



Advanced Electromagnetics:
21st Century Electromagnetics

Photonic Crystals

Lecture Outline

- Introduction
- Origin of the Band Gap
- Band Gap Engineering
- Devices Based on Band Gap Engineering
- Dispersion Engineering
- Devices Based on Dispersion Engineering
 - Self-Collimation

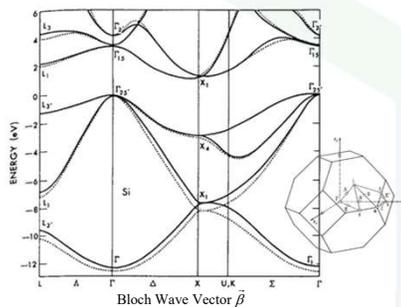
Introduction

Slide 3

Electromagnetic Bands

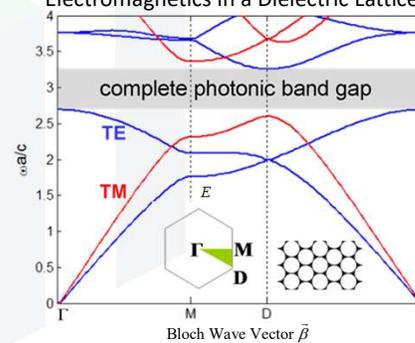
$$-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r})+V(\vec{r})\psi(\vec{r})=E\psi(\vec{r})$$

Electronic Band Diagram for Electrons in Semiconductors



$$\nabla^2\vec{E}(\vec{r})-k^2\vec{E}(\vec{r})=\vec{E}_{\text{src}}(\vec{r})$$

Photonic Band Diagram for Electromagnetics in a Dielectric Lattice



Due to the analogy between Schrödinger's equation and the wave equation, electromagnetic waves can be controlled inside periodic lattices like electrons are controlled in semiconductors.

Origin of the Band Gap

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The Bloch Theorem

The field inside of a periodic structure takes on the same symmetry and periodicity of that structure according to the Bloch theorem.

$$\vec{E}(\vec{r}) = \vec{A}(\vec{r}) e^{j\vec{\beta} \cdot \vec{r}}$$

Given the lattice translation vectors, the periodicity can be described mathematically.

$$\vec{A}(\vec{r} + \vec{t}) = \vec{A}(\vec{r}) \quad \vec{t} \equiv \text{lattice vector}$$

Slide 6

The Wave Equation in Periodic Structures

Maxwell's curl equations for *non-magnetic* materials are

$$\nabla \times \vec{E} = -j\omega\mu_0\vec{H} \quad \nabla \times \vec{H} = j\omega\varepsilon_0\varepsilon_r\vec{E}$$

The wave equation for the magnetic field \mathbf{H} can be derived by taking the curl of the second equation above and substituting the result in the first equation.

$$\nabla \times \frac{1}{\varepsilon_r} \nabla \times \vec{H} = k_0^2 \vec{H}$$

According to the Bloch theorem, the magnetic field is periodic as follows.

$$\vec{H}(\vec{r}) = \vec{H}_{\vec{\beta}}(\vec{r}) \cdot e^{j\vec{\beta} \cdot \vec{r}}$$

Substituting this into the wave equation leads to

$$(\nabla + j\vec{\beta}) \times \frac{1}{\varepsilon_r} (\nabla + j\vec{\beta}) \times \vec{H}_{\vec{\beta}} = \left(\frac{\omega_{\vec{\beta}}}{c_0} \right)^2 \vec{H}_{\vec{\beta}}$$

Solutions to the Wave Equation

The wave equation just derived is an eigen-value problem.

$$L\{\vec{H}_{\vec{\beta}}\} = v\vec{H}_{\vec{\beta}}$$

Eigen-value problems have discrete solutions (like modes in a waveguide) that are all orthogonal (very different from each other).

This means that electromagnetic waves in periodic structures only exist as discrete modes. These are called Bloch modes.

Fields can only exist as integer combinations of the eigen-modes, or Bloch modes, of the lattice.

$$\vec{H} = \sum_{\vec{\beta}} a_{\vec{\beta}} \vec{H}_{\vec{\beta}}$$

The variational theorem states that the lowest-order state satisfying the wave equation minimizes the following variational equation.

To minimize this equation, the denominator must be maximized. This happens when the most intense fields reside inside the high dielectric constant regions.

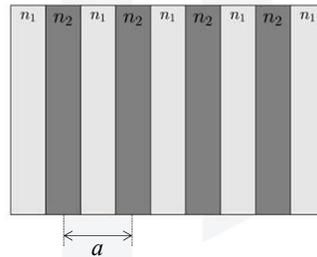
Conclusion – The field of the lowest order mode prefers to be in the high dielectric constant regions.

$$\left(\frac{\omega_{\vec{\beta}}}{c_0} \right)^2 \Rightarrow \min_{\Omega} \frac{\int_{\Omega} |(\nabla + j\vec{\beta}) \times \vec{E}_{\vec{\beta}}|^2 d\Omega}{\int_{\Omega} \varepsilon |\vec{E}_{\vec{\beta}}|^2 d\Omega}$$

The Electromagnetic Band Gap

To understand the origin of the band gap, recall three rules:

1. Bloch modes must have the same symmetry as the lattice.
2. Electric fields of the lowest order mode prefer to reside in higher index regions.
3. Modes must be orthogonal (very different).

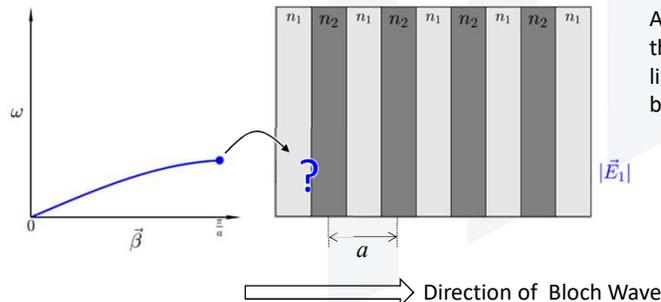


Consider a one-dimensional photonic crystal.

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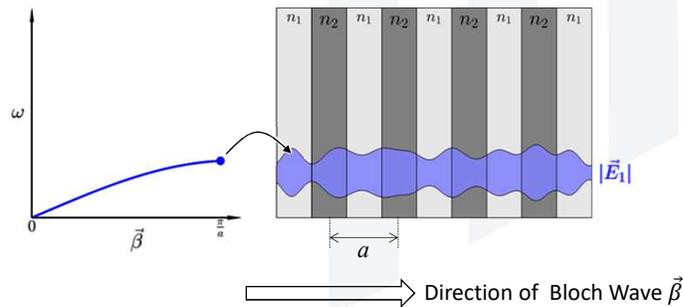
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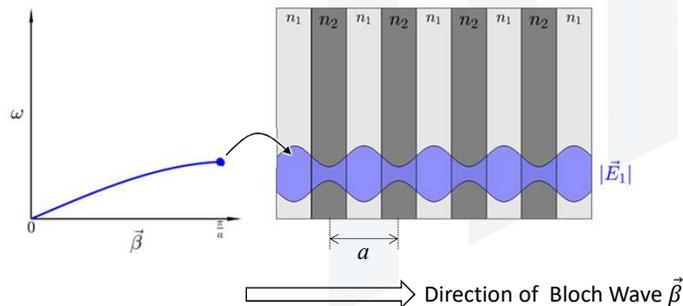


At $\beta = \pi/a$, what does the electric field look like for the lowest-order band?

The Electromagnetic Band Gap

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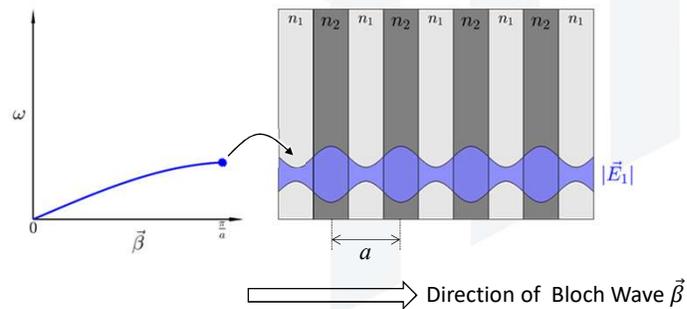


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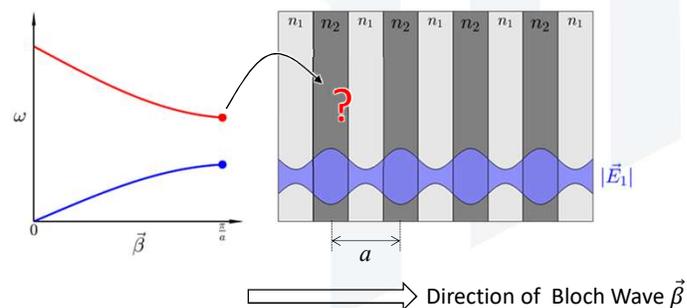


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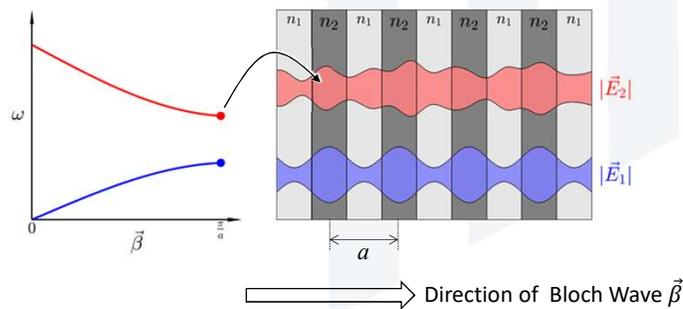


At $\beta = \pi/a$, what does the electric field look like for the second-order band?

