Advanced Electromagnetics:
21st Century Electromagnetics

Subwavelength Gratings

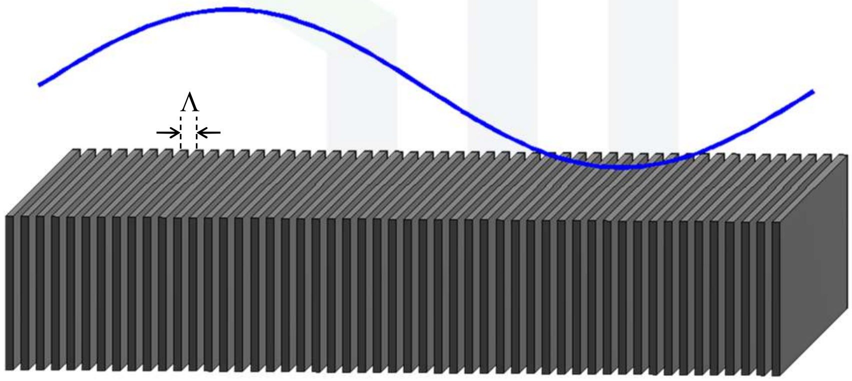
Lecture Outline

- Effective medium theory (EMT)
- Anti-reflection gratings
- Wave plates
- Chiral media
- Polarizers

What is a Subwavelength grating?

Assuming the grating period Λ is much smaller than the wavelength λ, the grating behaves like a homogeneous material because it does not diffract waves and the power is evenly distributed.

$$\Lambda << \lambda$$



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Effective Medium Theory (EMT)

for subwavelength gratings

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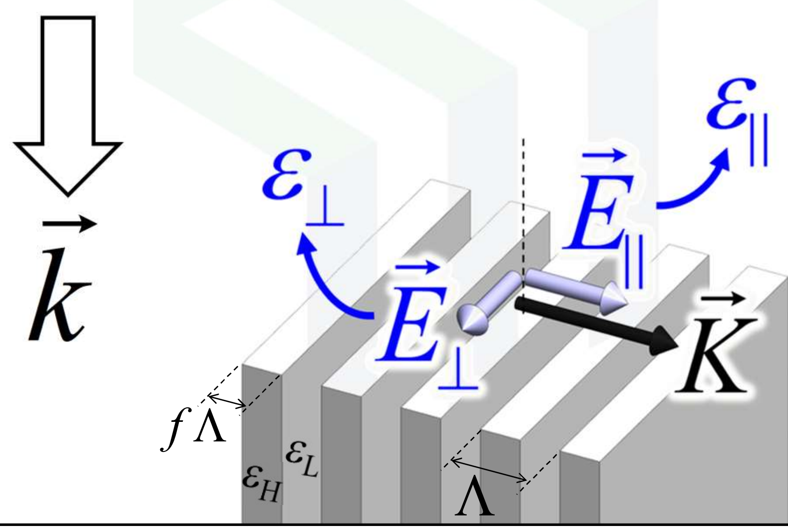
Relations Between Various Vectors



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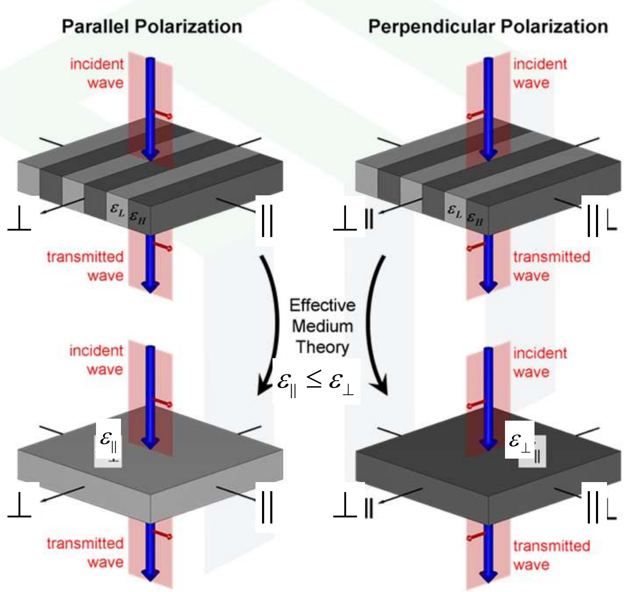
Effective Medium Theory Concept



$$\frac{1}{\varepsilon_\parallel} \cong \frac{\frac{1}{\varepsilon_L} + \frac{1}{\varepsilon_H}}{2}$$

$$\varepsilon_\perp \cong \frac{\varepsilon_L + \varepsilon_H}{2}$$

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Origin of Form Birefringence:
Perpendicular Polarization

<u>Boundary Conditions</u>

$\vec{E}$ is tangential to interfaces, so $\vec{E}$ is continuous across interfaces and $\vec{D}$ is discontinuous.

<u>Conclusions About Fields</u>

Boundary conditions make $\vec{E}$ constant throughout the grating.

This makes $\vec{D}$ discontinuous.

$$D_{\mathrm{H}} = \varepsilon_{\mathrm{H}} E_{\mathrm{avg}} \qquad D_{\mathrm{L}} = \varepsilon_{\mathrm{L}} E_{\mathrm{avg}}$$

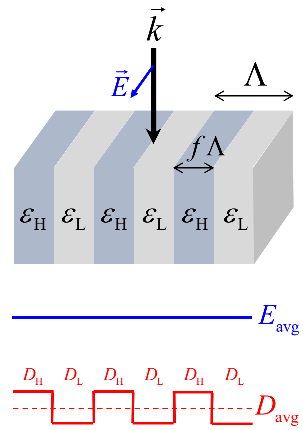
The weighted-average $\vec{D}$ field is then

$$\begin{aligned} D_{\mathrm{avg}} &= f D_{\mathrm{H}} + (1-f) D_{\mathrm{L}} \\ &= f \varepsilon_{\mathrm{H}} E_{\mathrm{avg}} + (1-f) \varepsilon_{\mathrm{L}} E_{\mathrm{avg}} \end{aligned}$$

<u>Effective Permittivity</u>

$$\varepsilon_{\mathrm{eff}} = \frac{D_{\mathrm{avg}}}{E_{\mathrm{avg}}} = f \varepsilon_{\mathrm{H}} + (1-f) \varepsilon_{\mathrm{L}}$$

$$\boxed{\varepsilon_{\perp} = f \varepsilon_{\mathrm{H}} + (1-f) \varepsilon_{\mathrm{L}}}$$



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Origin of Form Birefringence:
Parallel Polarization

<u>Boundary Conditions</u>

$\vec{E}$ is perpendicular to interfaces, so $\vec{E}$ is discontinuous across interfaces and $\vec{D}$ is continuous.

