

Problem #1: 1D Derivative Matrices

Write a MATLAB function to construct derivative matrices for first-order and second-order derivatives on a one-dimensional grid along the x -axis. Use the following equations for the finite-difference approximations and use Dirichlet boundary conditions.

$$\frac{df_i}{dx} \cong \begin{cases} \frac{f_{i+1}}{2\Delta x} & i=1 \\ \frac{f_{i+1}-f_{i-1}}{2\Delta x} & 2 \leq i \leq N-1 \\ -\frac{f_{i-1}}{2\Delta x} & i=N \end{cases} \quad \frac{d^2 f_i}{dx^2} \cong \begin{cases} \frac{f_{i+1}-2f_i}{(\Delta x)^2} & i=1 \\ \frac{f_{i+1}-2f_i+f_{i-1}}{(\Delta x)^2} & 2 \leq i \leq N-1 \\ \frac{-2f_i+f_{i-1}}{(\Delta x)^2} & i=N \end{cases}$$

Use the following header exactly for your function. Be sure your function can construct any size matrix defined by N_x . Save your function with the filename `fdder1d.m`. For this problem only, provide your code up front in your homework, not in an appendix. To verify your function, duplicate the results provided in the benchmarking document.

```
function [DX,DX2] = fdder1d(Nx,dx)
% FDDER1D      One-Dimensional Finite-Difference Derivative Operators
%              Using Dirichlet Boundary Conditions
%
% [DX,DX2] = fdder1d(Nx,dx);
%
% Nx is the size the grid.
%
% dx is the grid resolution.
%
% DX is a square matrix of size Nx-by-Nx that calculates a first-order
% derivative.
%
% DX2 is a square matrix of size Nx-by-Nx that calculates a second-order
% derivative.
```

Problem #2: Benchmark First-Order Derivative Matrix

Write a MATLAB script that calculates the first-order derivative of the $\cos(x)$ function in the interval $0 \leq x \leq 2\pi$. Your program should store the function in a column vector and use a derivative matrix from Problem #1 to perform the numerical differentiation. Your program should not modify the derivative matrices produced by `fdder1d()` in any way. Use 100 points. Plot all points of your function and it's first derivative on the same plot. Label the curves and be sure the figure is clean, clear, and professional. Explain any anomalies in your data. Explain any anomalies that may be present in your results. Include a signed graphics checklist at the end of your assignment.

Problem #3: Benchmark Second-Order Derivative Matrix

Write a MATLAB script that calculates the second-order derivative of the $\cos(x)$ function in the interval $0 \leq x \leq 2\pi$. Your program should store the function in a column vector and use a derivative matrix from Problem #1 to perform the numerical differentiation. Your program should not modify the derivative matrices produced by `fdder1d()` in any way. Use 100 points. Plot all points of your function and it's first derivative on the same plot. Label the curves and be sure the figure is clean, clear, and professional. Explain any anomalies in your data. Explain any anomalies that may be present in your results. Include a signed graphics checklist at the end of your assignment.