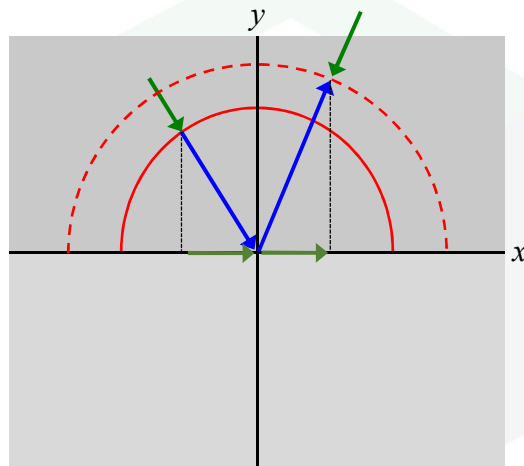
The system is left handed!

$$\vec{k} \perp \vec{E} \perp (-\vec{H})$$

$$-\vec{k} \perp \vec{E} \perp \vec{H}$$

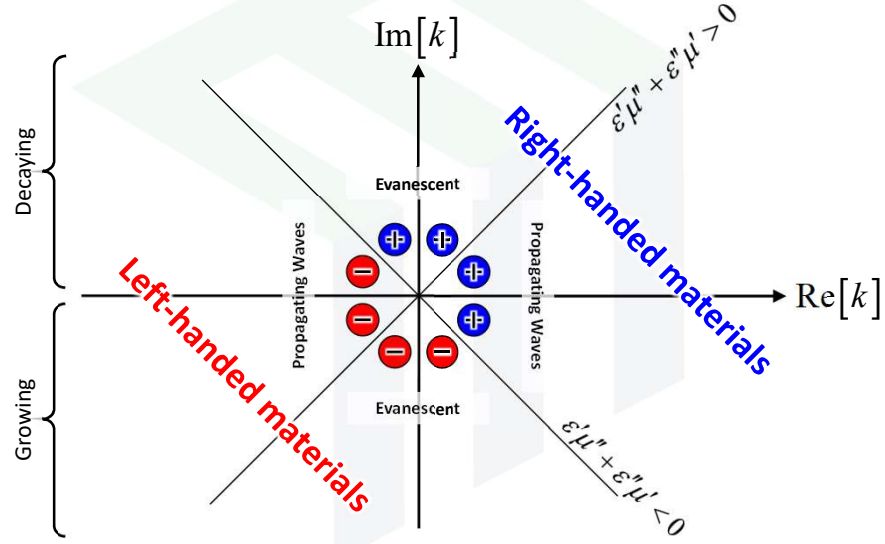
\vec{E} , \vec{H} , and \vec{k} form a "left-handed" system

LHMs Have a Negative Refractive Index

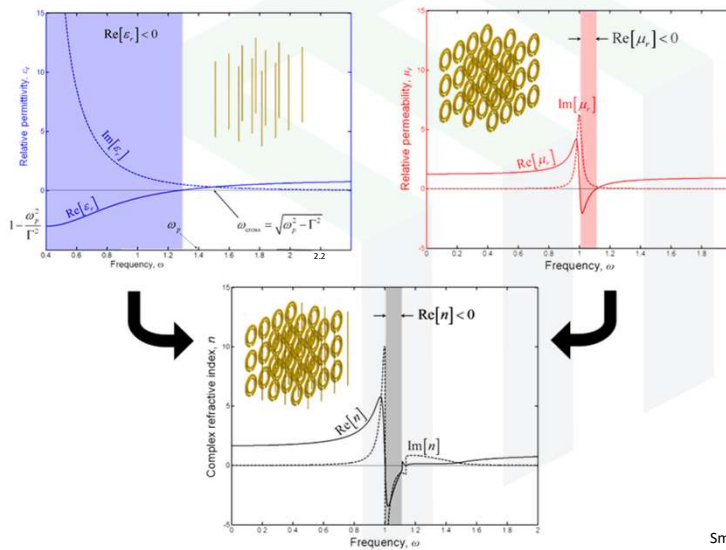


1. Wave is incident from ordinary right-handed material.
2. Transverse component of k is continuous across interface.
3. In the left-handed material, k_y must be negative to be consistent with sign convention.
4. Index ellipsoid for left-handed material must be negative, implying that the refractive index is negative.
5. Causality requires Poynting vector to correspond to "forward" travelling energy.

Conditions for Negative Refractive Index



How to Realize a Left-Handed Metamaterial



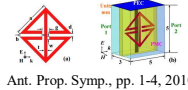
Smith et al., Phys. Rev. Lett. **84**, 4184–4187 (2000).

Low Loss LHMs

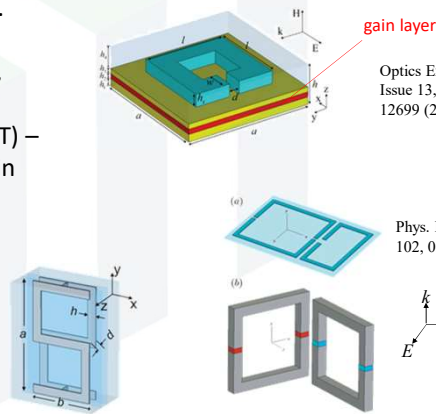
The negative permittivity and permeability due to electromagnetic resonance can be prohibitively lossy.

Approaches for minimizing loss:

1. Optimizing Geometry – Limited success so far.
2. Incorporate Gain – can achieve zero loss over broad frequency range, but difficult, complex, and expensive to realize in practice.
3. Electromagnetically Induced Transparency (EIT) – Very low loss, but do not maintain negativity in the real part of μ and ϵ . If realized, they are narrowband and involve using two different types of metals.
4. Alternate designs.