The Electromagnetic Band Gap

To understand the origin of the band gap, recall three rules:

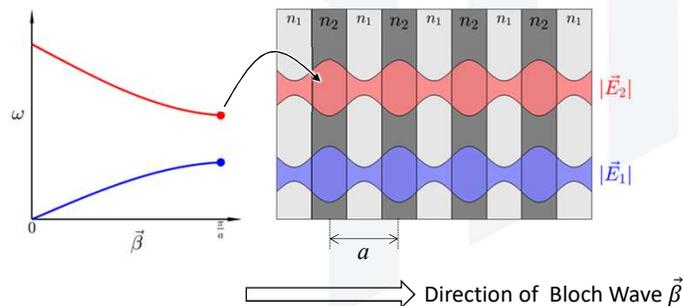
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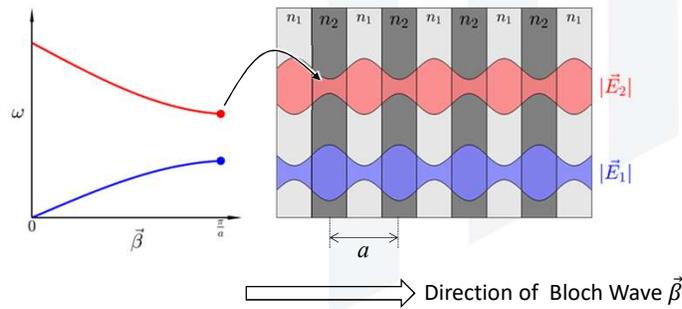
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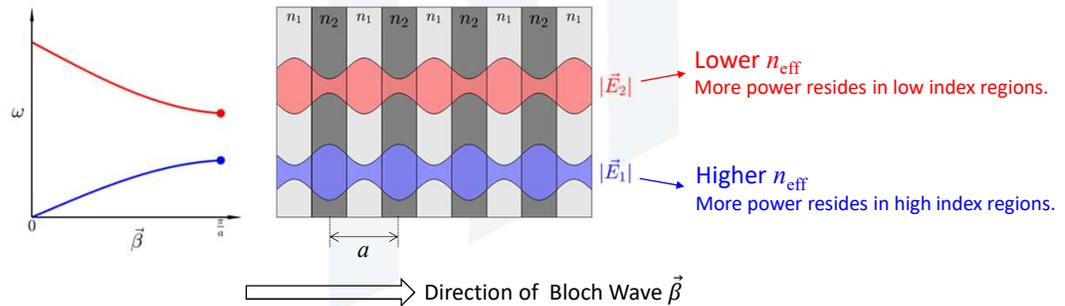
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At $\beta = \pi/a$, what does the electric field look like for the second-order band?

The Electromagnetic Band Gap

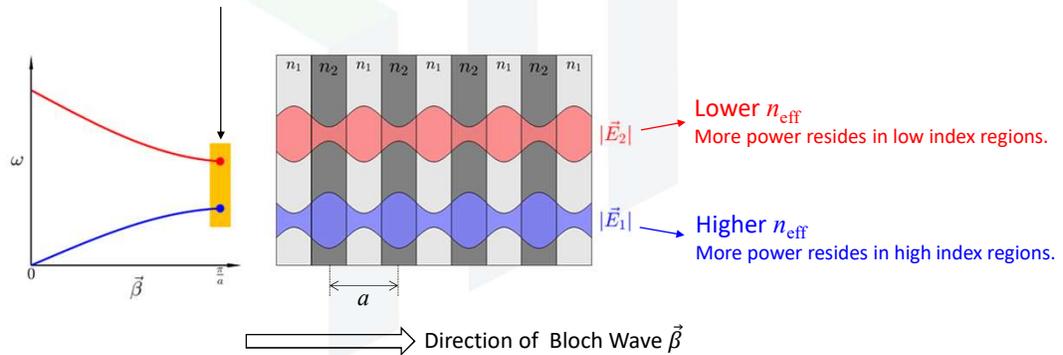
What about the effective refractive index n_{eff} of the modes?



The Electromagnetic Band Gap

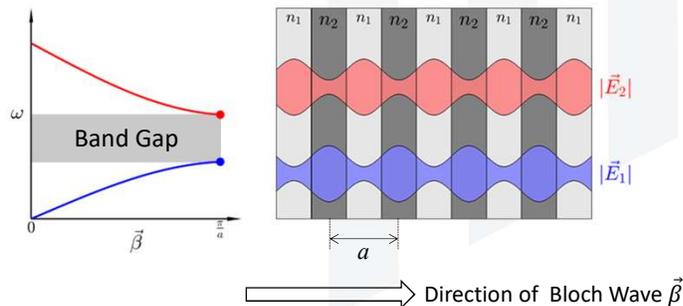
The two modes have the same wavelength but different effective refractive indices. How is this possible?

$\Delta\omega$ The modes must exist at different frequencies.



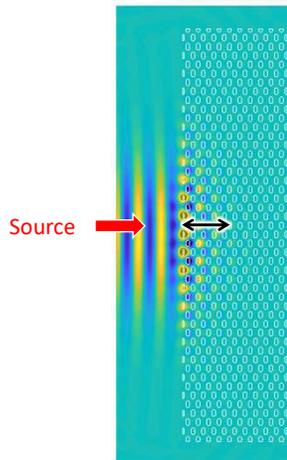
The Electromagnetic Band Gap

A band gap is defined as a range of frequencies over which no bands exist for all values of $\vec{\beta}$.



Behavior of Waves in the Band Gap

What if photonic crystal is illuminated by a wave at a frequency within its band gap?



The Bloch wave actually penetrates into the lattice by some distance.

Bloch waves still exist within the band gap. They are just cutoff and evanescent. This means they decay with distance into the lattice.

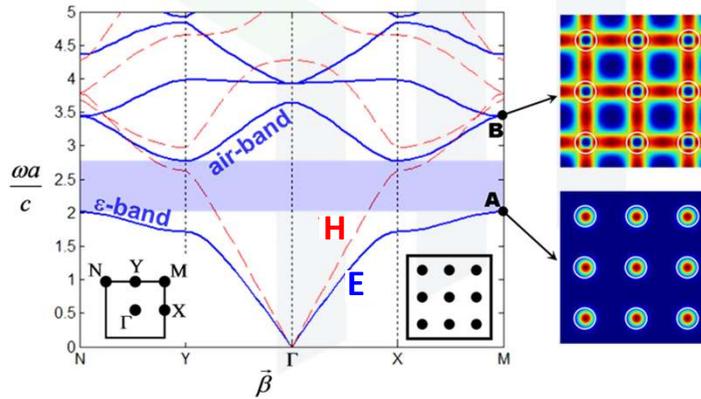
$$\vec{E}(\vec{r}) = \vec{A}(\vec{r}) \exp^{-\vec{\alpha} \cdot \vec{r}} \exp^{j\vec{\beta} \cdot \vec{r}}$$

Band Gap Engineering

Realizing a Complete 2D Band Gap (1 of 4)

The E modes have the electric field polarized perpendicular to the crystal plane so they can form isolated regions of high intensity surrounded by regions of low intensity.

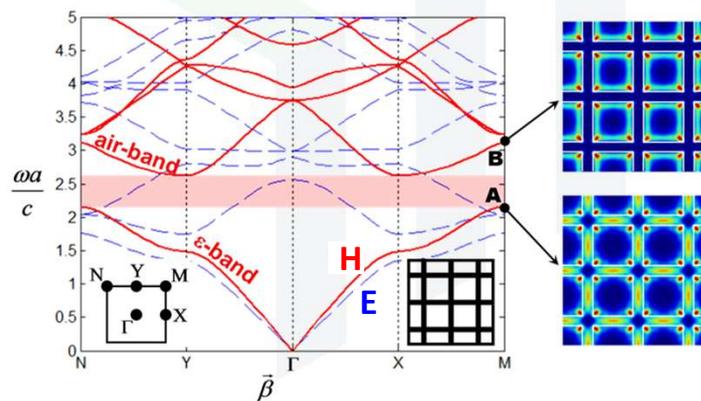
To maximize the disparity discussed previously, lattices supporting strong E -mode band gaps should be composed of isolated regions of high dielectric constant material.



Realizing a Complete 2D Band Gap (2 of 4)

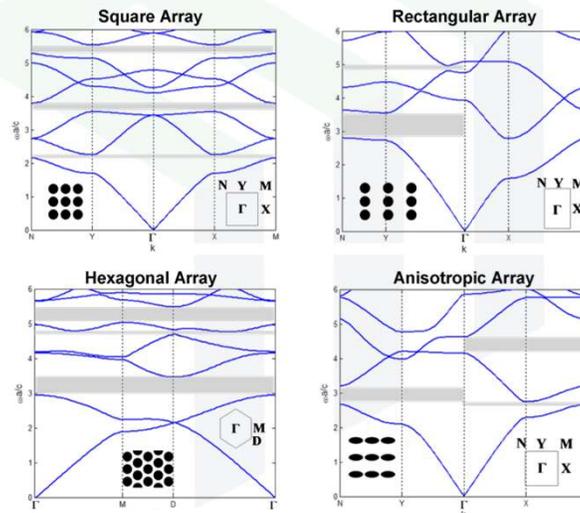
The H modes have the electric field polarized parallel to the crystal plane so it is a vector quantity and must form circular loops to satisfy Maxwell's equations.

To maximize the disparity discussed previously, lattices supporting strong H -mode band gaps should be composed of connected veins of high dielectric constant material.



Realizing a Complete 2D Band Gap (3 of 4)

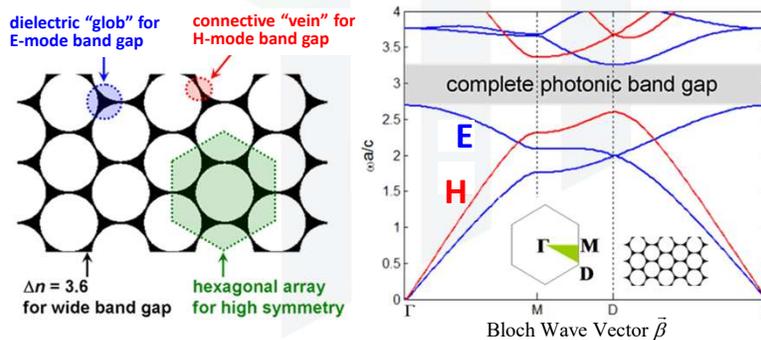
Lattices with greater symmetry support wider band gaps.



