Advanced Electromagnetics:
21st Century Electromagnetics

Slow Waves

Lecture Outline

- Review of phase and group velocity
- Slow waves
- Resonant structures for slow waves
- Materials for slow waves

Review of Phase and Group Velocity

Phase Velocity and Refractive Index

The *phase velocity* of a wave is the speed at which the phase of a single frequency wave propagates through space.

$$v_{\mathrm{p}} = \omega / k$$

We can characterize a medium by it phase refractive index n_{p}. This is the factor describing how much slower than the speed of light that the phase is propagating.

$$n_{\mathrm{p}} = c_0 / v_{\mathrm{p}}$$

Group Velocity

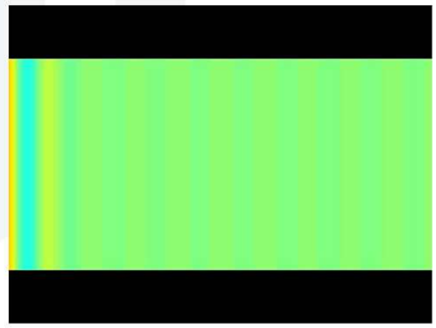
The group velocity is the speed and direction in which the envelope of the wave's amplitude propagates.

$$v_g = \frac{\partial \omega}{\partial k}$$

It follows that the group refractive index n_g is the factor describing how much slower than the speed of light that the envelop of the wave is propagating.

$$v_g = c_0 / n_g$$

Here, the wave appears to be very fast, but the overall "package" of energy propagate slowly.



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Phase Vs. Group Velocity

By their definitions, the phase velocity • applies only to a wave at a single frequency.

The group velocity • applies to a packet of waves covering some spectrum.

The phase and group velocities are often the same, but they can be different.

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Dispersive Materials (1 of 2)

In a dispersive material, the refractive index can vary. The dispersion relation is differentiated as follows.

$$\omega n = c_0 k$$

$$\omega dn + n d\omega = c_0 dk$$

Rearrange the terms to arrive at

$$\underbrace{\frac{c_0}{n}}_{\text{This is phase velocity, } v_{\mathrm{p}}} = \frac{\omega}{n}\frac{dn}{dk} + \underbrace{\frac{d\omega}{dk}}_{\text{This is group velocity, } v_{\mathrm{g}}}$$

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Dispersive Materials (2 of 2)

Now there is an expression relating group velocity v_{g} and phase velocity v_{p}.

$$v_{\mathrm{p}} = \frac{\omega}{n}\frac{dn}{dk} + v_{\mathrm{g}}$$

Solving this for group velocity v_{g} yields

$$v_{\mathrm{g}} = v_{\mathrm{p}} - \frac{\omega}{n}\frac{dn}{dk} = v_{\mathrm{p}}\left(1 - \frac{k}{n}\frac{dn}{dk}\right) \qquad k = \frac{\omega}{v_{\mathrm{p}}}$$

From the above equation, it is the dn/dk term that is responsible for $v_{\mathrm{g}} \neq v_{\mathrm{p}}$.

Any time the refractive index is not constant, the medium is said to have dispersion and the group velocity v_{g} will deviate from the phase velocity v_{p}.

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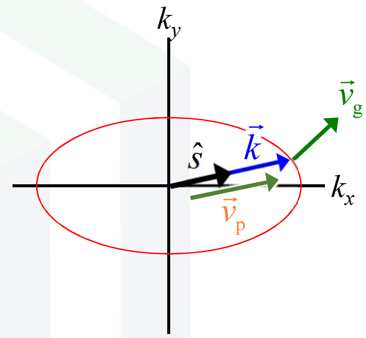
Summary of Phase, Group and Energy Velocity

<u>Phase Velocity</u>

Phase velocity describes the speed and direction of the phase of a wave.

$$\vec{v}_p = \frac{\omega}{\left|\vec{k}\right|}\hat{s} \qquad n_p = \frac{c_0}{v_p}$$