<u>Conclusions About Fields</u>

Boundary conditions make $\vec{D}$ constant throughout the grating.

This makes $\vec{E}$ discontinuous.

$$E_{\mathrm{H}} = \varepsilon_{\mathrm{H}}^{-1} D_{\mathrm{avg}} \qquad E_{\mathrm{L}} = \varepsilon_{\mathrm{L}}^{-1} D_{\mathrm{avg}}$$

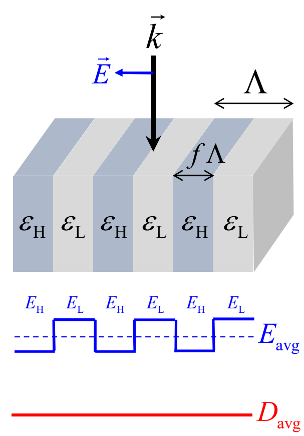
The weighted-average $\vec{E}$ field is then

$$\begin{aligned} E_{\mathrm{avg}} &= f E_{\mathrm{H}} + (1-f) E_{\mathrm{L}} \\ &= f \varepsilon_{\mathrm{H}}^{-1} D_{\mathrm{avg}} + (1-f) \varepsilon_{\mathrm{l}}^{-1} D_{\mathrm{avg}} \end{aligned}$$

<u>Effective Permittivity</u>

$$\varepsilon_{\mathrm{eff}} = \frac{D_{\mathrm{avg}}}{E_{\mathrm{avg}}} = \frac{1}{f \varepsilon_{\mathrm{H}}^{-1} + (1-f) \varepsilon_{\mathrm{l}}^{-1}}$$

$$\boxed{\varepsilon_{\parallel}^{-1} = f \varepsilon_{\mathrm{H}}^{-1} + (1-f) \varepsilon_{\mathrm{l}}^{-1}}$$



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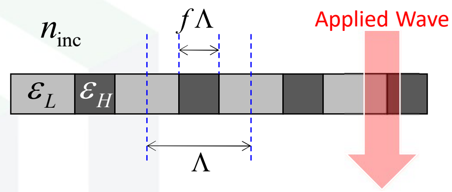
Effective Medium Theory Equations

Simplest Equations

$$\varepsilon_{\perp} \approx (1-f)\varepsilon_{\mathrm{L}} + f\varepsilon_{\mathrm{H}}$$

$$\frac{1}{\varepsilon_{\parallel}} \approx \frac{1-f}{\varepsilon_{\mathrm{L}}} + \frac{f}{\varepsilon_{\mathrm{H}}}$$



Higher Order Equations

$$\varepsilon_{\perp} \approx \varepsilon_{\mathrm{avg}} + \frac{(\Delta\varepsilon_{\mathrm{r}})^2}{2}\left(\frac{\Lambda}{\lambda}\right)^2 + 2\beta^2(\Delta\varepsilon_{\mathrm{r}})^2\left(\frac{\Lambda}{\lambda}\right)^4$$

$$\varepsilon_{\mathrm{avg}} = (\varepsilon_{\mathrm{L}} + \varepsilon_{\mathrm{H}})/2$$

$$\Delta\varepsilon = \varepsilon_{\mathrm{H}} - \varepsilon_{\mathrm{L}}$$

$$\beta = n_{\mathrm{inc}} \sin\theta_{\mathrm{inc}}$$

For more on higher accuracy equations, see

P. Lalanne, J. Hugonin, "High-order effective-medium theory of subwavelength gratings in classical mounting: application to volume holograms," J. Opt. Soc. Am. A 15(7), 1843–1851 (1998).

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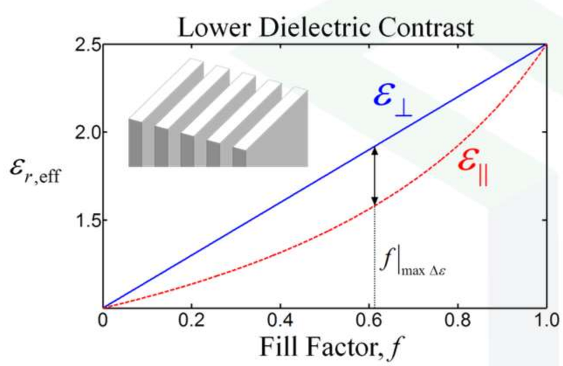
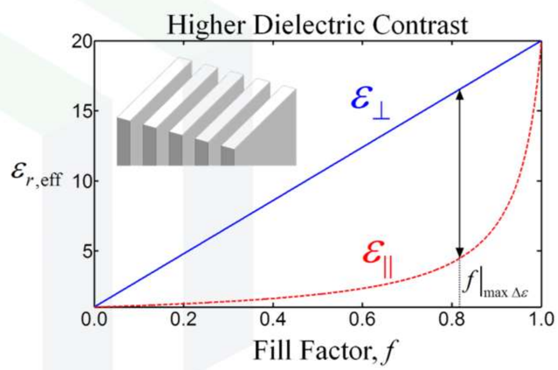
Design Trends





$$\varepsilon_{\perp} \approx (1-f)\varepsilon_{\mathrm{L}} + f\varepsilon_{\mathrm{H}}$$

$$\frac{1}{\varepsilon_{\parallel}} \approx \frac{1-f}{\varepsilon_{\mathrm{L}}} + \frac{f}{\varepsilon_{\mathrm{H}}}$$