Realizing a Complete 2D Band Gap (4 of 4)

To realize a COMPLETE photonic band gap, we now know we need the following ingredients:

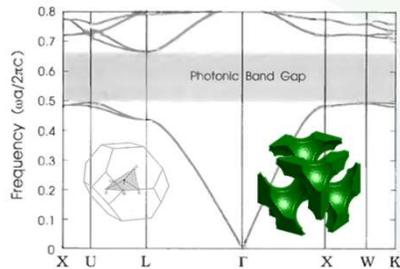
1. High contrast in the dielectric constant to maximize the disparity.
2. Isolated “globs” of high dielectric constant material for the E-modes.
3. Connected veins of high dielectric constant material for the H-modes.
4. High lattice symmetry for wider band gaps.



3D Band Gaps and Aperiodic Lattices

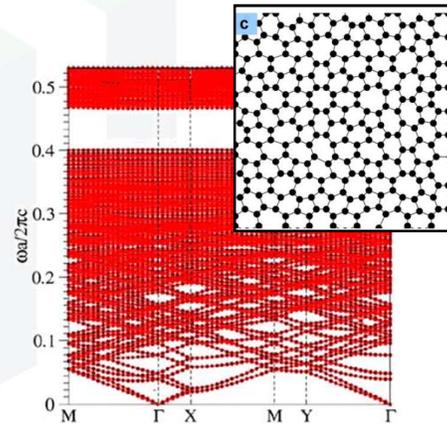
3D lattices are the only structures that can provide a true complete band gap.

The diamond lattice is known to have the strongest band gap of all 14 Bravais lattices.



J. Joannopoulos, "Photonic Crystals: Molding the Flow of Light," (Princeton University Press, ,1995).

Aperiodic lattices can have stronger band gaps than diamond.



M. Florescu, et al, "Complete band gaps in two-dimensional photonic quasicrystals," Phys. Rev. B 80, 155112 (2009).

Effects of Average Index and Index Contrast on Band Gap

Here the fractional bandwidth of the band gap is calculated as a function of average refractive index and the index contrast for a Bragg grating (1D photonic crystal).

$$n_1 = n_{\text{avg}} - \Delta n/2$$

$$n_2 = n_{\text{avg}} + \Delta n/2$$

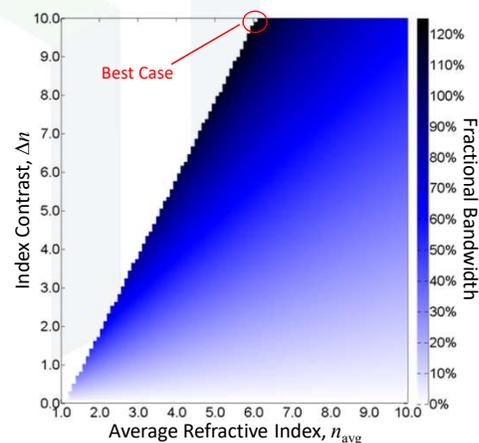
$$d_1 = \lambda_0/4n_1$$

$$d_2 = \lambda_0/4n_2$$

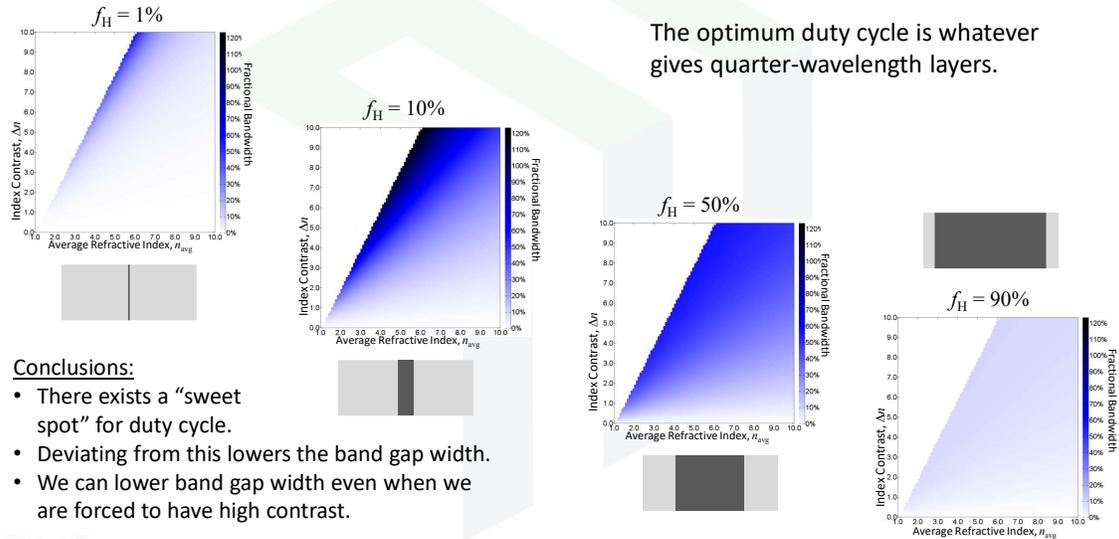
$$\text{FBW} = 2 \frac{k_{0,2} - k_{0,1}}{k_{0,2} + k_{0,1}}$$

Conclusions for Large Band Gaps:

- Want Δn as high as possible.
- Want n_{avg} as low as possible.



Effects of Duty Cycle on Width of Band Gap

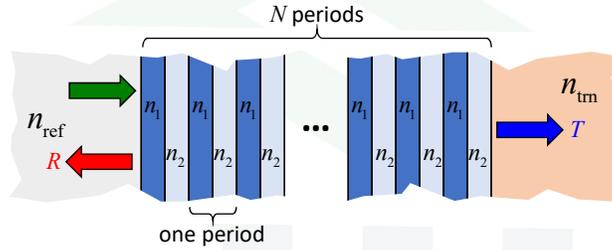


Conclusions:

- There exists a “sweet spot” for duty cycle.
- Deviating from this lowers the band gap width.
- We can lower band gap width even when we are forced to have high contrast.

Devices Based on Band Gap Engineering

Bragg Gratings

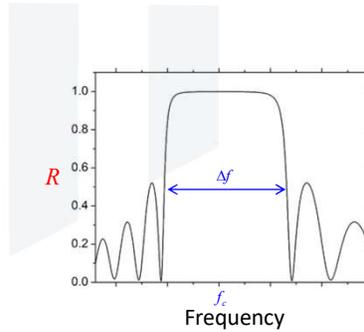


Peak Reflectivity

$$R(f_c) = \left(\frac{n_{\text{ref}} n_2^{2N} - n_{\text{tm}} n_1^{2N}}{n_{\text{ref}} n_2^{2N} + n_{\text{tm}} n_1^{2N}} \right)^2$$

Bandwidth of Stopband

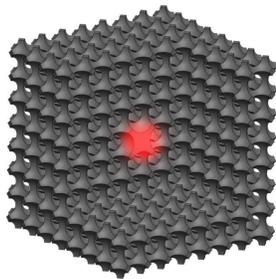
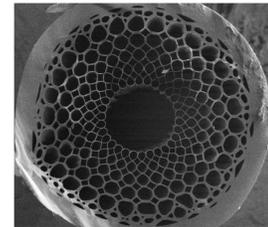
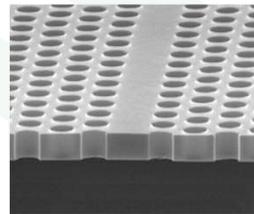
$$\frac{\Delta f}{f_c} = \frac{4}{\pi} \sin^{-1} \left(\frac{n_2 - n_1}{n_2 + n_1} \right)$$



Waveguides and Cavities Based on Electromagnetic Band Gaps

Waveguides

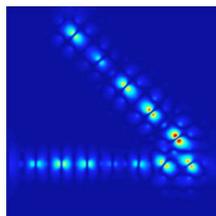
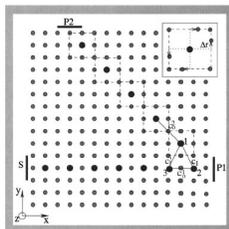
If light is forbidden to propagate inside a lattice, the lattice acts like a mirror throughout. This can be used to form waveguides.



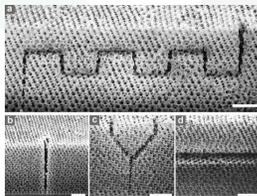
Cavities

If energy can be generated inside a lattice defect, it will be trapped and can be stored for long periods of time.

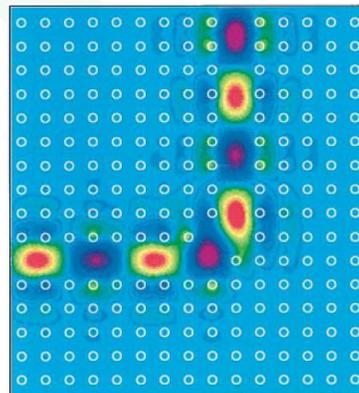
Tight Waveguide Bends



N. Malkove, C. Z. Ning, "Photonic crystal waveguides with acute bending angles," Appl. Phys. Lett. **87**, 161113 (2005).

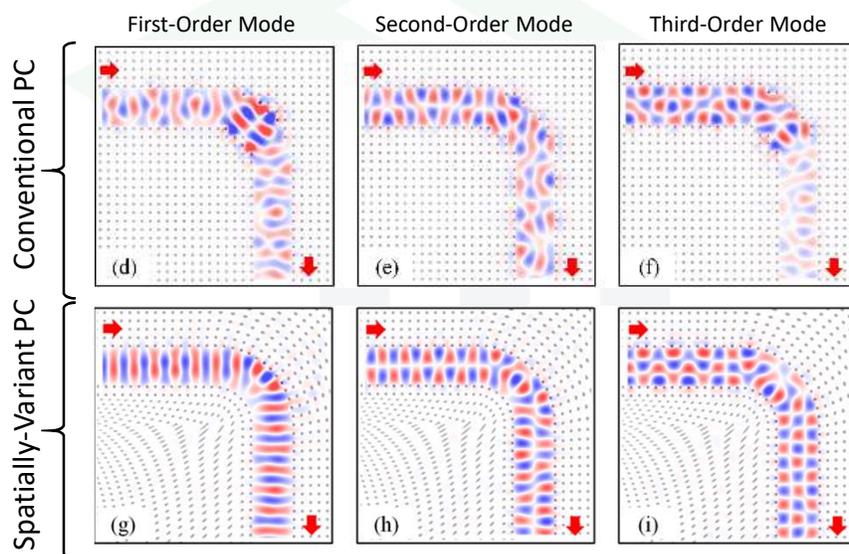


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A. Mekis *et al*, "High Transmission through Sharp Bends in Photonic Crystal Waveguides," Phys. Rev. Lett. **77**(18), 3787-3790 (1996).