<u>Group Velocity</u>

Group velocity describes the speed and direction of the envelope of a pulse.

$$\vec{v}_\mathrm{g} = \nabla_{\vec{k}}\omega\left(\vec{k}\right) \qquad n_\mathrm{g} = \frac{c_0}{v_\mathrm{g}}$$

$$\vec{v}_\mathrm{g} = \vec{v}_\mathrm{g} \quad \text{when no dispersion}$$

<u>Dispersion and Parameter Relations</u>

We can relate the velocities with the refractive indices to show that it is dispersion that causes phase and group velocities to differ.

$$v_\mathrm{g} = v_\mathrm{p} - \frac{\omega}{n}\frac{dn}{dk} = v_\mathrm{p}\left(1 - \frac{k}{n}\frac{dn}{dk}\right) \qquad n_\mathrm{g} = n_\mathrm{p} + \omega\frac{dn_\mathrm{p}}{d\omega} = n_\mathrm{p} - \lambda_0\frac{dn_\mathrm{p}}{d\lambda_0}$$

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Slow Waves

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What is a Slow Wave?

A slow wave is a wave with a very small group velocity.

A slow wave is NOT defined in terms of the phase velocity.

The phenomenon occurs because of the interaction with the medium in which the wave is propagating.

Slow waves cannot occur in free space because there is no medium.

How Do We Slow a Wave?

Recall that group velocity is determined by the dispersion.

$$\vec{v}_g = \vec{v}_p \left[1 - \frac{k}{n} \cdot \boxed{\nabla_k n} \right] \qquad n_g = n - \omega \boxed{\frac{dn}{d\omega}}$$

To slow a wave, dispersion must be introduced.

How can dispersion be introduced?

Slow Waves
- Material Induced Dispersion
- Structurally Induced Dispersion

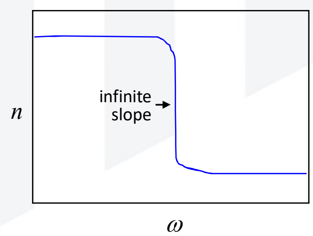
How Do We Stop a Wave?

After inspecting the equation for group refractive index,

$$n_{\mathrm{g}} = n - \omega \frac{dn}{d\omega}$$

A wave can be stopped if $n_{\mathrm{g}} = \infty$. The condition for this is

$$\frac{1}{dn/d\omega} = 0$$



Slow Wave Schemes

- Material Dispersion
  - Bose-Einstein condensates
  - Electromagnetically induced transparency
  - Coherent population oscillation
  - Stimulated scattering
- Structural Dispersion
  - Photonic crystals
  - Coupled resonators
  - Waveguides

Usually the preferred approach at room temperature due to strong temperature dependence on material based mechanisms and easier implementation.

Applications

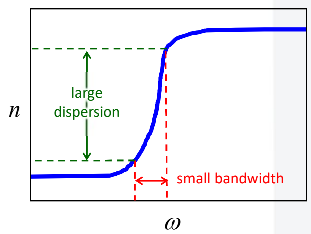
- Delay lines
- Noise reduction
- Optical switching and buffering
- Advanced time-domain signal processing
- Clock resynchronization
- Controlling information through a network
- Pulse compression
- More sensitive interferometers
- Miniaturization
- Enhancing linear and nonlinear effects
- Particle acceleration

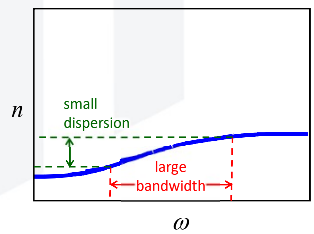
Important Metrics for Slow Waves

There are two important metrics to consider in slow wave structures.

1. **Frequency bandwidth** – We would like this metric to be large in order to handle more types of signals.
2. **Dispersion (Delay)** – We would like this metric to be large in order to produce slower waves.

There is a fundamental limit to how large both of these can be made at the same time. Increasing one virtually always decreases the other.





