Advanced Electromagnetics:
21st Century Electromagnetics

Photonic Crystals

Lecture Outline

• Introduction
• Origin of the Band Gap
• Band Gap Engineering
• Devices Based on Band Gap Engineering
• Dispersion Engineering
• Devices Based on Dispersion Engineering
  • Self-Collimation
Due to the analogy between Shrödinger’s equation and the wave equation, electromagnetic waves can be controlled inside periodic lattices like electrons are controlled in semiconductors.
Origin of the Band Gap

The Bloch Theorem

The field inside of a periodic structure takes on the same symmetry and periodicity of that structure according to the Bloch theorem.

\[ \vec{E}(\vec{r}) = \vec{A}(\vec{r}) e^{j \vec{p} \cdot \vec{r}} \]

Given the lattice translation vectors, the periodicity can be described mathematically.

\[ \vec{A}(\vec{r} + \vec{t}) = \vec{A}(\vec{r}) \quad \vec{t} \equiv \text{lattice vector} \]
The Wave Equation in Periodic Structures

Maxwell’s curl equations for non-magnetic materials are
\[ \nabla \times \vec{E} = -j\omega\mu_0\vec{H} \quad \nabla \times \vec{H} = j\omega\varepsilon_0\varepsilon_r\vec{E} \]

The wave equation for the magnetic field \( \vec{H} \) can be derived by taking the curl of the second equation above and substituting the result in the first equation.
\[ \nabla \times \nabla \times \vec{H} = k_0^2 \vec{H} \]

According to the Bloch theorem, the magnetic field is periodic as follows.
\[ \vec{H}(\vec{r}) = \vec{H}_\beta(\vec{r}) \cdot e^{j\beta \cdot \vec{r}} \]

Substituting this into the wave equation leads to
\[ (\nabla + j\beta) \times \frac{1}{\varepsilon_r}(\nabla + j\beta) \times \vec{H}_\beta = \left( \frac{\omega}{c_0} \right)^2 \vec{H}_\beta \]

Solutions to the Wave Equation

The wave equation just derived is an eigen-value problem.
\[ L \{ \vec{H}_\beta \} = \nu \vec{H}_\beta \]

Eigen-value problems have discrete solutions (like modes in a waveguide) that are all orthogonal (very different from each other).

This means that electromagnetic waves in periodic structures only exist as discrete modes. These are called Bloch modes.

Fields can only exist as integer combinations of the eigen-modes, or Bloch modes, of the lattice.
\[ \vec{H} = \sum_\beta a_\beta \vec{H}_\beta \]

The variational theorem states that the lowest-order state satisfying the wave equation minimizes the following variational equation.

To minimize this equation, the denominator must be maximized. This happens when the most intense fields reside inside the high dielectric constant regions.

Conclusion – The field of the lowest order mode prefers to be in the high dielectric constant regions.
The Electromagnetic Band Gap

To understand the origin of the band gap, recall three rules:

1. Bloch modes must have the same symmetry as the lattice.
2. Electric fields of the lowest order mode prefer to reside in higher index regions.
3. Modes must be orthogonal (very different).

Consider a one-dimensional photonic crystal.

At $\beta = \pi/a$, what does the electric field look like for the lowest-order band?
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At $\beta = \pi/a$, what does the electric field look like for the lowest-order band?
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At $\beta = \pi / a$, what does the electric field look like for the lowest-order band?

Direction of Bloch Wave $\vec{\beta}$

At $\beta = \pi / a$, what does the electric field look like for the second-order band?

Direction of Bloch Wave $\vec{\beta}$
The Electromagnetic Band Gap

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Direction of Bloch Wave $\vec{\beta}$
To understand the origin of the band gap, recall three rules:

1. **Bloch modes must have the same symmetry as the lattice.**
2. Electric fields of the lowest order mode prefer to reside in higher index regions.
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At $\beta = \pi / a$, what does the electric field look like for the second-order band?

What about the effective refractive index $n_{eff}$ of the modes?

- **Lower $n_{eff}$**
  - More power resides in low index regions.

- **Higher $n_{eff}$**
  - More power resides in high index regions.
The Electromagnetic Band Gap

The two modes have the same wavelength but different effective refractive indices. How is this possible?

\[ \Delta \omega \] The modes must exist at different frequencies.

\[ n_{\text{eff}} \]

- Higher \( n_{\text{eff}} \) \( \Rightarrow \) More power resides in high index regions.
- Lower \( n_{\text{eff}} \) \( \Rightarrow \) More power resides in low index regions.

Direction of Bloch Wave \( \vec{\beta} \)

The Electromagnetic Band Gap

A band gap is defined as a range of frequencies over which no bands exist for all values of \( \vec{\beta} \).
Behavior of Waves in the Band Gap

What if photonic crystal is illuminated by a wave at a frequency within its band gap?

The Bloch wave actually penetrates into the lattice by some distance.