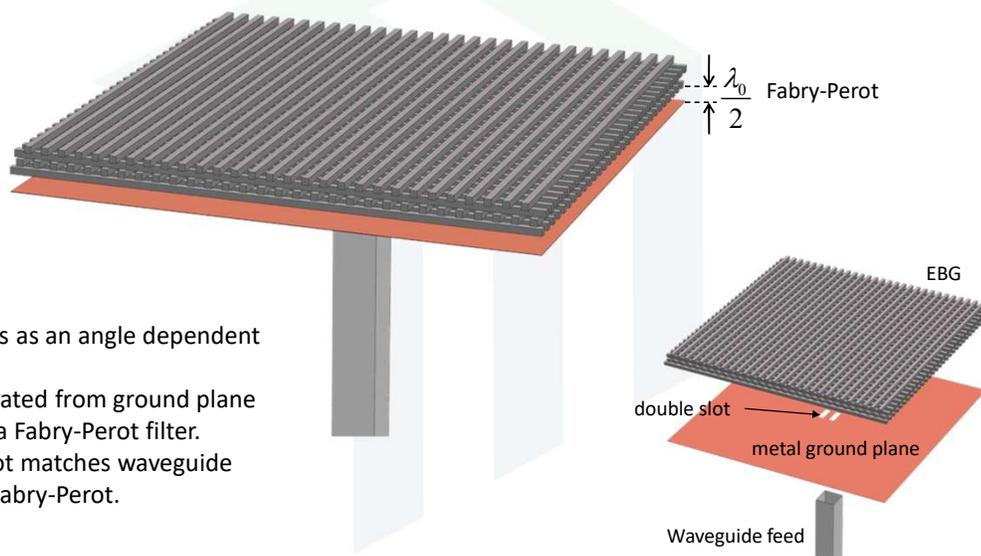
Tight Multimode Waveguide Bends



Antennas Utilizing EBGs

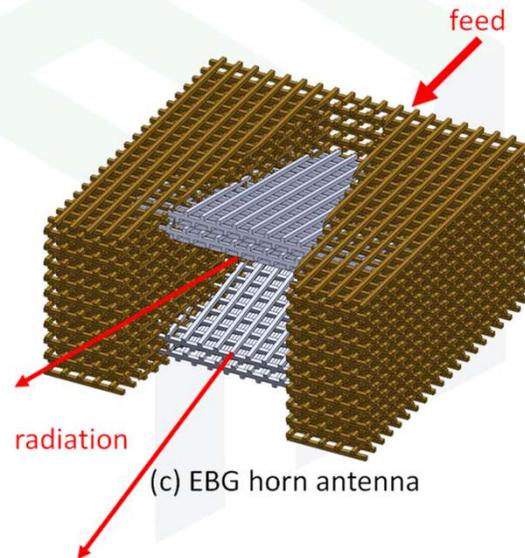
- Increase radiation efficiency by suppressing surface waves
- Use EBG material as an efficient reflector
- High directivity antennas based on angle dependent properties of EBGs
- All-dielectric “horn” antennas
- Additional degrees of freedom for polarization control

A Highly Directive EBG Antenna



1. EBG serves as an angle dependent mirror.
2. EBG separated from ground plane serves as a Fabry-Perot filter.
3. Double slot matches waveguide mode to Fabry-Perot.

All-Dielectric Horn Antenna



Dispersion Engineering

Phase, Group and Energy Velocity

Phase Velocity

Phase velocity describes the speed and direction of the phase of a wave.

$$\vec{v}_p = \frac{\omega}{|\vec{k}|} \hat{s} \quad n_p = \frac{c_0}{v_p}$$

Group Velocity

Group velocity describes the speed and direction of the envelope of a pulse.

$$\vec{v}_g = \nabla_{\vec{k}} \omega(\vec{k}) \quad n_g = \frac{c_0}{v_g}$$

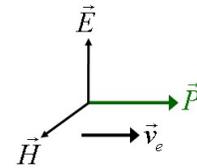
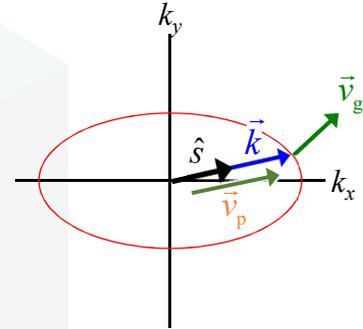
$$\vec{v}_g = \vec{v}_p \quad \text{for no dispersion}$$

Energy Velocity

Energy velocity describes the speed and direction of the energy.

$$\vec{v}_e = \frac{\vec{P}}{U} \quad n_e = \frac{c_0}{v_e}$$

$$\vec{v}_e = \vec{v}_g \quad \text{for linear materials}$$



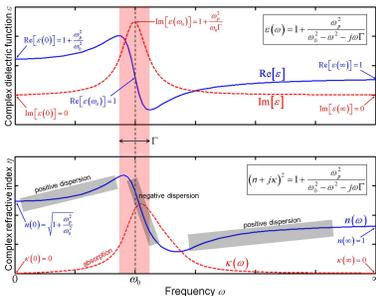
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Types of Dispersion

Material Dispersion

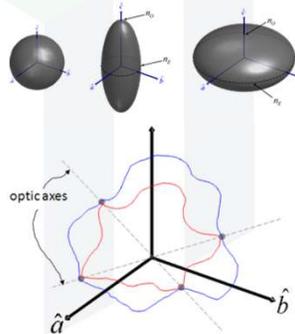
Waves of different frequency propagate at different speeds inside a material because the dielectric constant changes as a function of frequency.

$$\tilde{\epsilon}_r(\omega) = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - j\omega\Gamma}$$



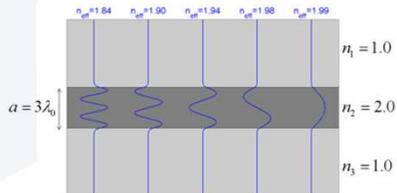
Spatial Dispersion

Waves travelling in different directions through anisotropic or periodic materials propagate at different speeds because the dielectric constant changes as a function of direction.



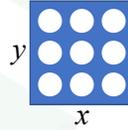
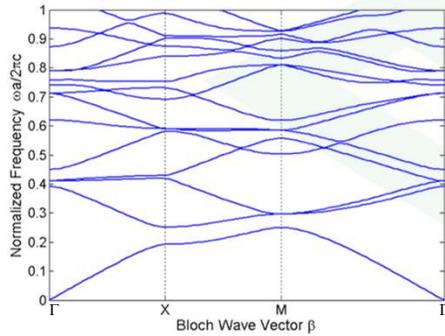
Modal Dispersion

Different modes inside a device typically propagate at different speeds because their energy is distributed differently inside the device leading to different effective refractive indices of the modes.



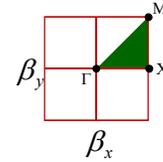
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The Band Diagram is Missing Information



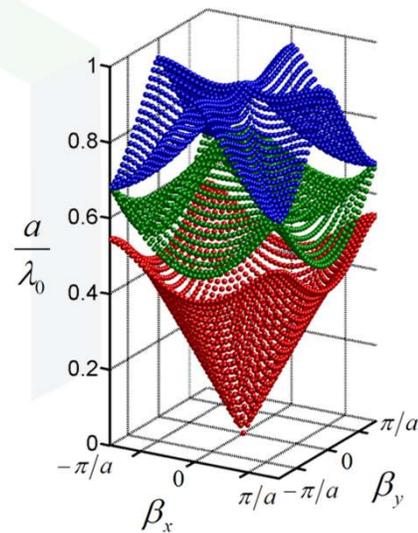
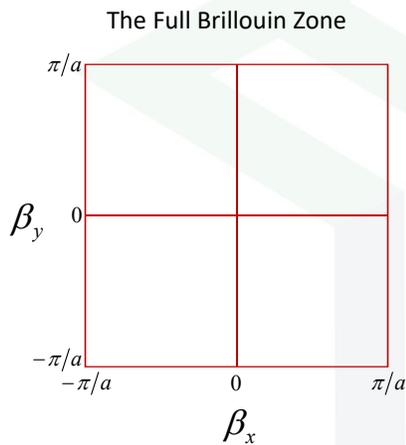
Direct lattice: We have an array of air holes in a dielectric with $n = 3.0$.

Reciprocal lattice: We construct the band diagram by marching around the perimeter of the irreducible Brillouin zone.



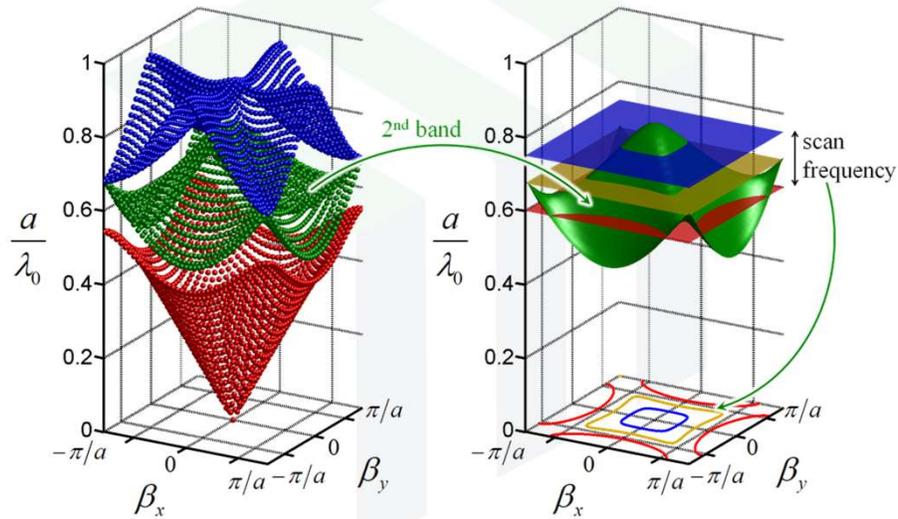
The band extremes “almost” always occur at the key points of symmetry.
But we are missing information from inside the Brillouin zone.

