Reading Assignment

Read Chapter 23.

Problem #1: Finite-Difference Approximations

Using the polynomial method, derive expressions for the following finite-difference approximations. You may do this by hand or use MATLAB in any way that you wish. Show your work. Report your equations in standard math format using Equation Editor, LaTeX, or whatever tool you wish. Do not report your equation using pure text or ASCII art style. Your finite-difference equations must be written in a way that conveys the location where the derivatives are being evaluated. They must also be written in terms of the given parameters. An example solution of a second-order derivative of \( f(x) \) with respect to \( x \), evaluated at the second point is

\[
\frac{df_2}{dx^2} \approx \frac{f_1 - 2f_2 + f_3}{(\Delta x)^2}
\]

**Part a**

\[
\frac{df(x)}{dx} \text{ at the first of 3 points}
\]

**Part b**

\[
\frac{df(x)}{dx} \text{ at the third of 4 points}
\]

**Part c**

\[
\frac{d^2 f(\theta)}{d\theta^2} \text{ at the midpoint of 4 points}
\]

**Part d**

\[
\frac{d^2 f(\theta)}{d\theta^2} \text{ at the second of 4 points}
\]

**Part e**

\[
\frac{d^3 f(t)}{dt^3} \text{ at the midpoint between the second and third points of 6 points}
\]
Problem #2: Numerical Differentiation

For this problem, you will numerically calculate the second-order derivative of the following function with respect to \( s \):

\[
f(s) \approx 1 + 2 \exp \left[ -\left( \frac{s - 6}{2} \right)^2 \right] \quad -10 \leq s \leq 10
\]

**Part a**

Derive all the finite-difference approximations you will need across the entire grid if all your finite-difference approximations are to use 5 points.

**Part b**

Using the finite-difference approximations you derived in Part a of this problem, write a MATLAB program to numerically calculate \( d^2 f(s)/ds^2 \) using 1000 points within the specified interval. DO NOT CALCULATE ANYTHING OUTSIDE OF THE SPECIFIED INTERVAL. On the same plot, show the original function and the approximated second-order derivative. Be sure your plot is of professional quality. Complete and sign a graphics checklist and attach it to this homework.