Bloch waves still exist within the band gap. They are just cutoff and evanescent. This means they decay with distance into the lattice.

$$\vec{E}(\vec{r}) = \vec{A}(\vec{r}) \exp^{-\alpha \mathbf{r}} \exp^{i \beta \mathbf{r}}$$
Realizing a Complete 2D Band Gap (1 of 4)

The $E$ modes have the electric field polarized perpendicular to the crystal plane so they can form isolated regions of high intensity surrounded by regions of low intensity.

To maximize the disparity discussed previously, lattices supporting strong $E$-mode band gaps should be composed of isolated regions of high dielectric constant material.

Realizing a Complete 2D Band Gap (2 of 4)

The $H$ modes have the electric field polarized parallel to the crystal plane so it is a vector quantity and must form circular loops to satisfy Maxwell’s equations.

To maximize the disparity discussed previously, lattices supporting strong $H$-mode band gaps should be composed of connected veins of high dielectric constant material.
Realizing a Complete 2D Band Gap (3 of 4)

Lattices with greater symmetry support wider band gaps.

Realizing a Complete 2D Band Gap (4 of 4)

To realize a COMPLETE photonic band gap, we now know we need the following ingredients:

1. High contrast in the dielectric constant to maximize the disparity.
2. Isolated “globs” of high dielectric constant material for the E-modes.
3. Connected veins of high dielectric constant material for the H-modes.
4. High lattice symmetry for wider band gaps.
**3D Band Gaps and Aperiodic Lattices**

3D lattices are the only structures that can provide a true complete band gap.

The diamond lattice is known to have the strongest band gap of all 14 Bravais lattices.

Aperiodic lattices can have stronger band gaps than diamond.


**Effects of Average Index and Index Contrast on Band Gap**

Here the fractional bandwidth of the band gap is calculated as a function of average refractive index and the index contrast for a Bragg grating (1D photonic crystal).

\[
\begin{align*}
    n_1 &= n_{\text{avg}} - \Delta n/2 \\
    n_2 &= n_{\text{avg}} + \Delta n/2 \\
    d_1 &= \lambda_0/4n_1 \\
    d_2 &= \lambda_0/4n_2 \\
    \text{FBW} &= 2 \left( \frac{k_{0,2} - k_{0,1}}{k_{0,2} + k_{0,1}} \right)
\]

**Conclusions for Large Band Gaps:**

- Want $\Delta n$ as high as possible.
- Want $n_{\text{avg}}$ as low as possible.
Effects of Duty Cycle on Width of Band Gap

The optimum duty cycle is whatever gives quarter-wavelength layers.

Conclusions:
- There exists a “sweet spot” for duty cycle.
- Deviating from this lowers the band gap width.
- We can lower band gap width even when we are forced to have high contrast.

Devices Based on Band Gap Engineering
Waveguides and Cavities Based on Electromagnetic Band Gaps

Waveguides
If light is forbidden to propagate inside a lattice, the lattice acts like a mirror throughout. This can be used to form waveguides.

Cavities
If energy can be generated inside a lattice defect, it will be trapped and can be stored for long periods of time.
Tight Waveguide Bends


Tight Multimode Waveguide Bends

First-Order Mode
Second-Order Mode
Third-Order Mode

Spatially-Variant PC
Conventional PC
Antennas Utilizing EBGs

- Increase radiation efficiency by suppressing surface waves
- Use EBG material as an efficient reflector
- High directivity antennas based on angle dependent properties of EBGs
- All-dielectric “horn” antennas
- Additional degrees of freedom for polarization control

A Highly Directive EBG Antenna

1. EBG serves as an angle dependent mirror.
2. EBG separated from ground plane serves as a Fabry-Perot filter.
3. Double slot matches waveguide mode to Fabry-Perot.
All-Dielectric Horn Antenna

(c) EBG horn antenna

Dispersion Engineering
Phase, Group and Energy Velocity

Phase Velocity
Phase velocity describes the speed and direction of the phase of a wave.
\[ \vec{v}_p = \frac{\omega}{|k|} \hat{s} \]
\[ n_p = \frac{c_0}{v_p} \]

Group Velocity
Group velocity describes the speed and direction of the envelope of a pulse.
\[ \vec{v}_g = \nabla_k \omega_k \]
\[ n_g = \frac{c_0}{v_g} \]
\[ \vec{v}_g = \vec{v}_p \] for no dispersion

Energy Velocity
Energy velocity describes the speed and direction of the energy.
\[ \vec{v}_e = \frac{\vec{P}}{U} \]
\[ n_e = \frac{c_0}{v_e} \]
\[ \vec{v}_e = \vec{v}_g \] for linear materials

Types of Dispersion

Material Dispersion
Waves of different frequency propagate at different speeds inside a material because the dielectric constant changes as a function of frequency.
\[ \varepsilon(\omega) = 1 + \frac{\alpha^2_{\omega}}{\alpha^2_{\omega} - \omega^2 - j\omega \alpha} \]

Spatial Dispersion
Waves travelling in different directions through anisotropic or periodic materials propagate at different speeds because the dielectric constant changes as a function of direction.