Ant. Prop. Symp., pp. 1-4, 2010.

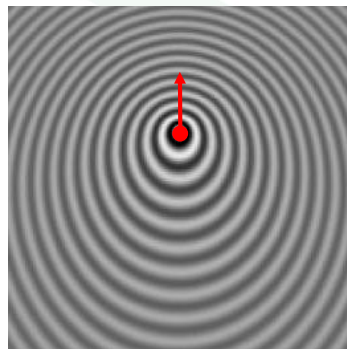


Optics Express, Vol. 19, Issue 13, pp. 12688-12699 (2011).

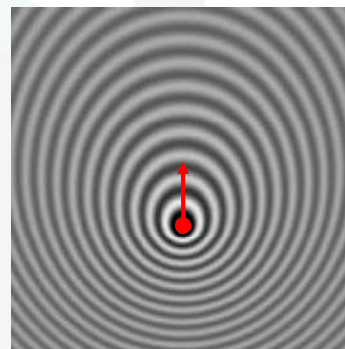
Phys. Rev. Lett., Vol. 102, 051901, 2009.

Doppler Shift in LHMs

Doppler shift is reversed in a left-handed metamaterial.



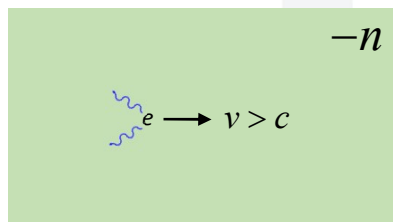
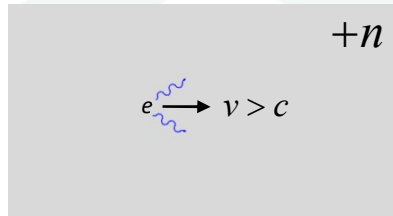
Right-handed material



Left-handed material

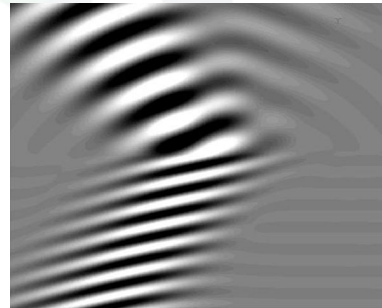
Cerenkov Radiation in LHMs

Cerenkov radiation is reversed.



Refraction in LHMs

Left-handed materials produce negative refraction.



Evanescent Waves in LHMs

Evanescent fields grow in amplitude.

Evanescent "wave" in ordinary material

$$E(z) = E_0 e^{j(k' + jk'')nz} = E_0 \cdot \underbrace{e^{-k''nz}}_{\text{evanescent component (decays)}} \cdot \underbrace{e^{jk'nz}}_{\text{oscillating component}}$$

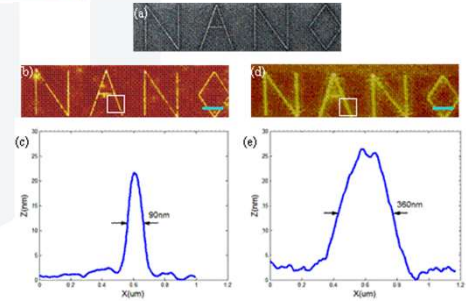
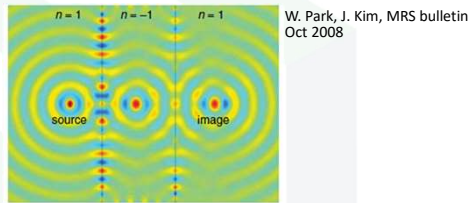
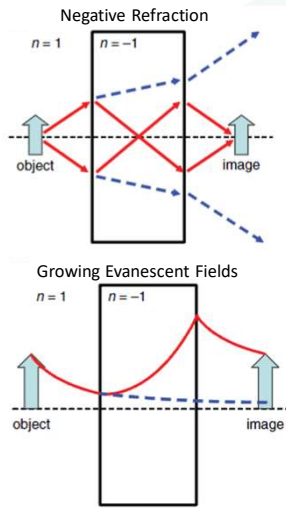
Evanescent "wave" in negative-index material

$$E(z) = E_0 e^{j(k' + jk'')(-\tilde{n})z} = E_0 \cdot \underbrace{e^{k''\tilde{n}z}}_{\text{evanescent component (grows)}} \cdot \underbrace{e^{-jk'\tilde{n}z}}_{\text{oscillating component}}$$



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Perfect Imaging and Super Lenses



N. Fang et al, Science 308, 534-537 (2005)

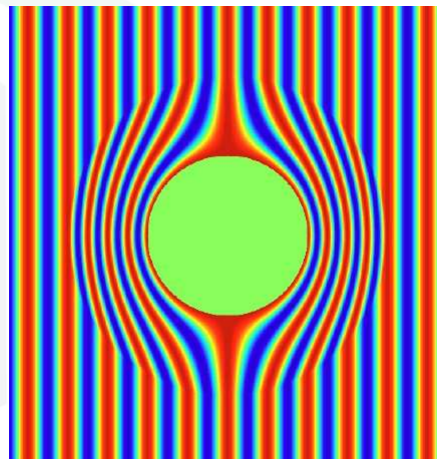
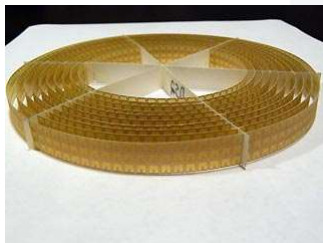
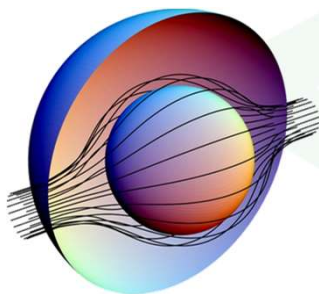


