Advanced Electromagnetics:
21st Century Electromagnetics

Examples Using the Grating Equation

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Example #1

An 11 mm wide diffraction grating has rulings of 550 lines per millimeter. What is the longest wavelength that forms an intensity maximum in the fifth order?

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Solution to Example #1 (1 of 3)

Write the grating equation and determine what is known.

n_{obs} is the refractive index where diffraction is being observed.

Assume reflection grating.
Assume air.
$n_{\text{obs}} = n_{\text{inc}} = 1.0$

λ_0 is the free space wavelength.

It is the unknow in this problem.

θ_{inc} is the angle of incidence of the applied wave.

Assume normal incidence.
$\theta_{\text{inc}} = 0°$

$$n_{\text{obs}} \sin\left[\theta(m)\right] = n_{\text{inc}} \sin\theta_{\text{inc}} - m\frac{\lambda_0}{\Lambda}\sin\phi$$

ϕ is the grating slant.

Assume $\phi = 90°$.

$\theta(m)$ is the angle of the mth diffraction order.

$\theta(\pm 5)$ must be purely real.

m is the diffraction order index.

It was given that $m = 5$.

Λ is the grating period.

$\Lambda = \dfrac{1\ \text{mm}}{550\ \text{lines}} = 1.82\ \mu\text{m}$

EMPossible

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Solution to Example #1 (2 of 3)

Write the grating equation and determine what is known.

$$1 \cdot \sin\left[\theta(5)\right] = 1 \cdot \sin 0° - 5\frac{\lambda_0}{1.82\ \mu m}\sin 90°$$

$$\vdots$$

$$\sin\left[\theta(5)\right] = \frac{\lambda_0}{364\ \text{nm}}$$

EMPossible

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Solution to Example #1 (2 of 3)

$$\sin\left[\theta(5)\right]=\frac{\lambda_0}{364\text{ nm}}$$

What wavelength λ_0 ensures that $\theta(5)$ is a purely real number?

$$\frac{\lambda_0}{364\text{ nm}}\leq 1 \qquad\qquad \lambda_0 \leq 364\text{ nm}$$

The final answer is

$$\boxed{\lambda_{\max}=364\text{ nm}}$$

EMPossible

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Common Assumptions for Poorly Defined Grating Problems

- Grating is in air.
  $n_{\text{obs}}=n_{\text{inc}}=1.0$
- Normal incidence.
  $\theta_{\text{inc}}=0°$
- Reflected diffraction orders are to be considered.

EMPossible

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Example #2

Light with wavelength 632 nm light is incident at 30° onto a diffraction grating having 3000 lines over 1 cm. How many diffraction orders will this grating produce?

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Solution to Example #2 (1 of 3)

Write the grating equation and determine what is known.

$$n_{obs} \sin\left[\theta(m)\right] = n_{inc} \sin\theta_{inc} - m\frac{\lambda_0}{\Lambda}\sin\phi$$

n_{obs} is the refractive index where diffraction is being observed.

Assume reflection grating.
Assume air.
$n_{obs} = n_{inc} = 1.0$

λ_0 is the free space wavelength.

$\lambda_0 = 632$ nm

θ_{inc} is the angle of incidence of the applied wave.

$\theta_{inc} = 30°$

ϕ is the grating slant.

Assume $\phi = 90°$.

$\theta(m)$ is the angle of the mth diffraction order.

$\theta(m)$ must be purely real for the mode to not be cut off.

m is the diffraction order index.

This is to be solved.

Λ is the grating period.

$$\Lambda = \frac{1\ \text{cm}}{3000\ \text{lines}} = 3.33\ \mu\text{m}$$

EMPossible

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Solution to Example #2 (2 of 3)

Write the grating equation and determine what is known.

$$1 \cdot \sin\left[\theta(m)\right] = 1 \cdot \sin 30° - m \frac{632 \text{ nm}}{3.33\ \mu\text{m}} \sin 90°$$

$$\vdots$$

$$\sin\left[\theta(m)\right] = 0.5 - m(0.1896)$$

EMPossible

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Solution to Example #2 (3 of 3)

$$\sin\left[\theta(m)\right] = 0.5 - m(0.1896)$$

This problem is solved by counting the number of solutions for m that give a real number for $\theta(m)$. Mathematically, this is found by solving

$$\left|0.5 - m(0.1896)\right| \leq 1$$

Check Negative Values of m

$$0.5 + (-m)(0.1896) \leq 1$$
$$(-m) \leq \frac{0.5}{0.1896}$$
$$(-m) \leq 2.64$$
$$m \geq -2.64$$
$$m \geq -2$$

Check Positive Values of m

$$m(0.1896) - 0.5 \leq 1$$
$$m \leq \frac{1.5}{0.1896}$$
$$m \leq 7.91$$
$$m \leq 7$$

$$-2 \leq m \leq 7$$
$$M = 7 - (-2) + 1$$
$$\boxed{M = 10}$$

EMPossible

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