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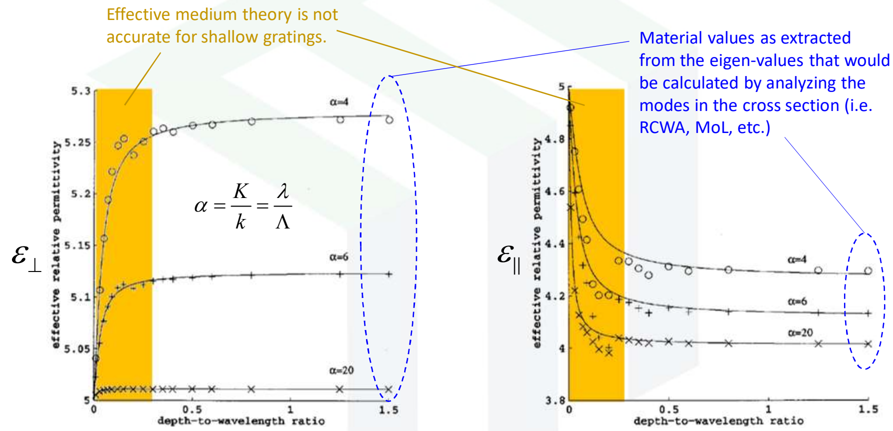
Depth Dependence of Effective Medium Theory



For shallow gratings, the effective index is highly dependant on the thickness of the grating.

P. Lalanne, D. Lemercier-Lalanne, "Depth dependence of the effective properties of subwavelength gratings," J. Opt. Soc. Am. A 14(2), 450–458 (1997).

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Notes About Subwavelength Gratings

- Period is much less than the wavelength so that only one diffracted mode is supported.
- Effective properties are independent of period, as long as it is sufficiently subwavelength.
- Properties are very broadband (DC up to cutoff)
- Properties are tolerant to structural deformations

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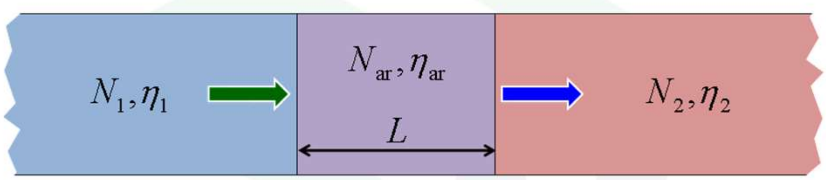
Anti-Reflection Gratings

Anti-Reflection Layers



General Case

$$\eta_{ar} = \sqrt{\eta_1 \eta_2}$$

$$L = \frac{\lambda_0}{4n_{ar}}$$

No magnetic response

$$n_{ar} = \sqrt{n_1 n_2}$$

$$L = \frac{\lambda_0}{4n_{ar}}$$

Note: this approach requires that dissimilar materials are bonded together.

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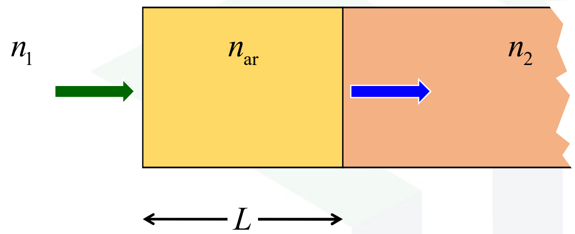
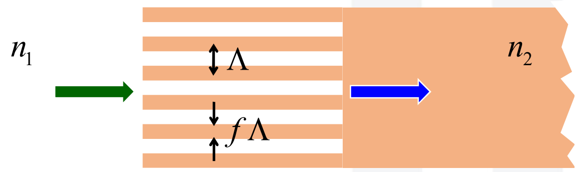
Anti-Reflection Gratings



Design Equations

$$n_{ar} = \sqrt{n_1 n_2}$$

$$L = \lambda_0 / (4 n_{ar})$$



Design Equations

$$n_{ar} = \sqrt{n_1 n_2}$$

$$\Lambda < \lambda / 10$$

$$f_\perp = \left(n_{ar}^2 - n_1^2\right) / \left(n_2^2 - n_1^2\right)$$

$$f_\parallel = \left(\frac{1}{n_{ar}^2} - \frac{1}{n_1^2}\right) \Big/ \left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$L = \lambda_0 / (4 n_{ar})$$

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Example #1 (1 of 3)

Suppose an anti-reflection grating is needed to minimize the reflections going from air into a ceramic material with $n = 3.0$ at a frequency of 2.4 GHz.

The effective index and depth of the grating layer should be

$$n_{ar} = \sqrt{(1.0)(3.0)} = 1.732 \qquad d = \frac{\lambda}{4} = \frac{c_0}{4 n_{ar} f_0} = \frac{3\times10^8 \ \frac{\text{m}}{\text{s}}}{4(1.732)(2.4\ \text{GHz})} = 1.8\ \text{cm}$$

Set the grating period to

$$\Lambda = \frac{\lambda}{20} = \frac{c_0}{20 n_{ar} f_0} = \frac{3\times10^8 \ \frac{\text{m}}{\text{s}}}{20(1.732)(2.4\ \text{GHz})} = 6.25\ \text{mm}$$

Set the fill factor to

$$f_\perp = \left[(1.732)^2 - (1.0)^2\right] \Big/ \left[(3.0)^2 - (1.0)^2\right] = 0.25 \qquad \text{“E mode”}$$

$$f_\parallel = \left[\frac{1}{(1.732)^2} - \frac{1}{(1.0)^2}\right] \Big/ \left(\frac{1}{(3.0)^2} - \frac{1}{(1.0)^2}\right) = 0.75 \qquad \text{“H mode”}$$

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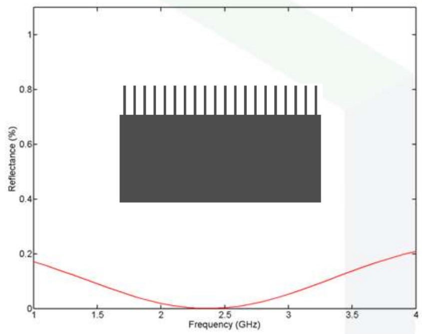
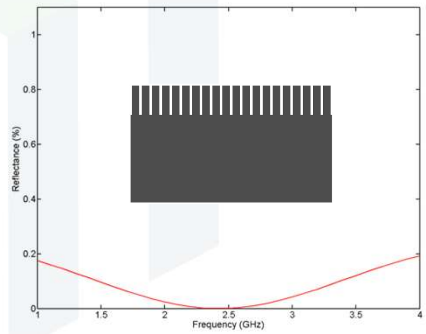
Example #1 (2 of 2)