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Refractive Index Less Than One

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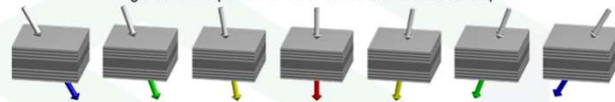
Cloaking and Invisibility



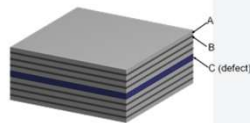
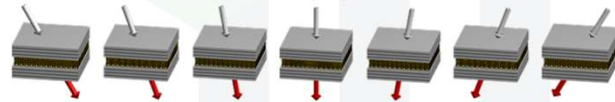
Slide 28

Zero-Thickness Devices

Angle sensitive performance of conventional diffractive optics

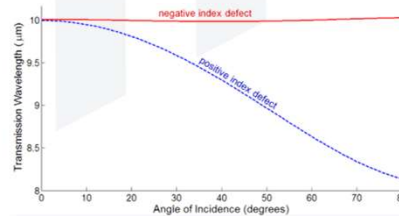


Omni-directional performance using negative refractive index materials



Positive Index Defect
 $n_x = 1.5$ $d_x = 1.50 \mu\text{m}$
 $n_y = 3.5$ $d_y = 0.64 \mu\text{m}$
 $n_z = 1.5$ $d_z = 0.59 \mu\text{m}$

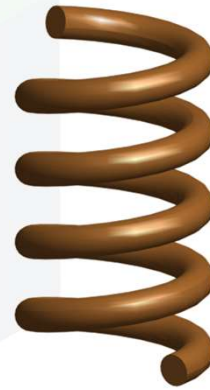
Negative Index Defect
 $n_x = 4.6$ $d_x = 1.63 \mu\text{m}$
 $n_y = 1.6$ $d_y = 1.63 \mu\text{m}$
 $n_z = -2.5$ $d_z = 2.01 \mu\text{m}$



Bi-Isotropic and Bi-Anisotropic Metamaterials

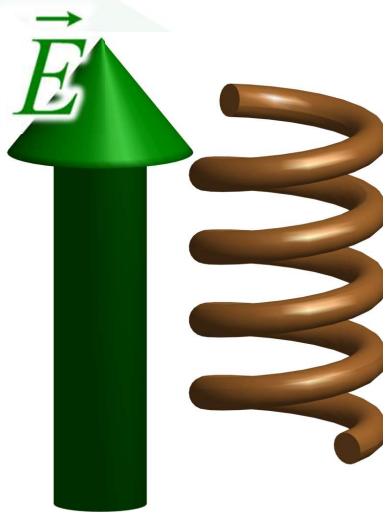
The Basic Bianisotropic Element (1 of 4)

A helix is one of the most basic and easiest to understand elements that produces a bianisotropic response.



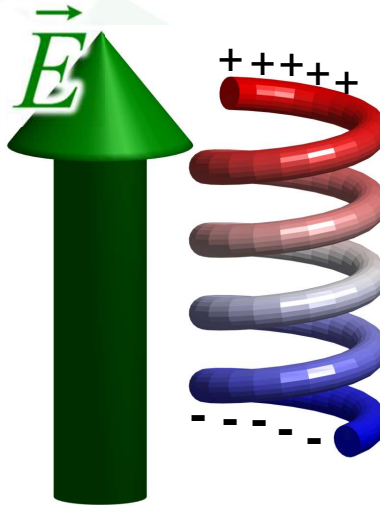
The Basic Bianisotropic Element (2 of 4)

Suppose an electric field is applied to the helix.



The Basic Bianisotropic Element (3 of 4)

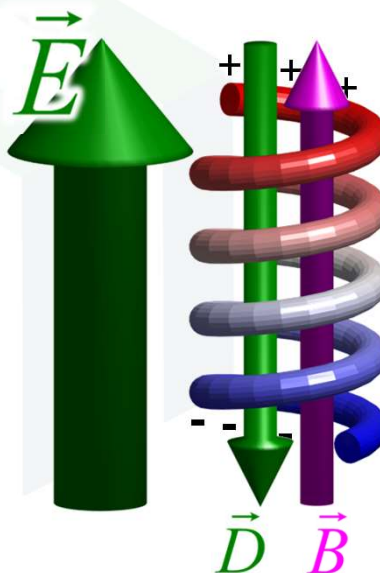
Charge along the wire will be displaced due to the electric field.



The Basic Bianisotropic Element (4 of 4)

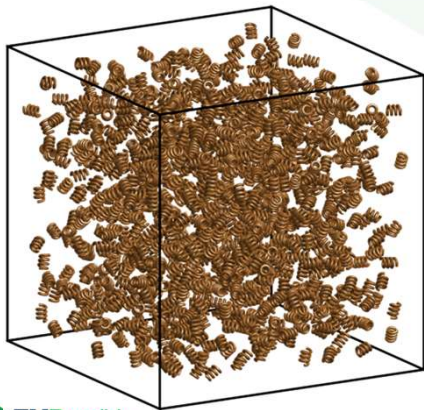
The displaced charge induces an electric dipole \vec{D} , while the current produced by the displaced charge induces magnetic flux \vec{B} .

\vec{E} has induced \vec{B} !