The more extreme values of n that can be achieve, the greater the bandwidth/dispersion can be realized.

Delay-Bandwidth Product

The delay-bandwidth product (DBP) is used to quantify the tradeoff between bandwidth and delay.

$$\text{DBP} = (\text{delay}) \cdot (\text{bandwidth}) = \Delta t \cdot \Delta f = \frac{L \Delta n}{\lambda_0}$$

$L \equiv$ length of device

$\Delta t \equiv$ delay through length L

$\Delta f \equiv$ operating bandwidth

$\Delta n \equiv$ change of n over bandwidth

$\lambda_0 \equiv$ operating wavelength

Often the normalized DBP is useful when comparing devices of different length L or operating at different wavelengths λ_0.

$$\widetilde{\text{DBP}} = n_g \left(\frac{\Delta f}{f} \right) \cong \Delta n$$

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Resonant Structures for Slow Waves

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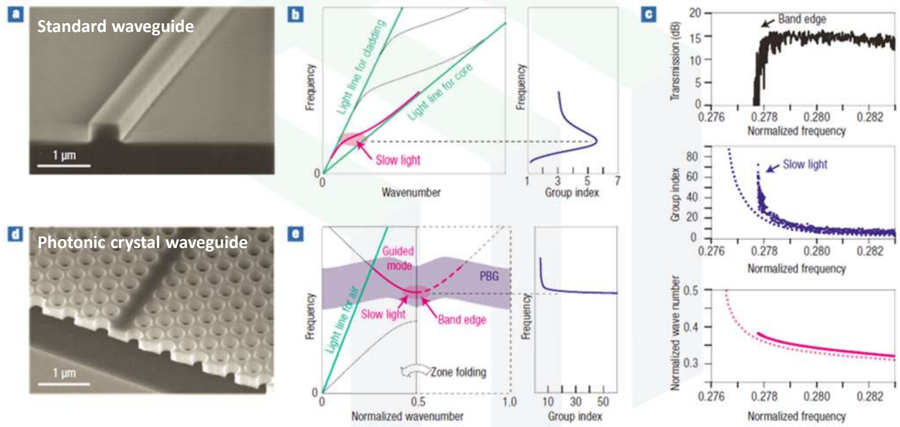
Photonic Crystal Waveguide

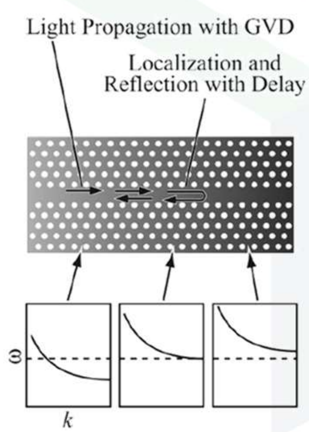


(a) Shows perhaps the first slow light device. Slow light occurs most strongly near the cutoff of the waveguide where its dispersion is highest. (d) Group index is greatly enhanced using a photonic crystal waveguide.

T. Baba, "Slow light in photonic crystal," Nature Photonics **2**, 465-473 (2008).

Dispersion Compensated Slow Wave Devices

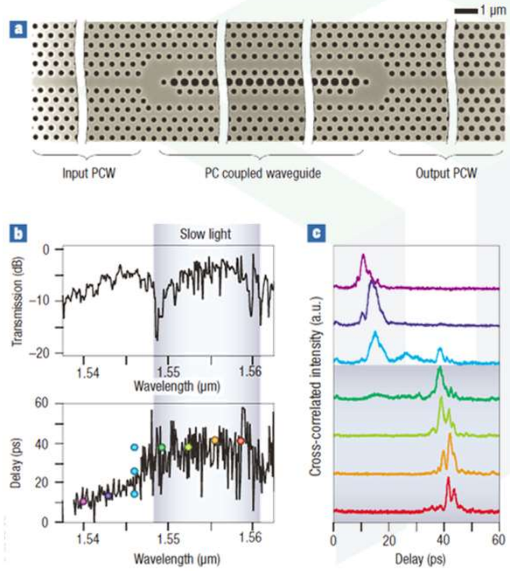
For many applications, the strong variation in group index with frequency is a problem. This is called group velocity dispersion (GVD) and can be compensated using chirped devices.