The simulated performance of these gratings is

E Mode (⊥ Polarization)



H Mode (|| Polarization)



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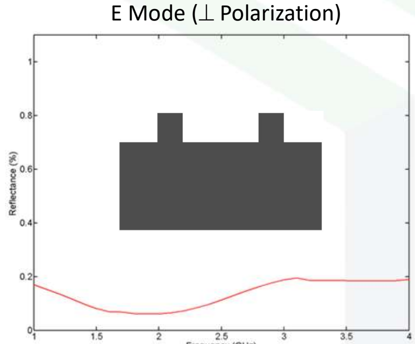
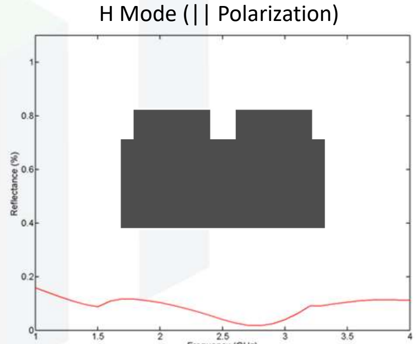
Example #1 (3 of 3)

What if we chose $\Lambda=\lambda/2$?

E Mode (⊥ Polarization)



H Mode (|| Polarization)



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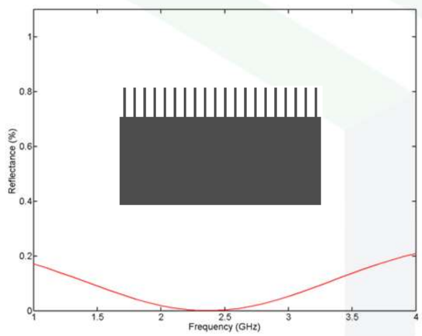
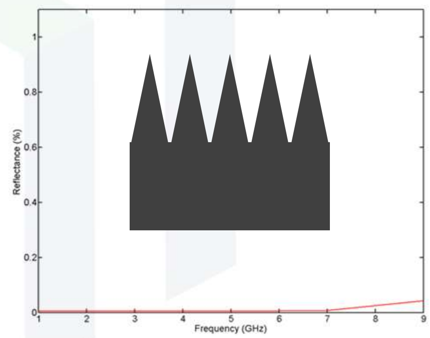
Broadband Anti-Reflection Gratings

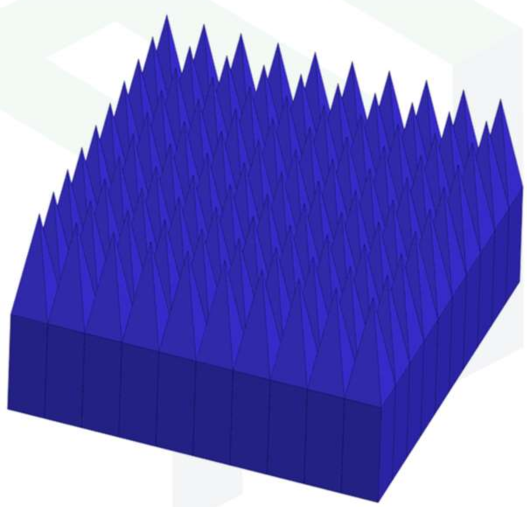
Binary Grating



Tapered Grating



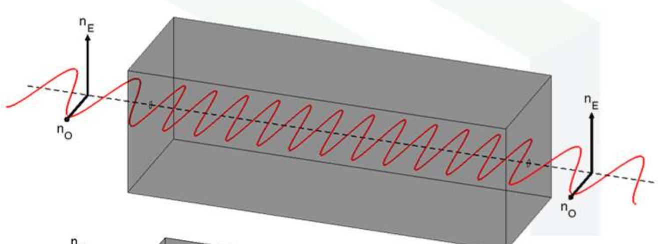
Polarization Independent Anti-Reflection Gratings



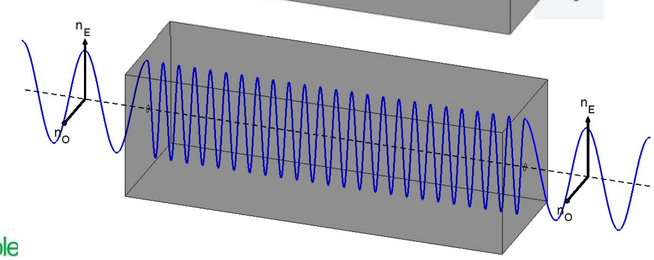
Wave Plates

Birefringence

A birefringent device is one that exhibits a different refractive index to different polarizations. Since a wave can always be decomposed into two orthogonal polarizations, a wave can see up to two refractive indices at the same time ("bi").



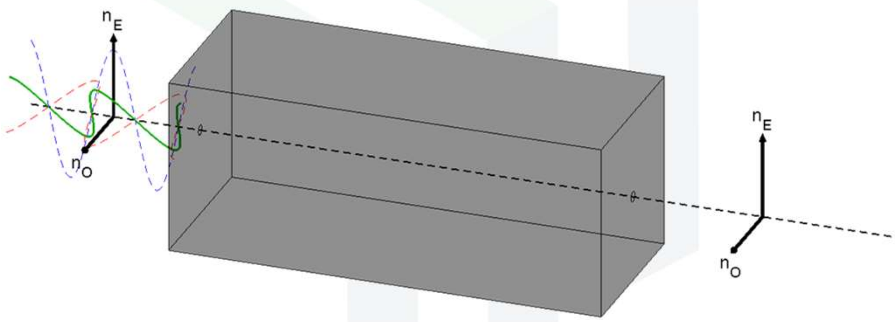
A wave polarized along n_O sees the lowest refractive index and propagates the fastest.



A wave polarized along n_E sees the highest refractive index and propagates the slowest.