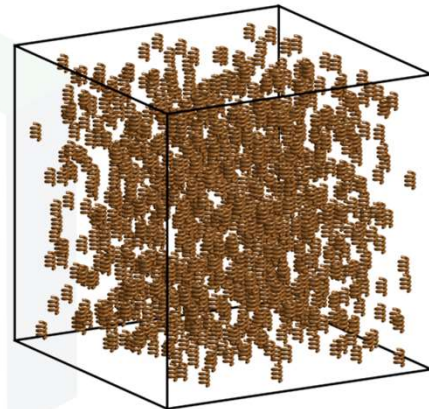


Arrangement of Elements

An array of aligned helices produces a bianisotropic medium.



EMPossible



An array of helices in random orientation produces an ordinary isotropic medium because the dipoles cancel.

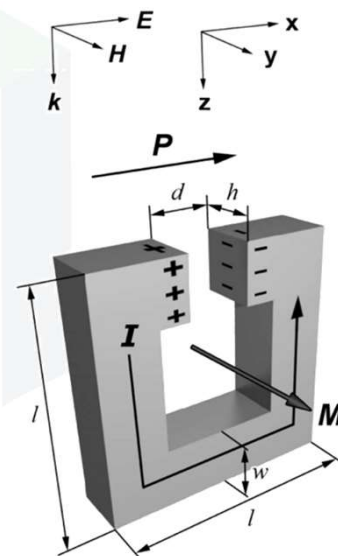
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Origin of Biisotropy and Bianisotropy

Biisotropy and bianisotropy occurs whenever a metamaterial lacks a center of inversion symmetry along the direction of propagation.

If this SRR had a split at the bottom and top, it would exhibit ordinary anisotropy.

Kriegler, Christine Eliane, et al. "Bianisotropic photonic metamaterials." Selected Topics in Quantum Electronics, IEEE Journal of 16.2 (2010): 367-375.



EMPossible

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Fishnet Metamaterial

Ku, Zahyun, Jingyu Zhang, and S. R. J. Brueck. "Bi-anisotropy of multiple-layer fishnet negative-index metamaterials due to angled sidewalls." *Optics express* 17.8 (2009): 6782-6789.

NOT BI



BI

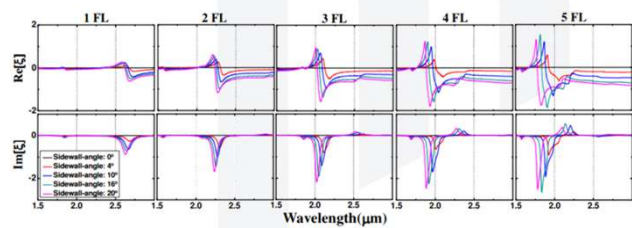
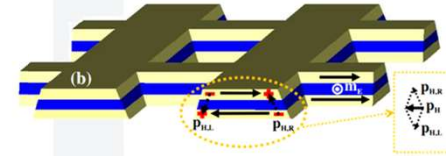
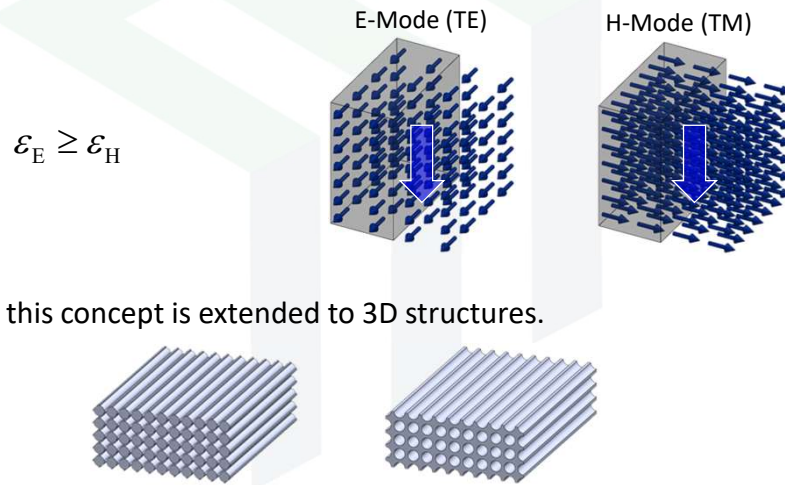


Fig. 5. Real and imaginary parts of the bi-anisotropic parameter (ζ) for one- up to five-functional layered fishnet NIMs with SWAs (0° , 4° , 10° , 16° , 20°).

Nonresonant Anisotropic Metamaterials

How to Produce Artificial Anisotropy in Metamaterials

Recall from a previous lecture that subwavelength gratings are artificially anisotropic media.



For metamaterials, this concept is extended to 3D structures.

Positive and Negative Birefringence

For uniaxial lattices, there is only an ordinary and an extraordinary dielectric constant, ϵ_o and ϵ_e .

Here the birefringence is defined as the difference between the extraordinary and ordinary dielectric constants.

$$\Delta\epsilon = \epsilon_e - \epsilon_o$$

It is possible to have a positive or a negative birefringence.

$$\epsilon_e < \epsilon_o \quad \text{negative birefringence } (\Delta\epsilon < 0)$$

$$\epsilon_e > \epsilon_o \quad \text{positive birefringence } (\Delta\epsilon > 0)$$

Metamaterials with Positive and Negative Birefringence

Positive Birefringence

These are equal due to symmetry. Therefore, these are the ordinary axes.

$\epsilon_e > \epsilon_0$

Anisotropy Cheat Sheet

Negative Birefringence

These are equal due to symmetry. Therefore, these are the ordinary axes.