- Material has a graded refractive index.
- Chirping holes size was less effective.
- Slow light is produced near band edge and chirping grades the wavelength of the band edge.

D. Mori, T. Baba, "Dispersion-controlled optical group delay device by chirped photonic crystal waveguides," Appl. Phys. Lett. **85**, 1101 (2004).

Coupled Photonic Crystal Waveguide



- More sophisticated dispersion compensation.
- $n_g \cong 40 - 60$

T. Baba, "Slow light in photonic crystal," Nature Photonics **2**, 465-473 (2008).

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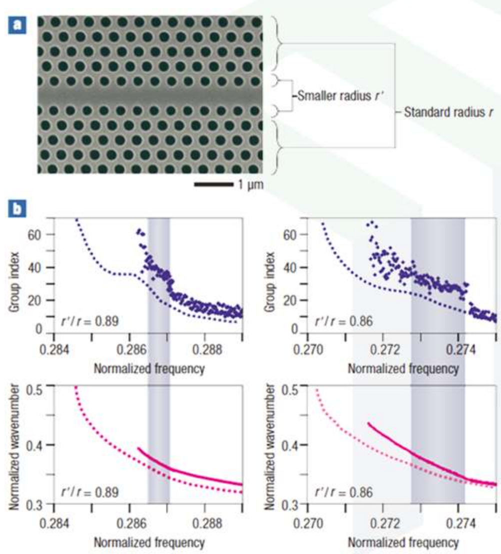
Zero-Dispersion Slow Waves



- Tapered hole size to flatten the dispersion.
- $n_g \cong 37 - 40$

T. Baba, "Slow light in photonic crystals," Nature Photonics **2**, 465-473 (2008).

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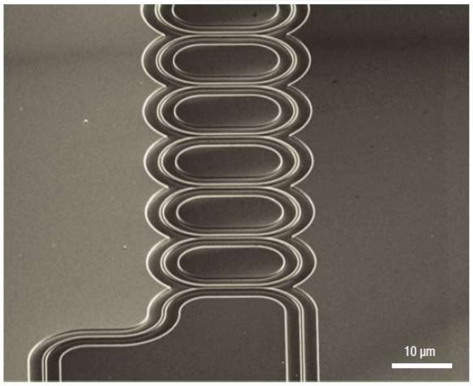
Coupled Resonator Optical Waveguides (CROW)

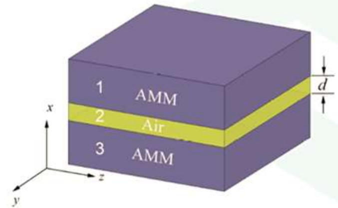


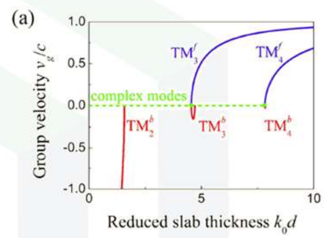
- Highly resonant structure slows the group velocity.
- Geometry is highly tunable due to strong resonances.

F. Xia, L. Sekaric, Y. Vlasov, “Ultracompact optical buffers on a silicon chip,” Nature Photonics **1**, 65-71 (2007).

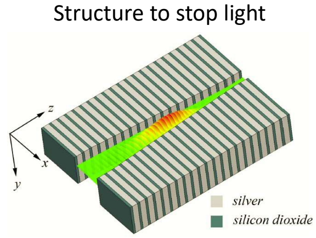
Air Waveguide with Anisotropic Metamaterial Cladding

