



Advanced Electromagnetics:  
21<sup>st</sup> Century Electromagnetics

# Applications of Diffraction Gratings



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## Lecture Outline

- Modes of operation
- The grating spectrometer
- Littrow gratings
- Patterned fanout gratings
- Diffractive optical elements

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# Modes of Operation

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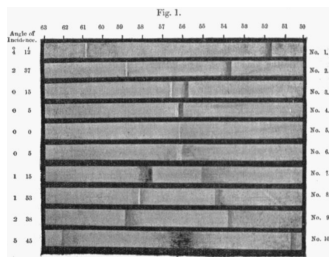
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## Wood's Anomalies

Robert W. Wood observed rapid variations in the spectrum of light diffracted by gratings which he could not explain. Later, Hessel explained them and identified two different types.

### Type 1 – Rayleigh Singularities

Rapid variation in the amplitudes of the diffraction orders that correspond to the onset or disappearance of other diffraction orders.



R. W. Wood, Phil. Mag. 4, 396 (1902)

### Type 2 – Resonance Effects

A resonance condition arising from leaky waves supported by the grating. Today, this is called guided-mode resonance.

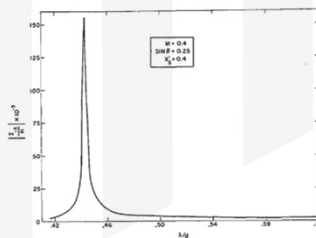
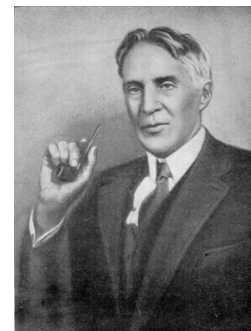


Fig. 23. Relative amplitude of the  $n = -3$  resonant spectral order vs  $\lambda/d$  for the case of four propagating spectral orders.

A. Hessel, A. A. Oliner, "A New Theory of Wood's Anomalies on Optical Gratings," Appl. Opt., Vol. 4, No. 10, 1275 (1965).



Robert Williams Wood  
1868 - 1955

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## Three Modes of Operation for 1D Gratings

### Bragg Grating

Couples power between counter-propagating waves.

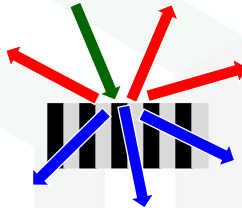


### Applications

- Thin film optical filters
- Fiber optic gratings
- Wavelength division multiplexing
- Dielectric mirrors
- Photonic crystal waveguides

### Diffraction Grating

Couples power between waves at different angles.



### Applications

- Beam splitters
- Patterned fanout gratings
- Laser locking
- Spectrometry
- Sensing
- Anti-reflection
- Frequency selective surfaces
- Grating couplers

### Long Period Grating

Couples power between co-propagating waves.



### Applications

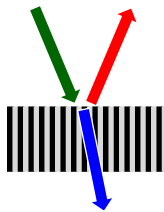
- Sensing
- Directional coupling

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## Some Applications of Gratings

### Subwavelength Gratings

Only the zero-order modes may exist.



### Applications

- Polarizers
- Artificial birefringence
- Form birefringence
- Anti-reflection
- Effective index media

### Littrow Gratings

Gratings in the Littrow configuration are a spectrally selective retroreflector.

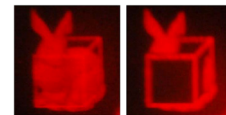


### Applications

- Sensors
- Lasers

### Patterned Fanout Gratings

Gratings diffract laser light to form images.

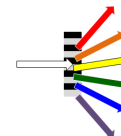


### Holograms

Holograms are stored as gratings.

### Spectrometry

Gratings separate broadband light into its component colors.