$\epsilon_e < \epsilon_0$

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Classes of Anisotropic Metamaterials

Isotropic

(a)

Uniaxial

+

(c)

-

(d)

Biaxial

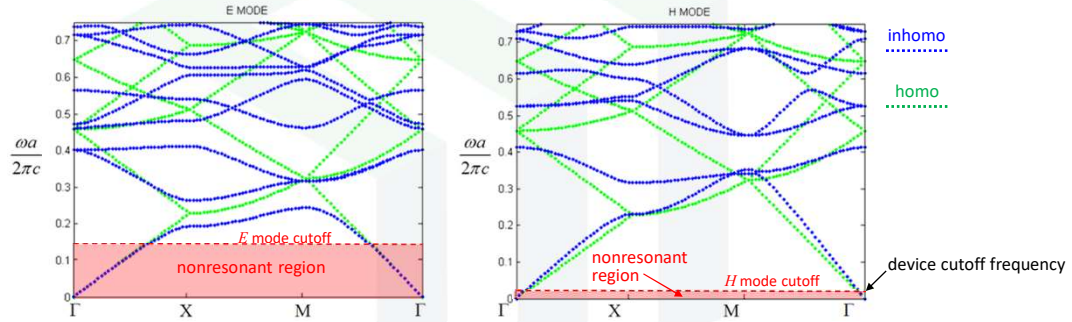
(e)

(f)

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Cutoff Frequency

Anisotropic metamaterials possess a cutoff frequency. They must operate below this frequency to ensure the structure is nonresonant.



$$n_{\text{homo}} = \frac{1}{V} \iiint_V n \, dv$$

$$\frac{1}{n_{\text{homo}}} \approx \frac{1}{V} \iiint_V \frac{1}{n} \, dv$$

Artificial anisotropy requires both \vec{E} and \vec{H} modes to be nonresonant. The cutoff frequency is taken as the lowest cutoff between the two. In this case, it is the \vec{H} mode.

Design of Negative Uniaxial Metamaterials by $\Delta\epsilon$

Limiting Values

$$\epsilon_{\min} \leq \epsilon \leq \epsilon_{\max} \quad \begin{aligned} \epsilon_{\min}^{-1} &= f\epsilon_1^{-1} + (1-f)\epsilon_2^{-1} \\ \epsilon_{\max} &= f\epsilon_1 + (1-f)\epsilon_2 \end{aligned}$$

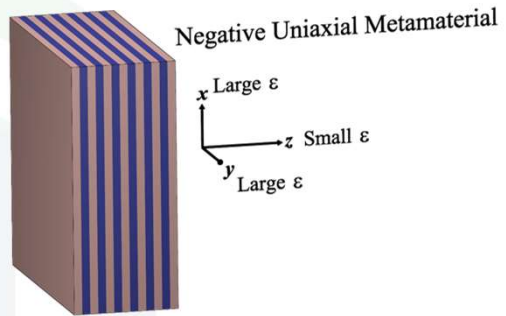
Strength of Anisotropy

$$\Delta\epsilon \leq \epsilon_{\max} - \epsilon_{\min} = \frac{(\epsilon_1 - \epsilon_2)^2}{\epsilon_1/f + \epsilon_2/(1-f)}$$

Optimum Fill Fraction for ϵ_1

$$f_{\max \Delta\epsilon} = \frac{1}{1 + \sqrt{\epsilon_2/\epsilon_1}}$$

$$\Delta\epsilon_{\max} = \epsilon_1 + \epsilon_2 - 2\sqrt{\epsilon_1\epsilon_2}$$

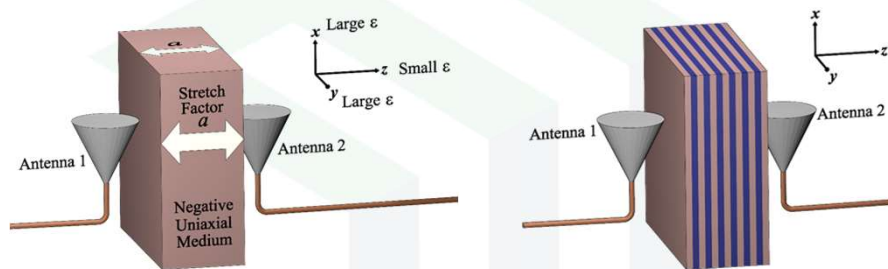


Negative Uniaxial Anisotropy

$$[\epsilon] = \begin{bmatrix} \epsilon_o & 0 & 0 \\ 0 & \epsilon_o & 0 \\ 0 & 0 & \epsilon_c \end{bmatrix} \quad \epsilon_o > \epsilon_c$$

$$\epsilon_o = \epsilon_{\max} \quad \epsilon_c = \epsilon_{\min}$$

Design of Negative Uniaxial Metamaterials by TO



Coordinate Transform

$$\begin{aligned} x' &= x \\ y' &= y \quad a > 1 \\ z' &= z/a \end{aligned}$$

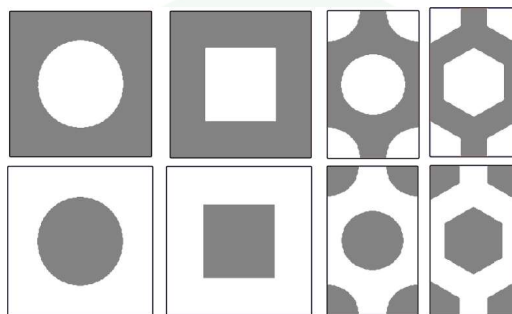
Negative Uniaxial Medium

$$[\epsilon] = \begin{bmatrix} \epsilon a & 0 & 0 \\ 0 & \epsilon a & 0 \\ 0 & 0 & \epsilon/a \end{bmatrix}$$