Structure to slow light





Structure to stop light



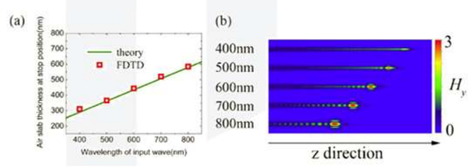
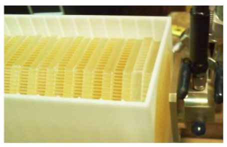
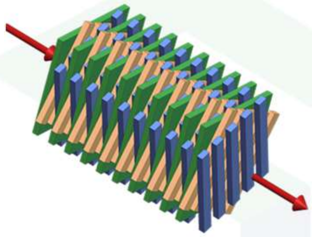


Fig. 5. (a) The relationship between the air slab thicknesses of the stop position and the excitation frequencies (material dispersion is not considered here). (b) Spatial field distributions in the tapered waveguide for light launched from the wider port, with the corresponding wavelengths marked on the left side.

T. Jiang, J. Zhao, Y. Feng, “Stopping light by an air waveguide with anisotropic metamaterial cladding,” Optics Express **17**(1), 102529 (2008).

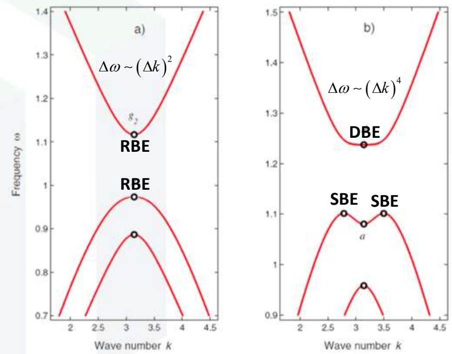
Degenerate Band Edge (DBE)





M. A. Fiddy, "Photonic crystals slow light for better sensing," SPIE Newsrom, 2006.

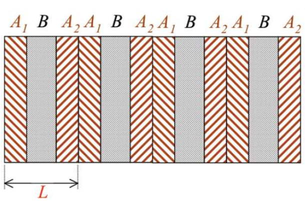
- Slow waves generated by a dramatic increase in the density of modes.
- Hyper coupling
- RBE can slow light, but transmission is poor.
- DBE requires strong anisotropy.



RBE – Regular band edge
SBE – Split band edge
DBE – Degenerate band edge

A. Figotin, I. Vitebskiy, "Phys. Rev. A **76**, 053839 (2007).

Simulated Transmission Through DBE Slow Wave Structure



… 55 periods

$L = 1.23$ cm

$d_{A1} = d_{A2} = 0.42L = 0.51$ cm

$d_B = 0.18L = 0.22$ cm

$\phi_z = 45°$

$\Lambda = 1$ mm

$f = 50\%$

$n_0 \cong 1.36$

$n_e \cong 1.21$

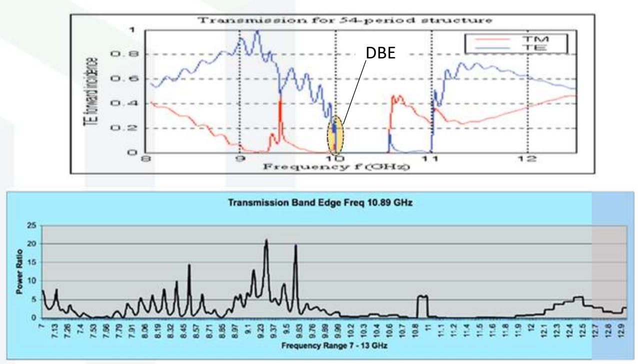


Figure 4: simulation of transmission spectrum through 54 periods (top) and measured data (lower)

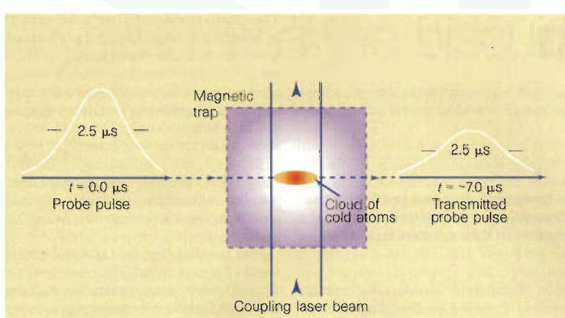
Y. Cao, J. Schenk, R. P. Ingel, M. A. Fiddy, K. Burbank, M. Graham, P. Sanger, W. Yang, "Form birefringent anisotropic photonic crystal exhibiting external field anomalies.