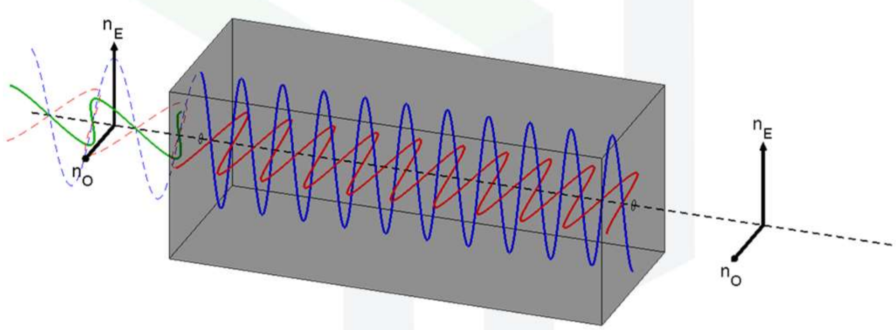
Typical Excitation of a Birefringent Device

It is common to excite a birefringent device at 45° with respect to the ordinary and extraordinary axes. This ensures that both polarizations are excited inside the device.



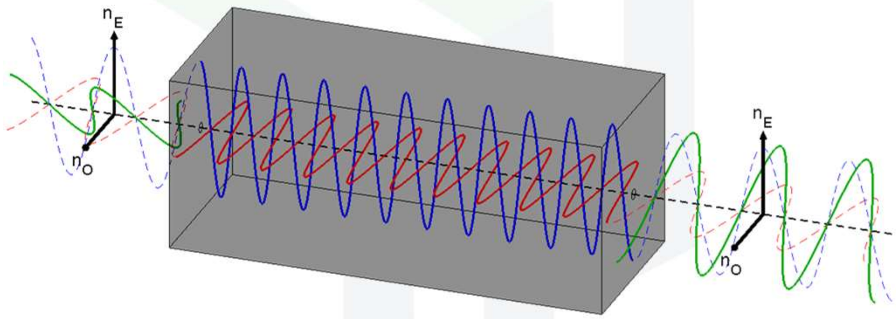
Propagation Through a Birefringent Device

Waves that enter a birefringent device in phase exit out of phase because they propagate at different speeds.



A New Polarization is Formed at the Output

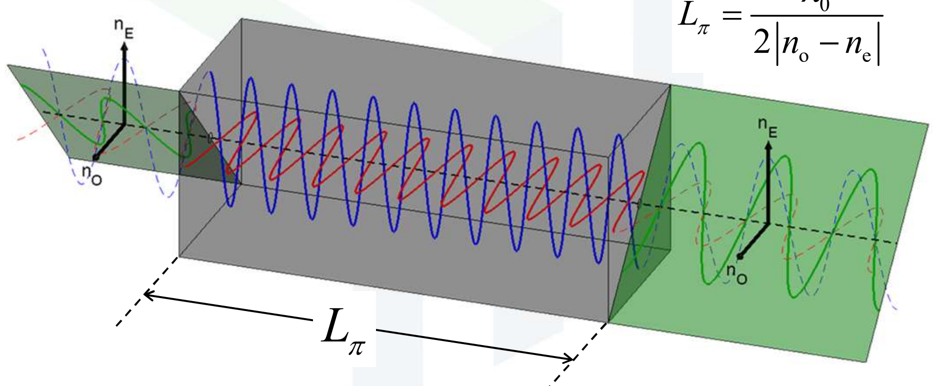
At the output of the device a new polarization is formed because the component waves are out of phase. The output polarization does change because both component waves travel at the same speed outside of the birefringent device.



Half-Wave Plates

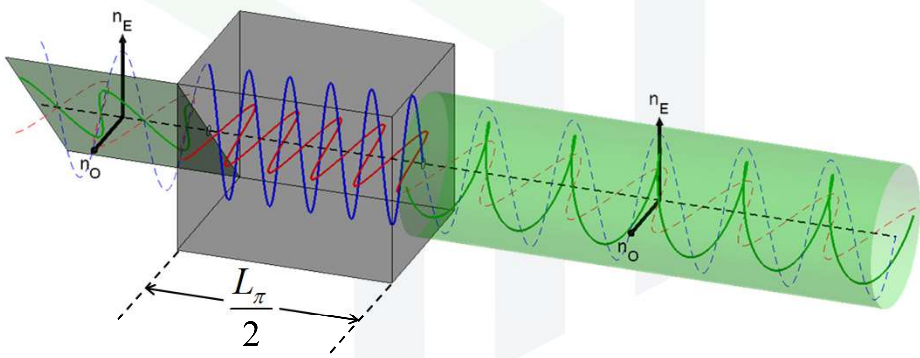
A half-wave plate is formed when 180° of phase difference is added between the component waves. This changes the handedness of the polarization.

$$L_{\pi} = \frac{\lambda_0}{2\left|n_{\mathrm{o}} - n_{\mathrm{e}}\right|}$$



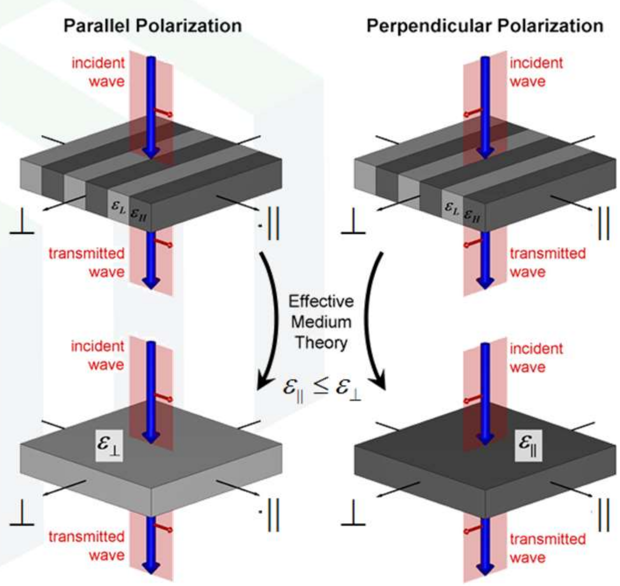
Quarter-Wave Plates

A quarter-wave plate is formed when 90 degrees of phase difference is added between the component waves. This converts between LP and CP.