The Complete Band Diagram

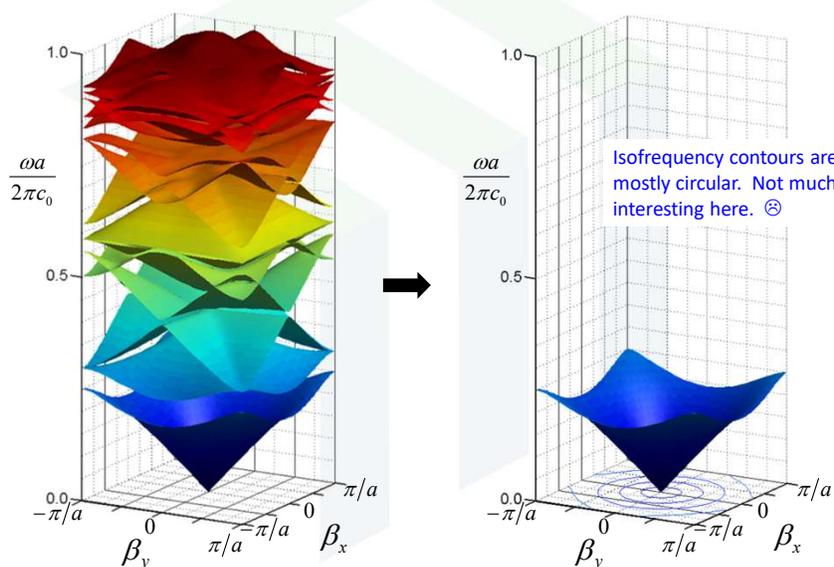


There is an infinite set of eigen-frequencies associated with each point in the Brillouin zone. These form “sheets” as shown at right.

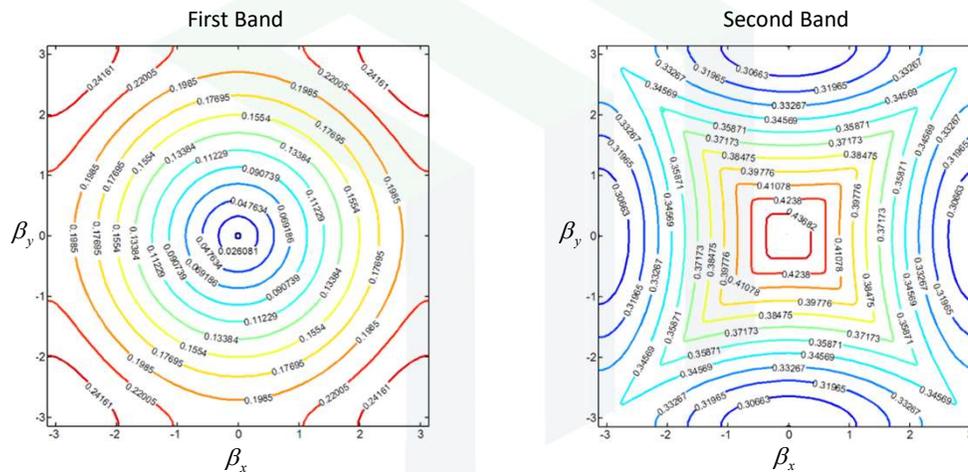
Constructing Isofrequency Contours (Index Ellipsoids)



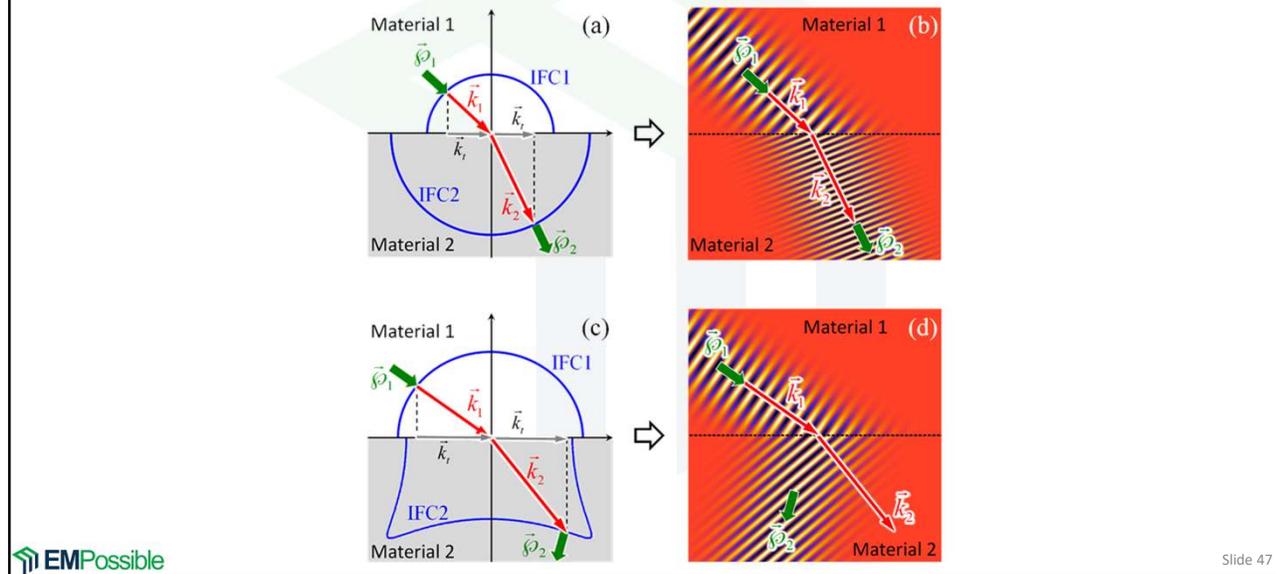
Isofrequency Contours From First-Order Band



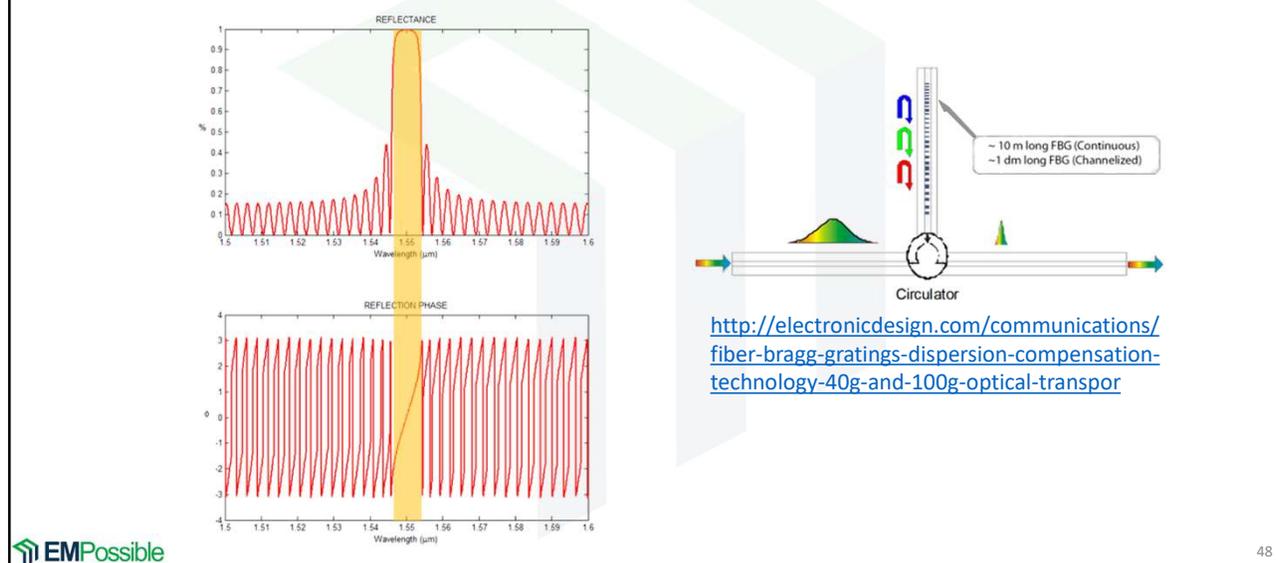
Standard View of Isofrequency Contours



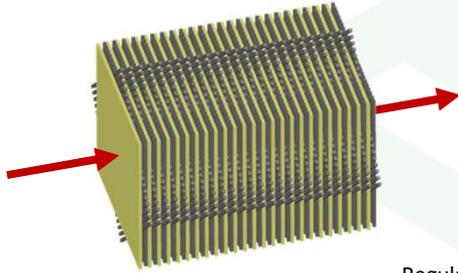
Negative Refraction Without Negative Refractive Index



Dispersion Compensation Using Chirped Bragg Gratings



Slow Wave Devices

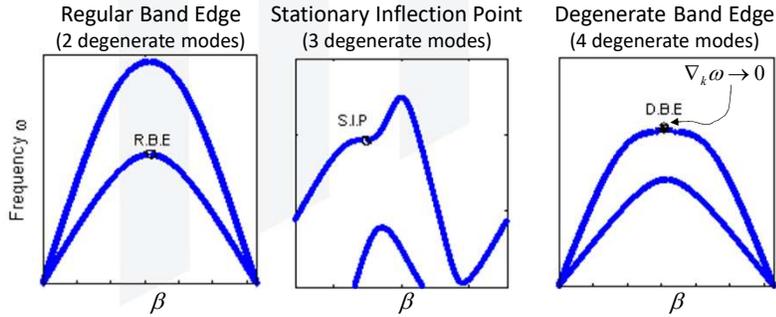


Degenerate modes have the same propagation constant so they easily exchange energy and are coupled as a result.

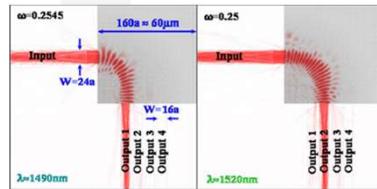
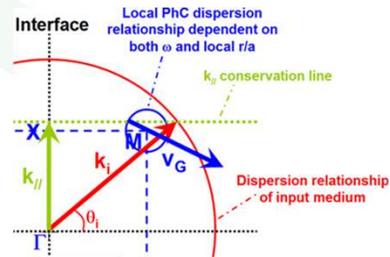
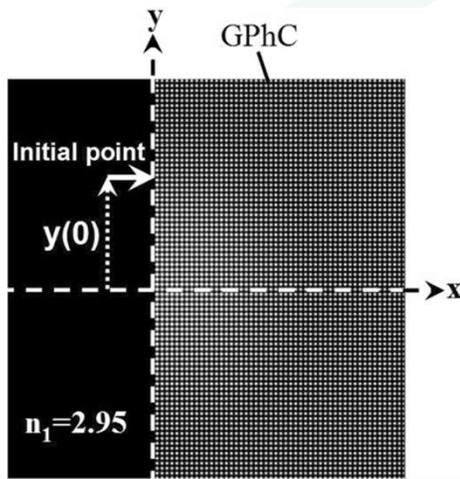
For a structure to support a DBE, all modes (TE and TM) are coupled at a single point. The slope of the band approaches zero, so group velocity does also.

$$\vec{v}_g = \nabla_{\vec{k}} \omega$$

Y. Cao, M. A. Fiddy, "Resonant Effect Analysis at Finite One Dimensional Anisotropic Photonic Crystal Band Edges," Proc. SPIE 6128 (2006).