Materials for Slow Waves

Bose-Einstein Condensates

Thermal energy is mechanical oscillations of atoms. Photons can impart momentum when absorbed by the atoms. Atom recoil occurs within nanoseconds with a velocity of a few mm/s. This concept can be used to supercool atoms to very near absolute zero (< 435 nK).

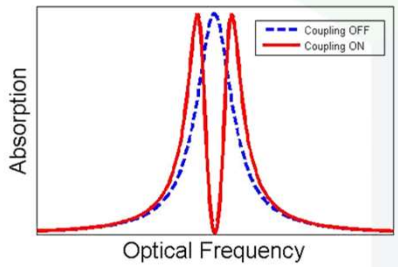
Slow light can occur in Bose-Einstein condensates (BECs).

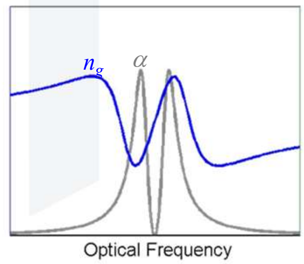


J. Marangos, "Slow light in cool atoms," Nature News, 6 (1999)

Electromagnetically Induced Transparency

Electromagnetically induced transparency (EIT) occurs in a material where a nonlinearity produces a narrow band of transparency in an absorption band. Extreme dispersion in the narrow band causes slow light.

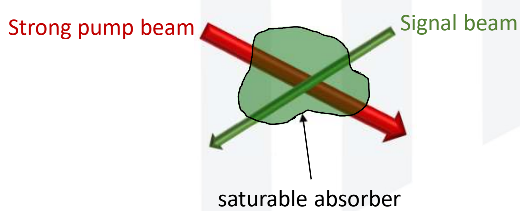




The technique involves delicate quantum interference. It has only been demonstrated at cryogenic temperatures to prevent collisions and dephasing phenomena.

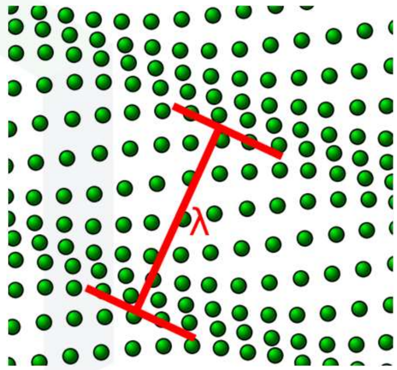
Coherent Population Oscillations

When signal and pump beams of slightly different wavelength interact in a saturable absorber, the ground state of the material oscillates coherently at the beat frequency of the two beams. Unlike EIT, this is highly insensitive to dephasing. The oscillations produce strong dispersion that lead to slow light.



Slow Light by Stimulated Scattering (1 of 2)

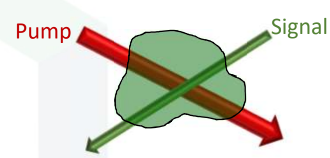
Brillouin Scattering – Light will scatter in a material when the lattice is vibrating. This varies the refractive index and induces a diffraction grating. Because the lattice vibrations are moving, there is a Doppler shift in the scattered wave that shifts the frequency of the scattered wave. This is called the Brillouin shift.



Raman Scattering – Raman scattering is similar to Brillouin scattering, but the oscillations are much faster and result from oscillations of individual atoms and molecules.

Slow Light by Stimulated Scattering (2 of 2)

The scattering processes can be stimulated by interfering two beams with slightly different frequencies. When the beat frequency matches the material oscillation, scattering is stimulated.



Energy transfers from the pump to the signal beam producing gain.

The amplification process is also accompanied by a spectrally varying refractive index that produces slow light.