TO Parameters

$$\begin{aligned} \epsilon_o &= \epsilon a & \epsilon_c &= \epsilon/a \\ \epsilon &= \sqrt{\epsilon_o \epsilon_c} & a &= \sqrt{\epsilon_o / \epsilon_c} \end{aligned}$$

Optimization of $\Delta\epsilon$ in a Positive Uniaxial Metamaterial



C. R. Garcia, J. Correa, D. Espalin, J. H. Barton, R. C. Rumpf, R. Wicker, V. Gonzalez, "3D Printing of Anisotropic Metamaterials," PIER Lett, Vol. 34, pp. 75-82, 2012.

Fractal unit cells may provide slightly higher anisotropy, but will be more difficult to manufacture.

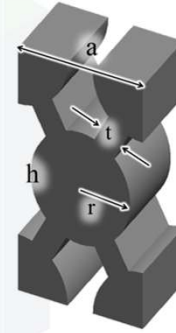
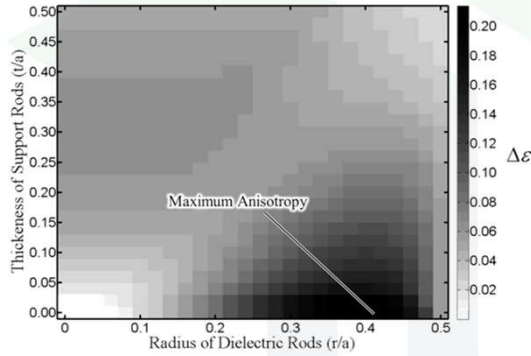
A close second is a hexagonal array of circular rods.

Much easier to manufacture.

Best geometry is a hexagonal array of hexagonal shaped rods with high ϵ_r surrounded by low ϵ_r .

Hexagonal shaped rods is more difficult to manufacture.

Optimization of $\Delta\epsilon$ of a Positive Uniaxial Metamaterial



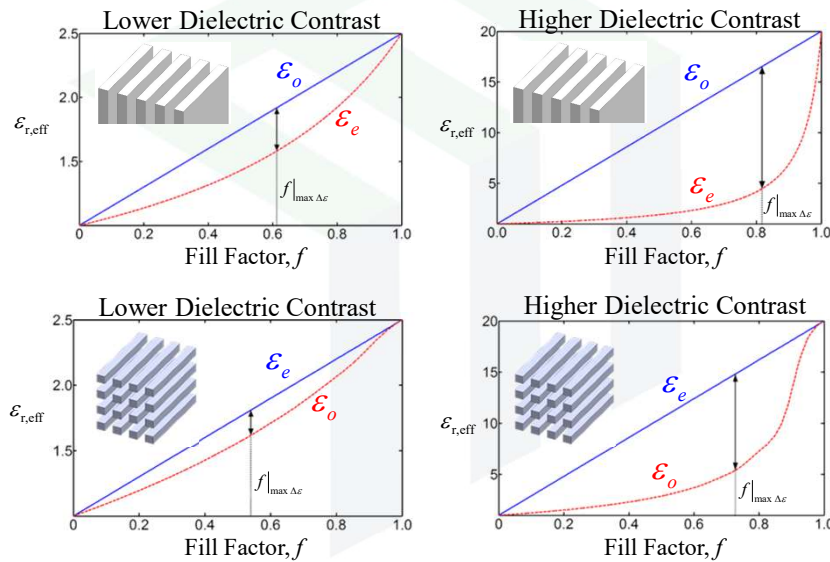
- Support rods decrease the anisotropy, but may be needed for support.
- Radius of cylinders should be $\sim 42\%$ of lattice constant. This seems to be consistent regardless of dielectric constant used.
- It may be true that uniaxial symmetry provides the strongest anisotropy.

C. R. Garcia, J. Correa, D. Espalin, J. H. Barton, R. C. Rumpf, R. Wicker, V. Gonzalez, "3D Printing of Anisotropic Metamaterials," PIER Lett, Vol. 34, pp. 75-82, 2012.



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Study of Strength of Anisotropy in Uniaxial Structures



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Optimizing a in Negative Uniaxial Metamaterials

Applying Effective Medium Theory

$$\frac{1}{\varepsilon_e} = \frac{f}{\varepsilon_1} + \frac{1-f}{\varepsilon_2} \quad \varepsilon_o = f\varepsilon_1 + (1-f)\varepsilon_2$$

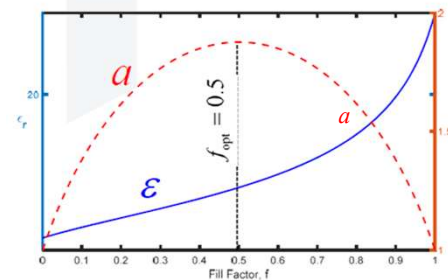
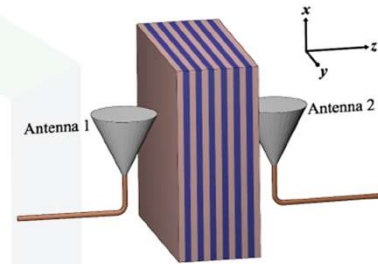
TO Parameters

$$\varepsilon = \sqrt{\frac{f\varepsilon_1 + (1-f)\varepsilon_2}{f\varepsilon_1^{-1} + (1-f)\varepsilon_2^{-1}}} \quad a = \sqrt{1 + f(1-f) \frac{(\varepsilon_1 - \varepsilon_2)^2}{\varepsilon_1\varepsilon_2}}$$