Graded Photonic Crystals



Cassan, E., K. V. Do, C. Caer, D. Marris-Morini, and L. Vivien, "Short-wavelength light propagation in graded photonic crystals," Journal of Lightwave Technology, Vol. 29, 1937-1943, 2011.

Superprism Devices

H. Kosaka, et al, "Superprism phenomena in photonic crystals," Phys. Rev. B, Vol. 58, No. 16, 1998.

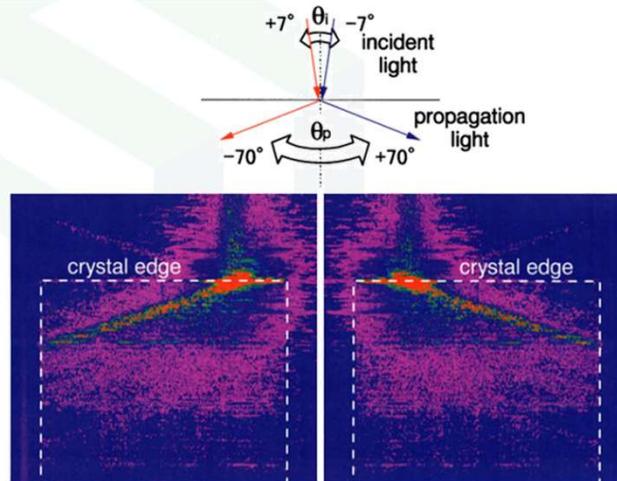
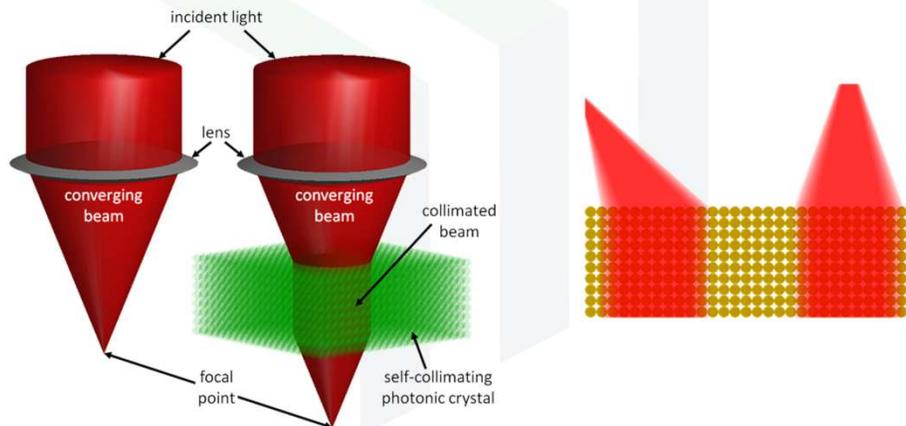


FIG. 1. (Color) Photographs showing light-beam swing inside the photonic crystal. The tilting angle of the incident light was slightly altered from $+7^\circ$ (left) to -7° (right). Both paths show negative bending. The incident light has a wavelength corresponding to the normalized frequency $\Omega = 0.33$ (defined later) with the TM polarization. The angles were measured from normal to the crystal edge ($\Gamma - M$ crystal direction). The crystal size is $500 \mu\text{m} \times 500 \mu\text{m}$.

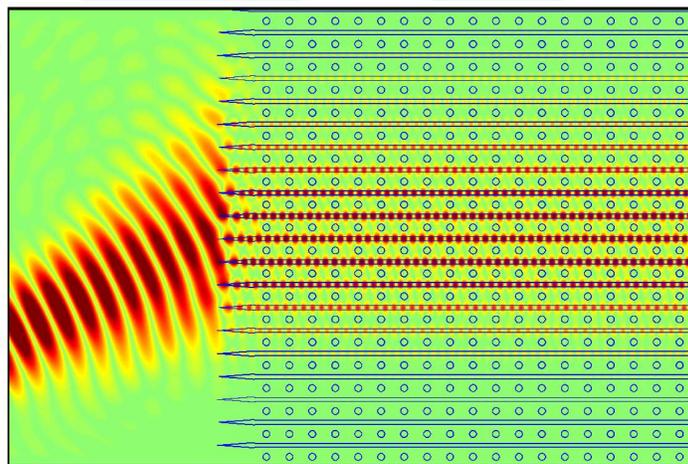
Self-Collimation

Self-Collimation

Self-collimation is a property of some periodic structures where a beam appears to remain collimated indefinitely almost independently of the source beam.

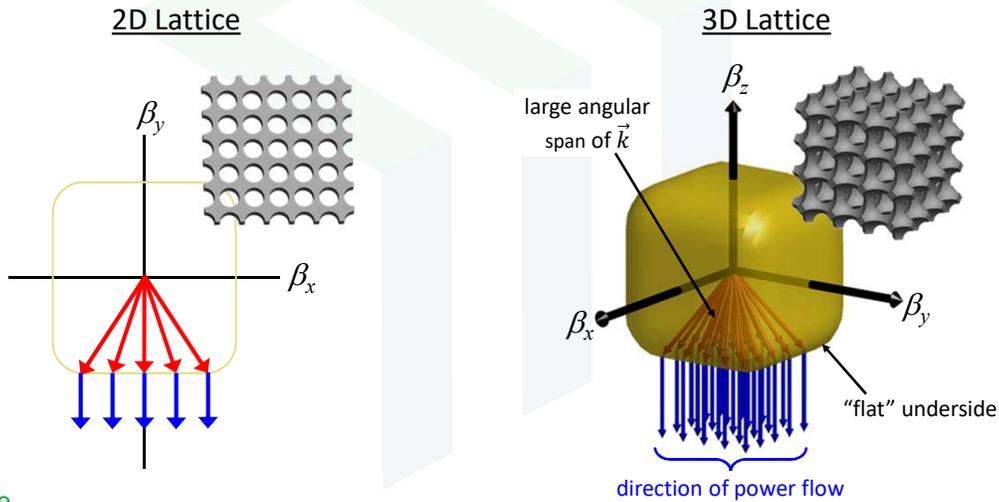


Example Simulation



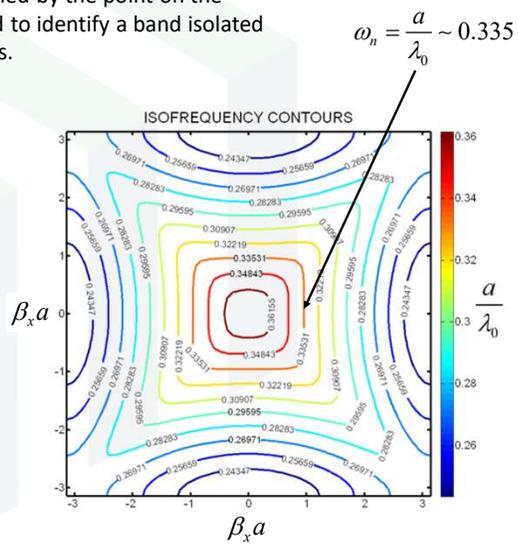
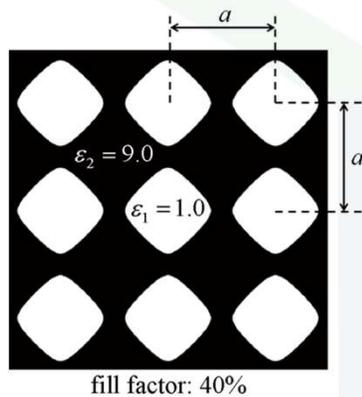
Conditions for Self-Collimation

Self-Collimation occurs whenever the index ellipsoids have flat surfaces.



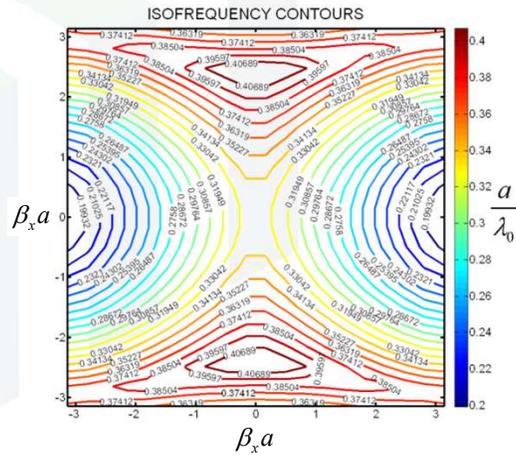
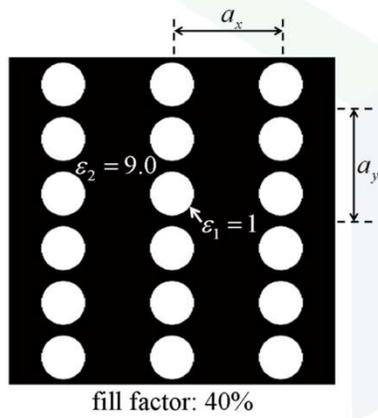
Identifying the Self Collimation Conditions

The frequency of self-collimation is traditionally identified by the point on the isofrequency contour that is flat. Further, it is designed to identify a band isolated from other bands to prevent coupling into other modes.