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# The Grating Spectrometer

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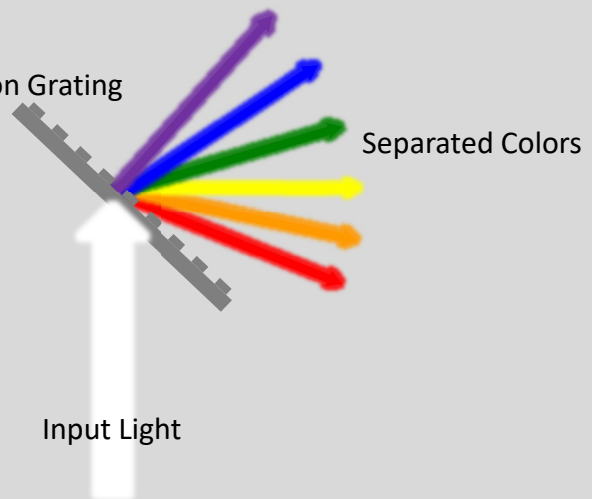
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## The Grating Spectrometer

The angle of a diffraction order  $\theta(m)$  depends on wavelength. Each wavelength  $\lambda_0$  will diffract into a slightly different direction. This can be used to separate a spectrum.

$$n_{\text{obs}} \sin [\theta(m)] = n_{\text{inc}} \sin \theta_{\text{inc}} - m \frac{\lambda_0}{\Lambda} \sin \phi$$

Diffraction Grating



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## Spectral Sensitivity

Start with the grating equation.

$$n_{\text{avg}} \sin[\theta(m)] = n_{\text{inc}} \sin \theta_{\text{inc}} - m \frac{\lambda_0}{\Lambda_x}$$

Define *spectral sensitivity* as how much the diffracted angle  $\theta(m)$  changes with respect to wavelength  $\lambda_0$ .

$$\frac{\partial \theta(m)}{\partial \lambda_0} = - \frac{m}{\Lambda_x n_{\text{avg}} \cos \theta_m}$$

$$\Delta \theta(m) \cong \frac{m}{\Lambda_x n_{\text{avg}} \cos[\theta(m)]} \Delta \lambda_0$$

This equation shows how to maximize sensitivity.

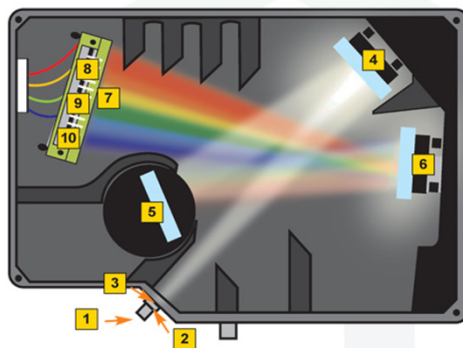
1. Diffract into higher order modes ( $\uparrow m$ ).
2. Use shorter period gratings ( $\downarrow \Lambda_x$ ).
3. Diffract into larger angles ( $\uparrow \theta(m)$ ).
4. Diffract into lower refractive index materials ( $\downarrow n_{\text{avg}}$ ).



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## Ocean Optics HR4000 Grating Spectrometer

[http://www.oceanoptics.com/Products/benchoptions\\_hr.asp](http://www.oceanoptics.com/Products/benchoptions_hr.asp)



- |                   |                       |                     |                         |
|-------------------|-----------------------|---------------------|-------------------------|
| 1 SMA Connector   | 4 Collimating Mirror  | 7 Collection Lenses | 10 Optional UV Detector |
| 2 Entrance Slit   | 5 Diffraction Grating | 8 Detector Array    |                         |
| 3 Optional Filter | 6 Focusing Mirror     | 9 Optional Filter   |                         |



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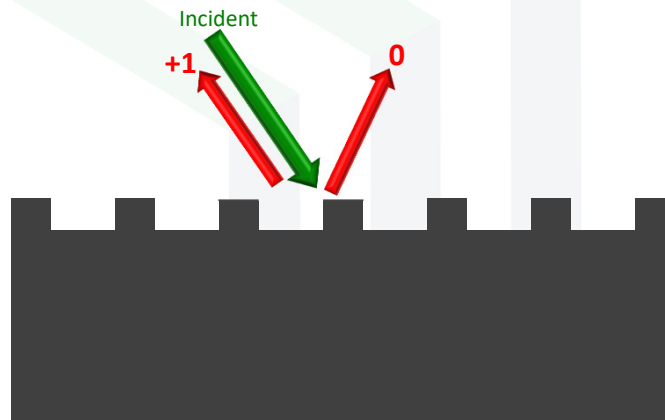
# Littrow Gratings

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## Littrow Configuration

In the Littrow configuration, the +1 order reflected mode is parallel to the incident wave vector. This forms a spectrally selective mirror.