Modal Dispersion
Different modes inside a device typically propagate at different speeds because their energy is distributed differently inside the device leading to different effective refractive indices of the modes.
The Band Diagram is Missing Information

Direct lattice: We have an array of air holes in a dielectric with $n = 3.0$.

Reciprocal lattice: We construct the band diagram by marching around the perimeter of the irreducible Brillouin zone.

The band extremes “almost” always occur at the key points of symmetry.

But we are missing information from inside the Brillouin zone.

The Complete Band Diagram

The Full Brillouin Zone

There is an infinite set of eigen-frequencies associated with each point in the Brillouin zone. These form “sheets” as shown at right.
Constructing Isofrequency Contours (Index Ellipsoids)

Index ellipsoids are “isofrequency contours” in $k$-space.

Isofrequency Contours From First-Order Band

Isofrequency contours are mostly circular. Not much interesting here.
Standard View of Isofrequency Contours

First Band

Second Band

Devices Based on Dispersion Engineering
Negative Refraction Without Negative Refractive Index

Dispersion Compensation Using Chirped Bragg Gratings

Degenerate modes have the same propagation constant so they easily exchange energy and are coupled as a result.

For a structure to support a DBE, all modes (TE and TM) are coupled at a single point. The slope of the band approaches zero, so group velocity does also.

\[
\vec{v}_g = \nabla_k \omega
\]

Regular Band Edge (2 degenerate modes)  
Stationary Inflection Point (3 degenerate modes)  
Degenerate Band Edge (4 degenerate modes)

Superprism Devices


![Superprism Devices Diagram](image)

**Self-Collimation**
Self-Collimation

Self-collimation is a property of some periodic structures where a beam appears to remain collimated indefinitely almost independently of the source beam.

Example Simulation
Conditions for Self-Collimation

Self-Collimation occurs whenever the index ellipsoids have flat surfaces.

2D Lattice

3D Lattice

Identifying the Self Collimation Conditions

The frequency of self-collimation is traditionally identified by the point on the isofrequency contour that is flat. Further, it is designed to identify a band isolated from other bands to prevent coupling into other modes.

$$\omega_n = \frac{a}{\lambda_0} \approx 0.335$$

fill factor: 40%
Tailoring the Self-Collimation Conditions

Tailoring the symmetry and pattern within the unit cell of the lattice can have a profound effect on the isofrequency contours. This can be used to “tune” the self-collimation effect or other properties of the lattice.

Self-Collimation Vs. Graded Index
Metrics for Self-Collimation

Fractional Bandwidth

\[ FBW = 2 \frac{\omega(\beta_2) - \omega(\beta_1)}{\omega(\beta_2) + \omega(\beta_1)} \]

Normalized Acceptance Angle

\[ \theta_y = \frac{\tan^{-1}\left(\beta_{12}/\beta_{22}\right)}{90^\circ} \]

Strength Metric

\[ S = 1 - \left| \frac{2\lambda}{\pi} \beta_y - 1 \right| \]

Overall Figure-of-Merit

\[ FOM = \sqrt{FBW \cdot \theta_y \cdot S} \]

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Strength Metric

The center frequency of self-collimation should be as far away from the band edges as possible.

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Optimization of Self-Collimation

$\begin{align*}
E_{13} & = 2.51 \\
E_{12} & = 1.00 \\
f & = 13\% \\
r & = 0.2a
\end{align*}$

$\begin{align*}
E_{r1} & = 1.00 \\
E_{r2} & = 3.93 \\
f & = 45.5\% \\
r & = 0.38a
\end{align*}$


"Best" Self-Collimator (1 of 3)

$\begin{align*}
TE & \\
n & = 3.5 \\
r & = 0.421a
\end{align*}$

This lattice self-collimates and the curvature has an inflection point, but it operates over a narrow frequency band.

This lattice self-collimates over a much broader range of frequencies, but the dispersions surfaces are never perfectly flat (no inflection point).


This lattice combines the waveguide array for broadband self-collimation and inserts dielectric globs to get an inflection point.

Waveguide Bends


Autocloned Graded Lattices


Fig. 3. Light intensity map of Gaussian beam incident at grad-wavelike 2DPC with wavelength of λ = 1407 nm and inside angle of 12°. The black arrows represent the incident light directions. (a) TM mode, (b) TE mode.
Super Lensing


Beam Splitter

Beam Bending with Defects

Fig. 3. Routing capability of a material having a square-shaped equifrequency dispersion contour. (a) Three-dimensional FDTD simulation of light guided through a PhC lattice and routed by reflection from a mirror. (b) Image of the scattered light as it is reflected by the mirror, where $\lambda = 1432$ nm.

Spatially Variant Self-Collimation to Control the Flow of Waves


World’s Tightest Bend of Unguided Optical Beam

- World’s tightest unguided bend ($R = 6.7 \lambda_0$).
- Utilized very low refractive index (SU-8, $n \approx 1.59$).
- Operated at $\lambda_0 = 2.94 \ \mu m$.