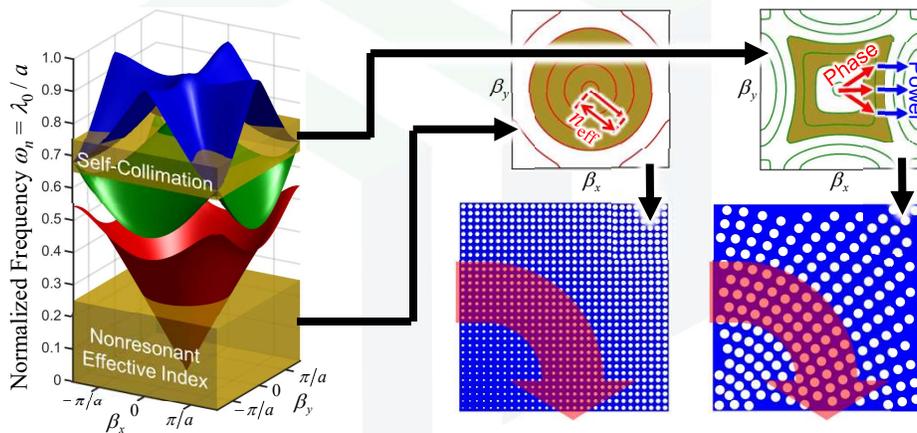


Tailoring the Self-Collimation Conditions

Tailoring the symmetry and pattern within the unit cell of the lattice can have a profound effect on the isofrequency contours. This can be used to “tune” the self-collimation effect or other properties of the lattice.



Self-Collimation Vs. Graded Index



Metrics for Self-Collimation

Fractional Bandwidth

$$\text{FBW} = 2 \frac{\omega(\beta_{x2}) - \omega(\beta_{x1})}{\omega(\beta_{x2}) + \omega(\beta_{x1})}$$

Normalized Acceptance Angle

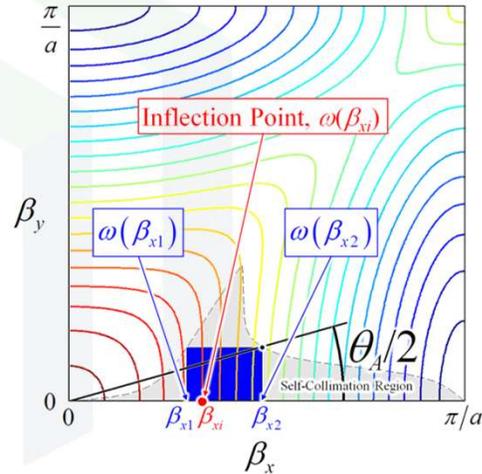
$$\theta_A = \frac{\tan^{-1}(\beta_{y2}/\beta_{x2})}{90^\circ}$$

Strength Metric

$$S = 1 - \left| \frac{2\Lambda_x}{\pi} \beta_{xi} - 1 \right|$$

Overall Figure-of-Merit

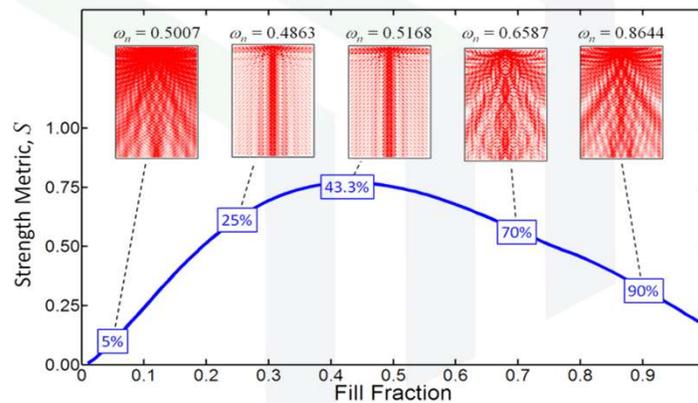
$$\text{FOM} = \sqrt[3]{\text{FBW} \cdot \theta_A \cdot S}$$



Raymond C. Rumpf and Javier J. Pazos, "Optimization of planar self-collimating photonic crystals," J. Opt. Soc. Am. A, Vol. 30, No. 7, pp. 1297-1304, 2013.

Strength Metric

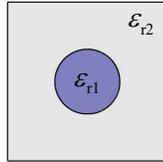
The center frequency of self-collimation should be as far away from the band edges as possible.



Raymond C. Rumpf and Javier J. Pazos, "Optimization of planar self-collimating photonic crystals," J. Opt. Soc. Am. A, Vol. 30, No. 7, pp. 1297-1304, 2013.

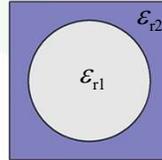
Optimization of Self-Collimation

Optimized Unit Cell for E Mode

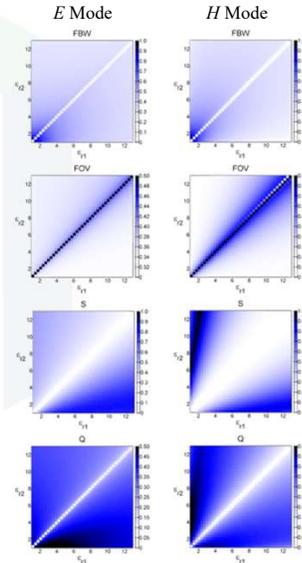


$$\begin{aligned} f &= 13\% \\ r &= 0.2a \\ \epsilon_{r1} &= 2.51 \\ \epsilon_{r2} &= 1.00 \end{aligned}$$

Optimized Unit Cell for H Mode

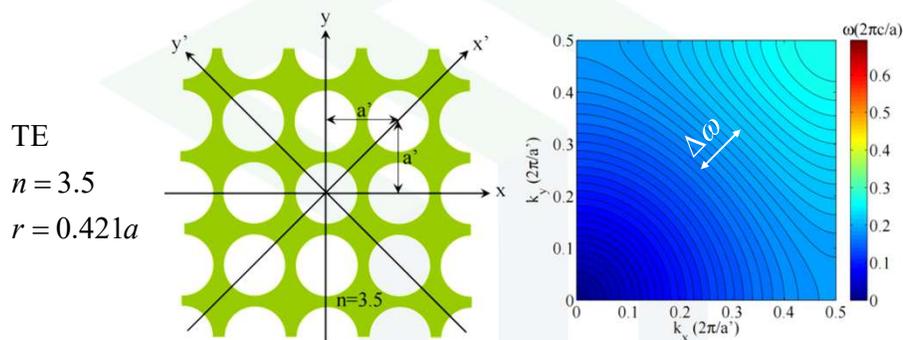


$$\begin{aligned} f &= 45.5\% \\ r &= 0.38a \\ \epsilon_{r1} &= 1.00 \\ \epsilon_{r2} &= 3.93 \end{aligned}$$



Raymond C. Rumpf and Javier J. Pazos, "Optimization of planar self-collimating photonic crystals," J. Opt. Soc. Am. A, Vol. 30, No. 7, pp. 1297-1304, 2013.

"Best" Self-Collimator (1 of 3)

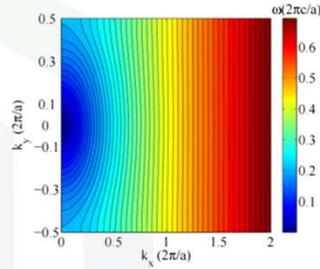
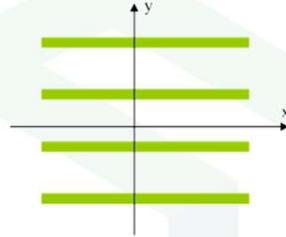


This lattice self-collimates and the curvature has an inflection point, but it operates over a narrow frequency band.

Rafif E. Hamam, et al, "Broadband super-collimation in a hybrid photonic crystal structure," Optics Express, Vol. 17, No. 10, pp. 8109-8118, 2009.

“Best” Self-Collimator (2 of 3)

TM
 $n = 3.5$
 $t = 0.2a$



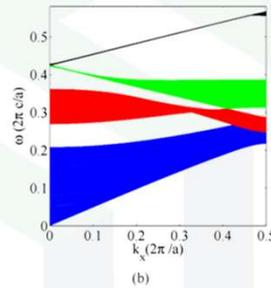
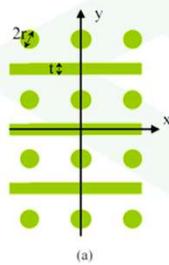
“A hint as to how to achieve this feature can be inferred from the flatness of tight-binding bands for electrons in solids [20]. The fact that tight-binding bands arise from the weak overlap between sub atomic orbitals inspires us to consider the simple, well-known waveguide array...”

This lattice self-collimates over a much broader range of frequencies, but the dispersions surfaces are never perfectly flat (no inflection point)

Rafif E. Hamam, et al, “Broadband super-collimation in a hybrid photonic crystal structure,” Optics Express, Vol. 17, No. 10, pp. 8109-8118, 2009.

“Best” Self-Collimator (3 of 3)

TM
 $n = 3.5$
 $r = 0.16a$
 $t = 0.2a$

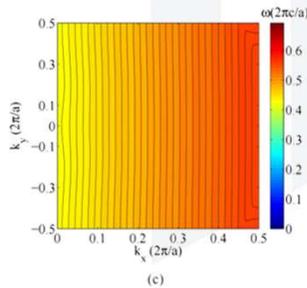


Metrics

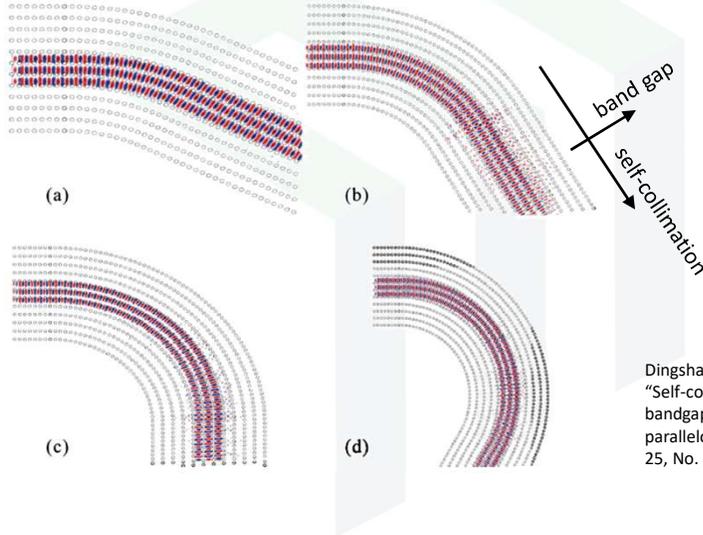
FBW = 40.7%
 $\theta_A = 1$
 $S = 1$

Rafif E. Hamam, et al, “Broadband super-collimation in a hybrid photonic crystal structure,” Optics Express, Vol. 17, No. 10, pp. 8109-8118, 2009.