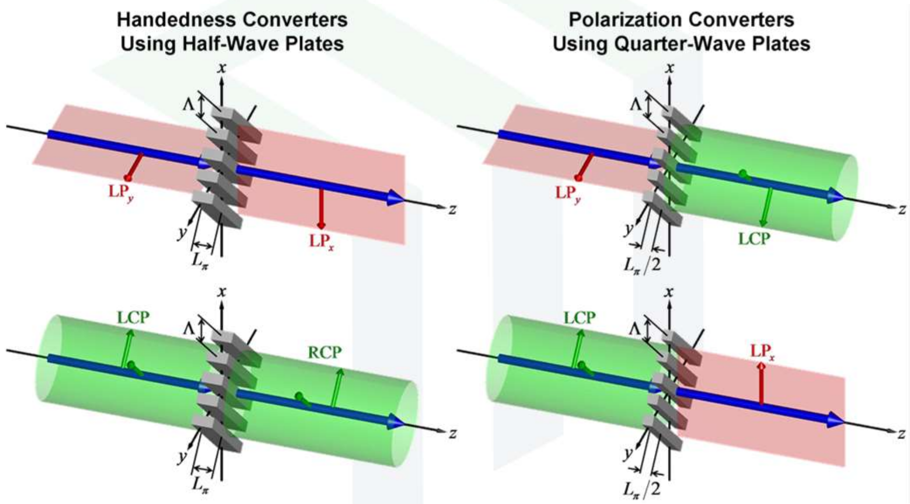


Subwavelength Gratings are Uniaxial Metamaterials

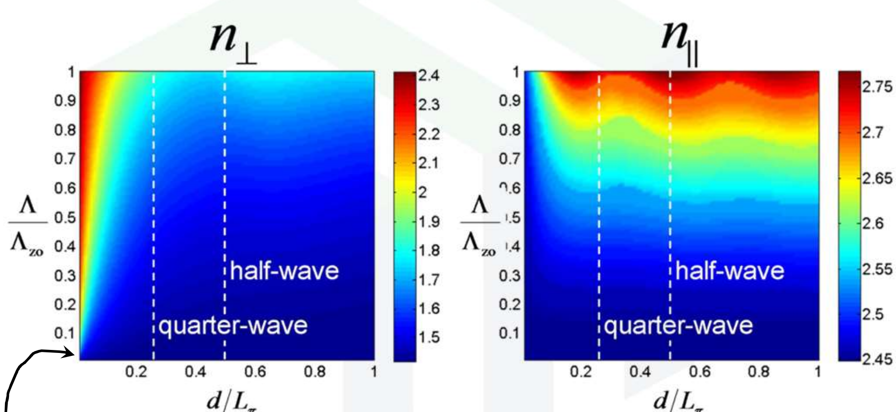
A subwavelength grating is a metamaterial. They are "materials" that derive their properties from how they are structured, not just their chemical makeup.



Subwavelength Gratings Can Function as Artificial Wave Plates



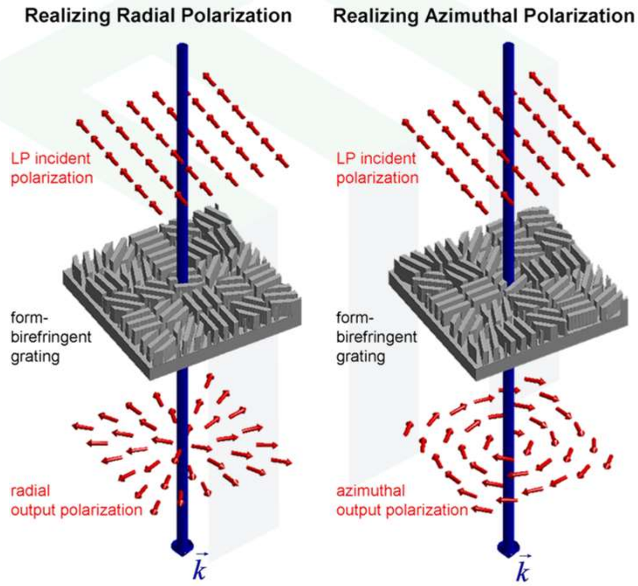
Optimizing Their Performance Using Parameter Sweeps



Effective refractive indices are more stable for thicker gratings (recall depth dependence). This implies half-wave designs are more robust.

Highly subwavelength gratings provide more consistent effective refractive indices.

Form-Birefringent Devices



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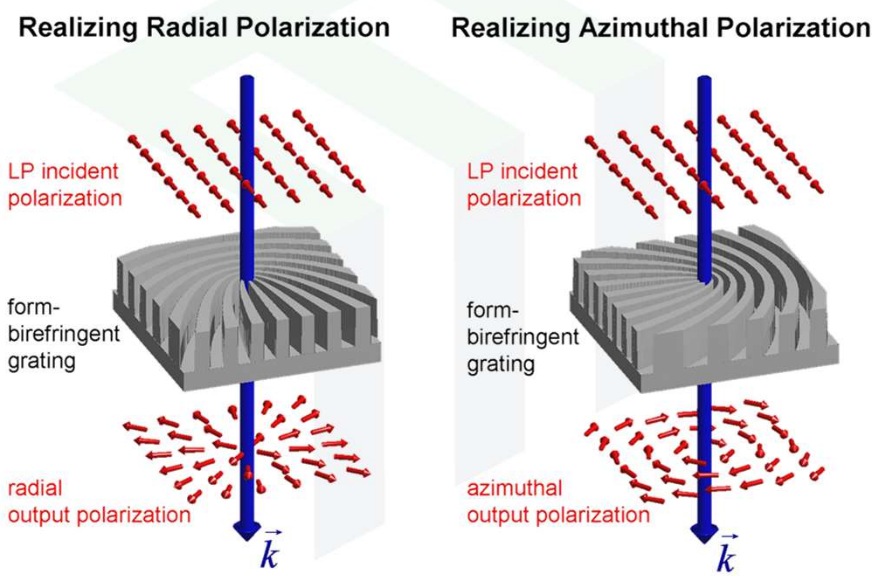
Continuous Form-Birefringent Devices