Maximizing a

$$f_{\text{opt}} = 0.5 \quad \Rightarrow \quad \varepsilon(f_{\text{opt}}) = \sqrt{\varepsilon_1\varepsilon_2}$$

This means that optimized SVAMs can be designed without knowing ε_1 and ε_2 .

$$a(f_{\text{opt}}) = \frac{\varepsilon_1 + \varepsilon_2}{2\sqrt{\varepsilon_1\varepsilon_2}}$$


Nonmagnetic SVAMs

Result Provided by TO

$$\begin{aligned} x' &= x \\ y' &= y \\ z' &= z/a \end{aligned} \quad \Rightarrow \quad [\mu] = [\varepsilon] = \begin{bmatrix} \varepsilon a & 0 & 0 \\ 0 & \varepsilon a & 0 \\ 0 & 0 & \varepsilon/a \end{bmatrix}$$

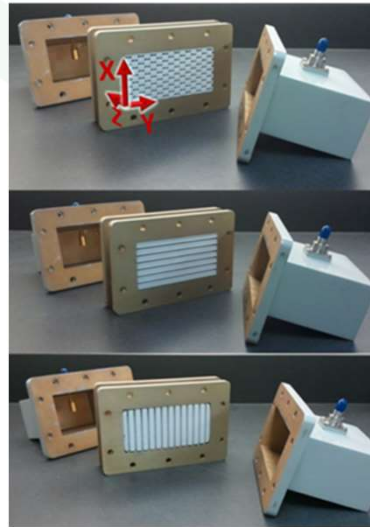
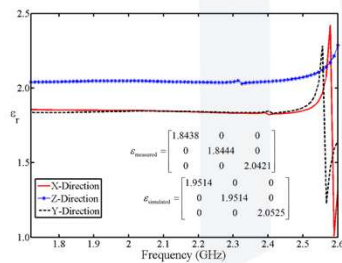
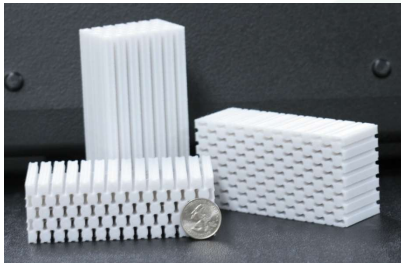
Note that to realize a stretching factor of a , permeability and permittivity must be equal.

Nonmagnetic Result?

$$[\mu] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \neq [\varepsilon] = \begin{bmatrix} \varepsilon a & 0 & 0 \\ 0 & \varepsilon a & 0 \\ 0 & 0 & \varepsilon/a \end{bmatrix}$$

Maybe $a \rightarrow \sqrt{a}$???

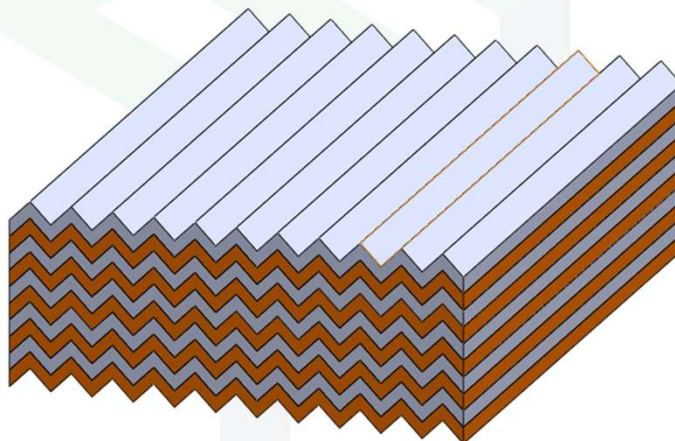
Anisotropic Metamaterials Manufactured by 3D Printing



C. R. Garcia, J. Correa, D. Espalin, J. H. Barton, R. C. Rumpf, R. Wicker, V. Gonzalez, "3D Printing of Anisotropic Metamaterials," PIER Lett, Vol. 34, pp. 75-82, 2012.

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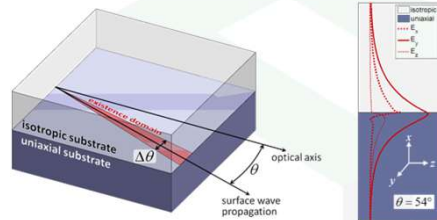
Anisotropic Metamaterials Fabricated by Autoclaving



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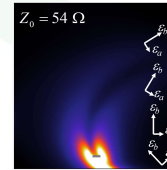
Some Applications

Dyakonov Surface Waves



O. Takayama, et al. "Dyakonov Surface Waves: A Reivew,"
Electromagnetics **28**, 126-145 (2008).

Near-Field Sculpting



Hyperbolic Metamaterials

Recall the Dispersion Surfaces of Uniaxial Crystals

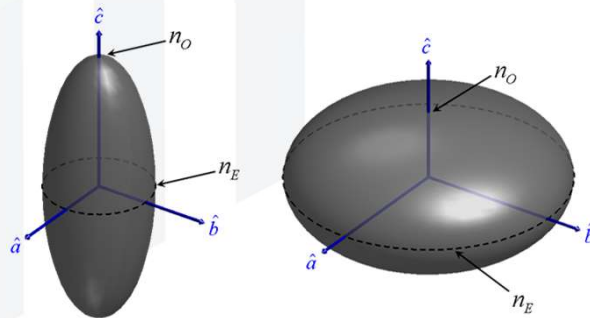
Dielectric Tensor

$$\begin{bmatrix} \epsilon_o & 0 & 0 \\ 0 & \epsilon_o & 0 \\ 0 & 0 & \epsilon_e \end{bmatrix}$$

Dispersion Relation (Ellipsoids)

$$\frac{k_x^2 + k_y^2}{\epsilon_e} + \frac{k_z^2}{\epsilon_o} = \left(\frac{\omega}{c_0} \right)^2$$

These ellipsoids are closed surfaces.
Possible values of \vec{k} are constrained.