This lattice combines the waveguide array for broadband self-collimation and inserts dielectric globs to get an inflection point.



Waveguide Bends



Dingshan Gao, Zhiping Zhou, David S. Citrin, "Self-collimated waveguide bends and partial bandgap reflection of photonic crystals with parallelogram lattice," J. Opt. Soc. Am. A, Vol. 25, No. 3, pp. 791-795, 2008.

Autocloned Graded Lattices

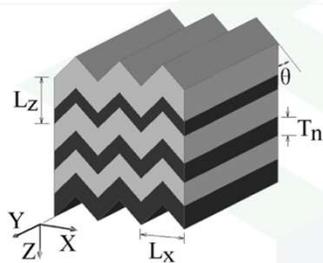


Fig. 1. Geometry of a 2DPC made of wavelike thin films with graded filling factor. The light gray area and dark gray area represent SiO₂ and Si, respectively.

Yi-Yu Li, et al, "Graded wavelike two-dimensional photonic crystal made of thin films," Applied Optics, Vol. 47, No. 13, pp. C70-C74, 2008.

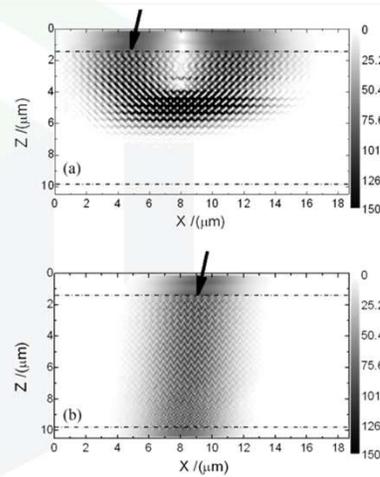
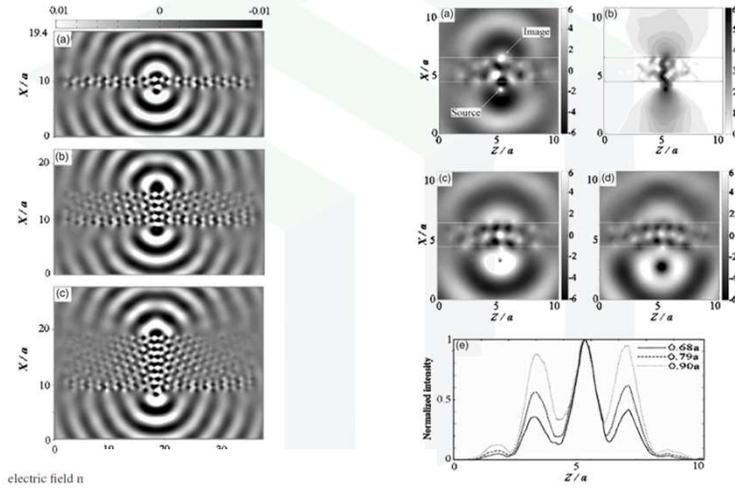


Fig. 3. Light intensity map of Gaussian beam incident at grad wavelike 2DPC with wavelength of $\lambda = 1467$ nm and incidence angle of 12°. The black arrows represent the incident light directions. (a) TM mode, (b) TE mode.

Super Lensing



electric field n

Yi-Yu Li, et al, "Self-collimation and superlensing in wavy-structured two-dimensional photonic crystals," Applied Physics Letters, Vol. 88, 151911 (2006).



Beam Splitter

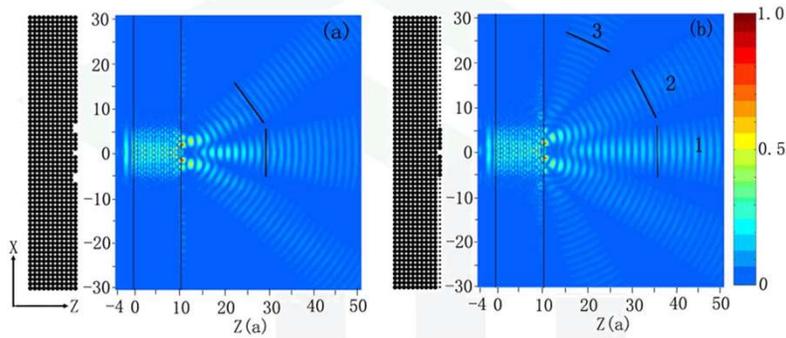
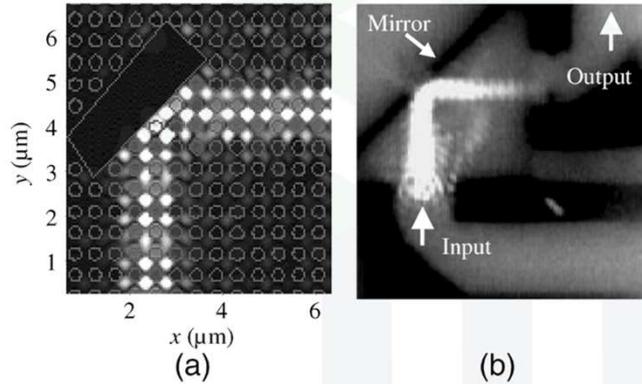


Fig. 4. The 2D PC structure and the spatial distribution of the Poynting vector. (a) One-to-three beam splitter. The radius r_o of four cylinders at $(10a, 3a)$, $(10a, 4a)$ and $(10a, -4a)$, as well as the radius r_c of the center cylinder of the output surface at $(10a, 0)$, are changed to be $r_o=r_c=0$. Two short black lines denote the positions of two detectors. (b) One-to-five beam splitter. Letting $r_c=0$, and keeping every five cylinders next to $(10a, 0)$ at both sides unchanged, the radius of the rest output surface cylinders is symmetrically changed to be $r_o=0.25a$. 1, 2 and 3 denote the split beams and the three short black lines denote the positions of the detectors.

W. Y. Liang, J. W. Dong, H. Z. Wang, "Directional emitter and beam splitter based on self-collimation effect," Optics Express, Vol. 15, No. 3, pp. 1234-1239, 2007.



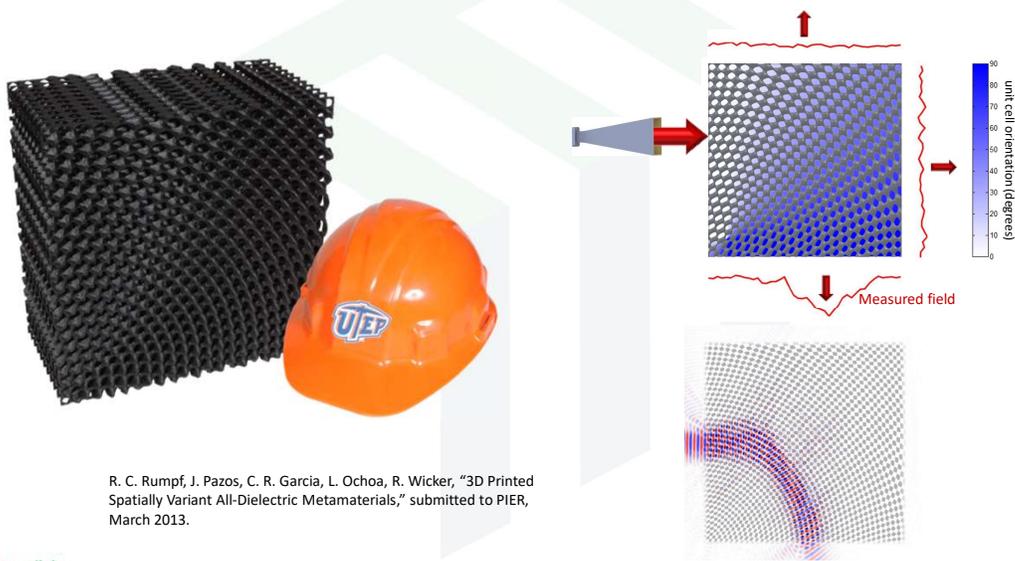
Beam Bending with Defects



Dennis W. Prather, et al, "Dispersion-based optical routing in photonic crystals," Optics Letters, Vol. 29, No. 1, pp. 50-52, 2004.

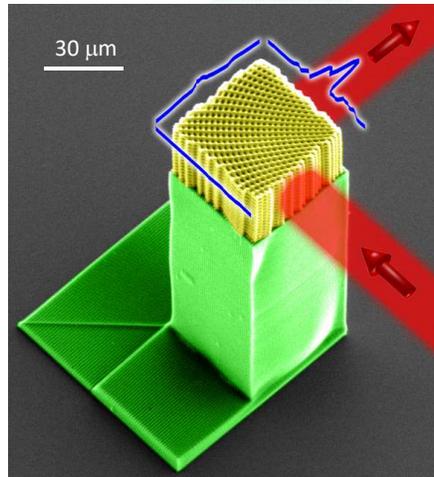
Fig. 3. Routing capability of a material having a square-shaped equifrequency dispersion contour. (a) Three-dimensional FDTD simulation of light guided through a PhC lattice and routed by reflection from a mirror. (b) Image of the scattered light as it is reflected by the mirror, where $\lambda = 1432$ nm.

Spatially Variant Self-Collimation to Control the Flow of Waves



R. C. Rumpf, J. Pazos, C. R. Garcia, L. Ochoa, R. Wicker, "3D Printed Spatially Variant All-Dielectric Metamaterials," submitted to PIER, March 2013.

World's Tightest Bend of Unguided Optical Beam



- World's tightest unguided bend ($R = 6.7\lambda_0$).
- Utilized very low refractive index (SU-8, $n \cong 1.59$).
- Operated at $\lambda_0 = 2.94 \mu\text{m}$.

J. L. Digaum, J. J. Pazos, J. Chiles, J. D'Archangel, G. Padilla, A. Tatulian, R. C. Rumpf, S. Fathpour, G. D. Boreman, and S. M. Kuebler, "Tight Control of Light Beams in Photonic Crystals with Spatially-Variant Lattice Orientation," *Optics Express*, Vol. 22, Issue 21, pp. 25788-25804, 2014.