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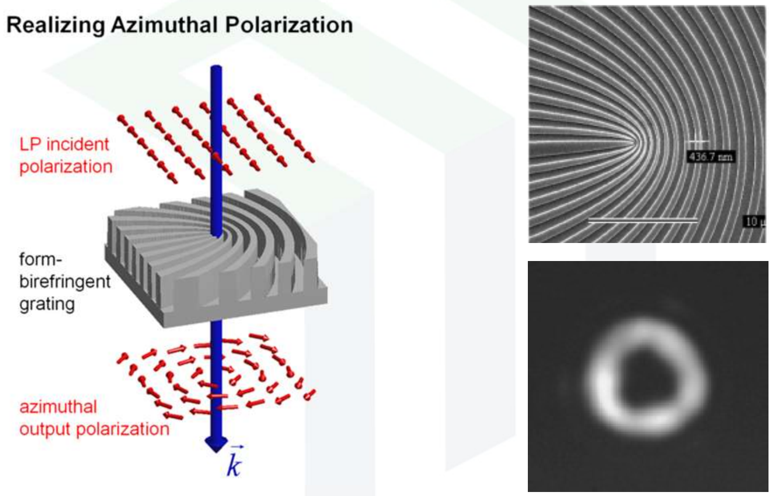
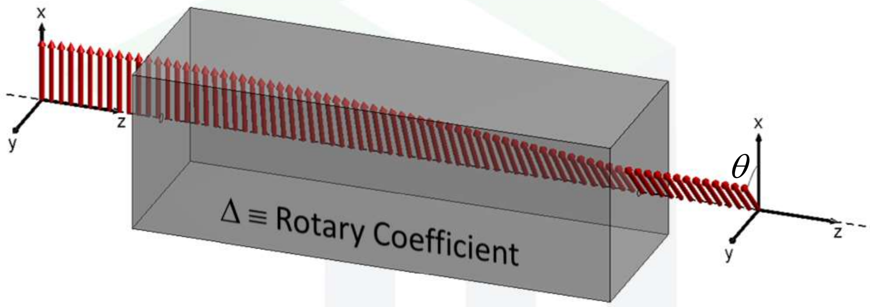
Form Birefringent Structures for Selective Mode Excitation



A. Mehta, J. D. Brown, P. Srinivasan, R. C. Rumpf, E. G. Johnson, "Spatially polarizing autocloned elements," Opt. Lett. **32**, 1935–1937 (2007) .

Chiral Media

Optical Activity



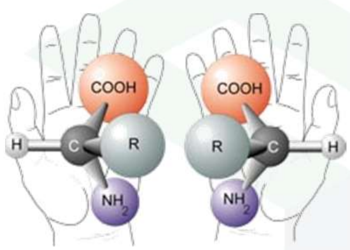
Field Polarization Inside Media

$$\vec{E}(z) = E_0\left[\hat{x}\cos(\Delta \cdot z) + \hat{y}\sin(\Delta \cdot z)\right] \qquad \Delta = \tfrac{1}{2}k_0 n \,\mathrm{Im}\left[\chi_m\right]$$

Optically active materials are birefringent to circularly polarized waves. The sum of RCP and LCP is a linearly polarized wave where the phase between the CPs defines the orientation of the LP wave.

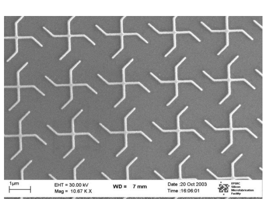
Chiral Materials Produce Optical Activity

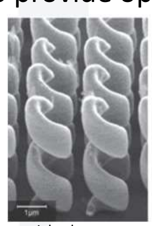
A chiral material does not have a plane of symmetry so its mirror image looks different.



http://upload.wikimedia.org/wikipedia/commons/8/87/Chirality_with_hands.jpg

Chiral metamaterials can be made to provide optical activity and perform many functions.





http://users.ecs.soton.ac.uk/dmb/chiralmetamaterials.php

http://www.nanophotonics.org.uk/niz/niz/

Polarizers

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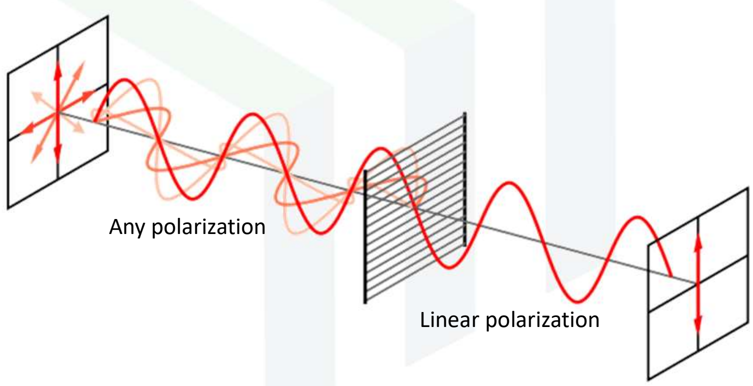
Concept of a Polarizer

A polarizer is a device that allows only one specific polarization to pass through. All other polarizations are reflected.



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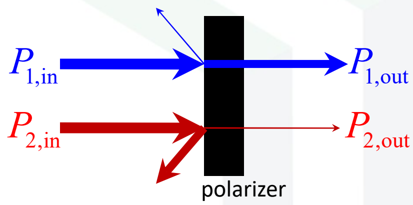
Extinction Ratio

The extinction ratio is often used to quantify the filtering performance of a polarizer. It is a measure of how well it suppresses the undesired polarization compared to the desired polarization.



$$\xi = \frac{P_{1,out}}{P_{2,out}}$$

Typical polarizers have extinction ratios exceeding 100:1. High performance applications can require extinction ratios exceeding 1000:1 or even 10,000:1.