Hyperbolic Tensors

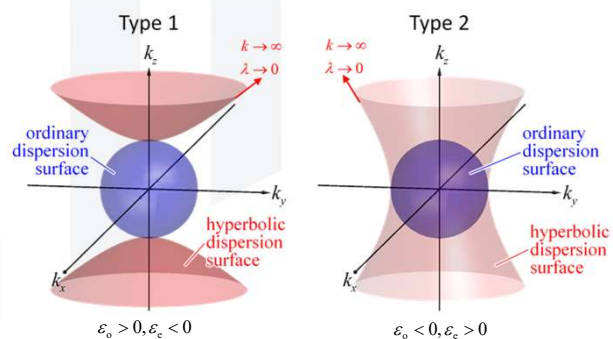
Suppose the tensor elements ϵ_o and ϵ_e had opposite sign.

$$\epsilon_o \cdot \epsilon_e < 0$$

The dispersion relation now describes open surfaces that extend to infinity.

$$\frac{k_x^2 + k_y^2}{\epsilon_e} + \frac{k_z^2}{\epsilon_o} = \left(\frac{\omega}{c_0} \right)^2$$

These hyperbolas are open surfaces.
Possible values of \vec{k} extend to infinity.



What is New and Magical?

Values of \vec{k} in hyperbolic materials can go to infinity.

This means there can exist propagating waves in hyperbolic materials with extremely large values of \vec{k} . For a given frequency, these will have extremely small wavelength.

$$|\vec{k}| = \frac{2\pi}{\lambda} \quad |\vec{k}| \rightarrow \infty \text{ means } \lambda \rightarrow 0$$

These waves would be cutoff, or evanescent, in ordinary materials.

These “high- k ” waves enable many things. Miniaturization is a big one!

Types of Hyperbolic Metamaterials

Effective Metals

$$\begin{bmatrix} -\varepsilon_a & 0 & 0 \\ 0 & -\varepsilon_b & 0 \\ 0 & 0 & -\varepsilon_c \end{bmatrix}$$

All waves evanescent.
Not really hyperbolic.

Type I

$$\begin{bmatrix} -\varepsilon_a & 0 & 0 \\ 0 & \varepsilon_b & 0 \\ 0 & 0 & \varepsilon_c \end{bmatrix} \text{ or } \begin{bmatrix} \varepsilon_a & 0 & 0 \\ 0 & -\varepsilon_b & 0 \\ 0 & 0 & \varepsilon_c \end{bmatrix} \text{ or } \begin{bmatrix} \varepsilon_a & 0 & 0 \\ 0 & \varepsilon_b & 0 \\ 0 & 0 & -\varepsilon_c \end{bmatrix}$$

Low loss, good impedance match to air because they are predominantly dielectric like.

Type II

$$\begin{bmatrix} -\varepsilon_a & 0 & 0 \\ 0 & -\varepsilon_b & 0 \\ 0 & 0 & \varepsilon_c \end{bmatrix} \text{ or } \begin{bmatrix} -\varepsilon_a & 0 & 0 \\ 0 & \varepsilon_b & 0 \\ 0 & 0 & -\varepsilon_c \end{bmatrix} \text{ or } \begin{bmatrix} \varepsilon_a & 0 & 0 \\ 0 & -\varepsilon_b & 0 \\ 0 & 0 & -\varepsilon_c \end{bmatrix}$$

High loss, high impedance mismatch because they are predominantly metal like.

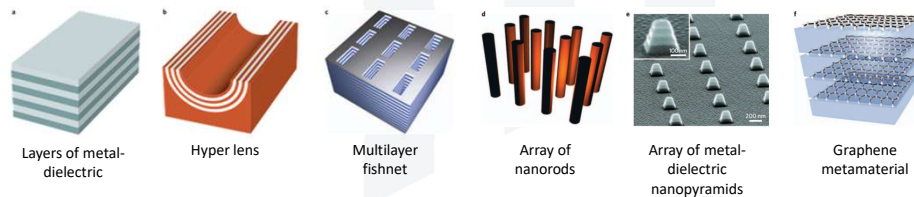
How to Make Hyperbolic Metamaterials

Metals have a dielectric constant that is negative below the plasma frequency.

A negative dielectric constant implies the movement of electrons is constrained in that direction.

In hyperbolic materials, not all tensor elements are negative. This means the continuity of the metal has to be broken in the direction of positive tensor elements.

So, any structure where metal is continuous in some directions but not others will be hyperbolic.



Phenomena and Applications

- Negative Refraction (dispersion effect, not NRI)
- Lensing
- Spontaneous emission engineering (Purcell-factor enhancement)
- Nanophotonics
- Minaturization
- Tunable active devices
- Heat transfer engineering
 - Efficiency can exceed the Stefan-Boltzmann law
 - Broadband emission can exceed the Planck law.

Future Steps

- Wide spectral and wave vector ranges.
- Losses must be mitigated.
- Performance should be maintained for different orientations and polarizations.
- Improved methods for coupling from air into the hyperbolic metamaterials