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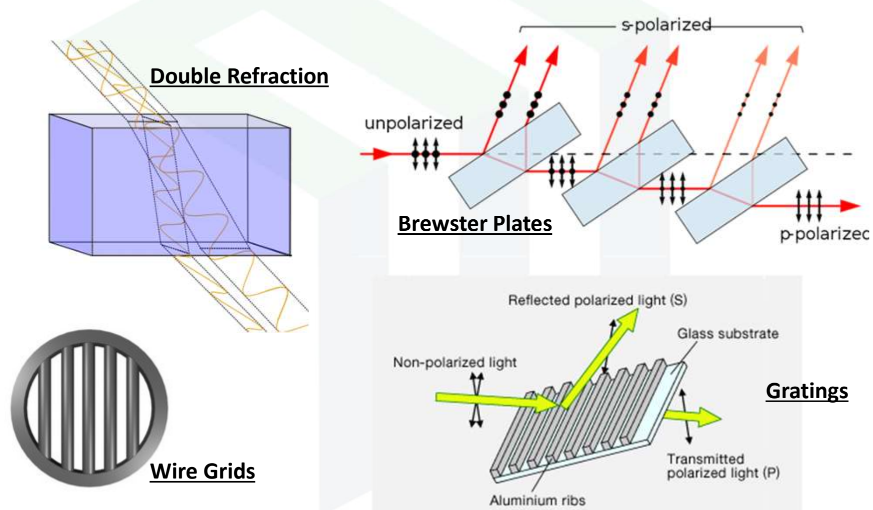
Techniques for Making Linear Polarization Filters

A polarizer can be made from anything that behaves differently to different polarizations.



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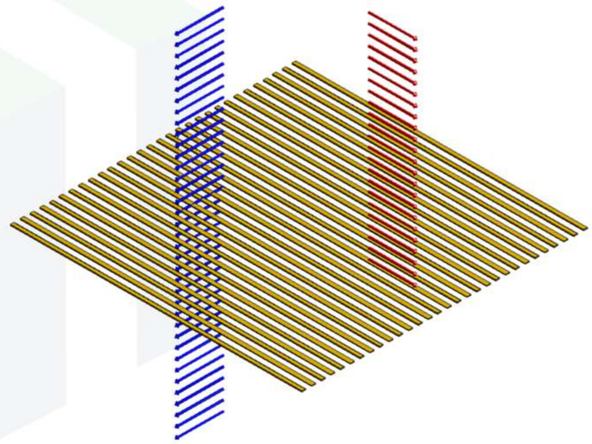
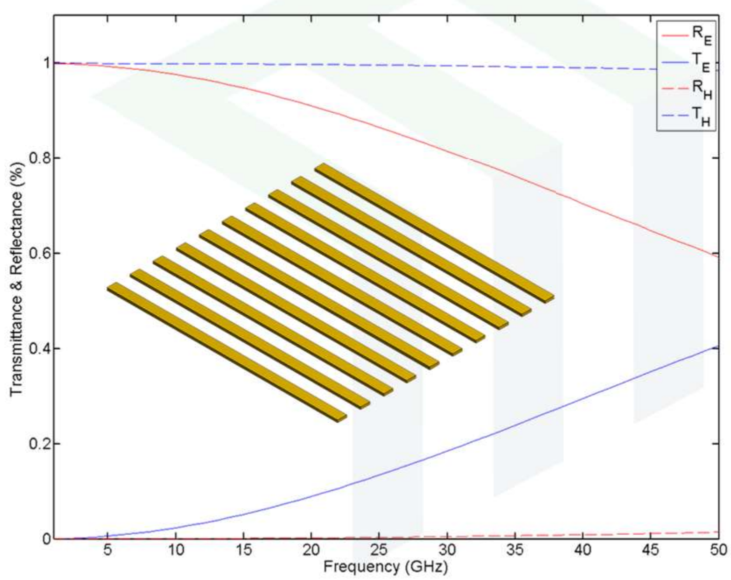
Wire Grid Polarizers

Fields oscillating parallel to the grooves are able to push charges freely along the length of the wires. In this case, the array acts like a solid sheet of metal and reflects waves of this polarization.

Fields oscillating perpendicular try to push charges across the short section of the wires. The charges are not free to move very far and behave more like bound charges. In this configuration, the array acts like a dielectric sheet and allows waves to transmit.



FDFD Simulation of a Wire Grid Polarizer



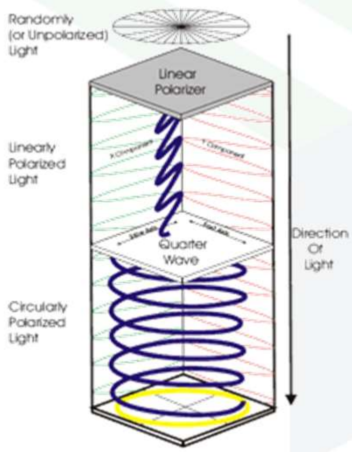
$\Lambda = 2\ \text{mm}$

$t = 0.2\ \text{mm}$

$f = 0.25$

What about circular polarization filters?

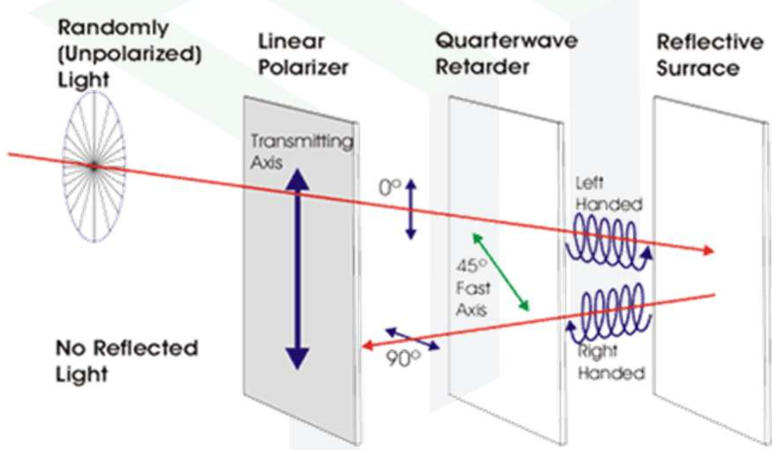
Circular polarization filters can be made using wave plates and linear polarization filters. The wave plate is usded to convert CP to LP and then filter the LP polarizations.



http://www.apioptics.com/circular-polarizers.html

Isolators Using Polarizers and Wave Plates

We can make “one way” devices called isolators using wave plates and polarizers.



http://www.apioptics.com/circular-polarizers.html