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## Conditions for the Littrow Configuration

The grating equation for the Littrow configuration is

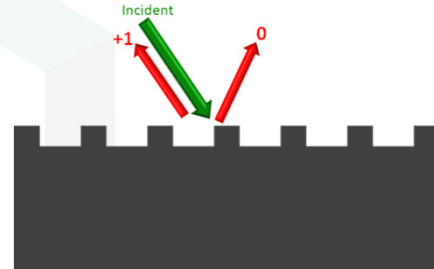
$$n \sin[\theta(m)] = n \sin \theta_{\text{inc}} - m \frac{\lambda_0}{\Lambda_x}$$

The Littrow configuration occurs when

$$\theta(+1) = -\theta_{\text{inc}}$$

The condition for the Littrow configuration is found by replacing  $\theta(m)$  in the grating equation with  $-\theta_{\text{inc}}$ . This gives

$$2n \sin \theta_{\text{inc}} = \frac{\lambda_0}{\Lambda_x}$$



## Spectral Selectivity

Typically, a cone of angles  $\Delta\theta$  reflected from a grating is detected.

It is desired to figure out what range of wavelengths  $\Delta\lambda_0$  is captured within the cone  $\Delta\theta$ .

The grating equation for the  $m = +1$  diffraction order is

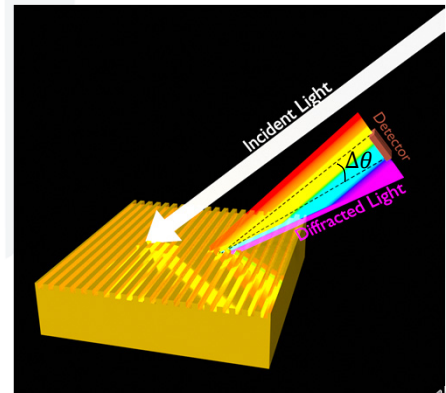
$$\sin \theta_{+1} = \sin \theta_{\text{inc}} - \frac{\lambda_0}{n\Lambda_x}$$

This equation is differentiated to obtain  $d\lambda_0/d\theta_{+1}$ .

$$\cos \theta_{+1} d\theta_{+1} = -\frac{1}{n\Lambda_x} d\lambda_0 \quad \longrightarrow \quad \frac{d\lambda_0}{d\theta_{+1}} = -n\Lambda_x \cos \theta_{+1}$$

In the Littrow configuration  $\theta_{+1} = -\theta_{\text{inc}}$ .

$$\frac{\Delta\lambda_0}{\Delta\theta} = n\Lambda_x \cos \theta_{\text{inc}} \quad \longrightarrow \quad \Delta\lambda_0 = n\Lambda_x \Delta\theta \cos \theta_{\text{inc}}$$



## Example (1 of 2)

Design a metallic grating in air that is to be operated in the Littrow configuration at 10 GHz at an angle of  $45^\circ$ . What bandwidth of frequencies is captured inside a  $5^\circ$  cone?

### Solution

Right away, it is known that

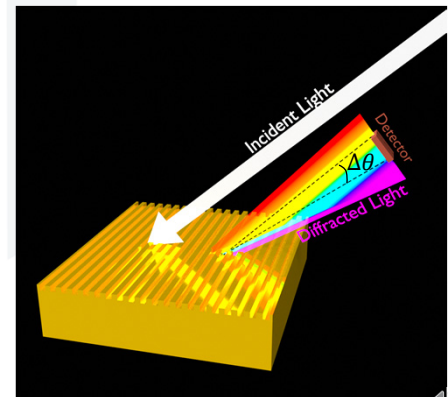
$$n = 1.0$$

$$\theta_{\text{inc}} = 45^\circ$$

$$\lambda_0 = \frac{c_0}{f} = \frac{3 \times 10^8 \frac{\text{m}}{\text{s}}}{10 \text{ GHz}} = 3.00 \text{ cm}$$

The grating period is then found to be

$$\Lambda_x = \frac{\lambda_0}{2n \sin \theta_{\text{inc}}} = \frac{3.00 \text{ cm}}{2(1.0) \sin(45^\circ)} = 2.12 \text{ cm}$$



## Example (2 of 2)

### Solution continued

The equation for linewidth  $\Delta\lambda_0$  must be converted to calculate bandwidth  $\Delta f$ .

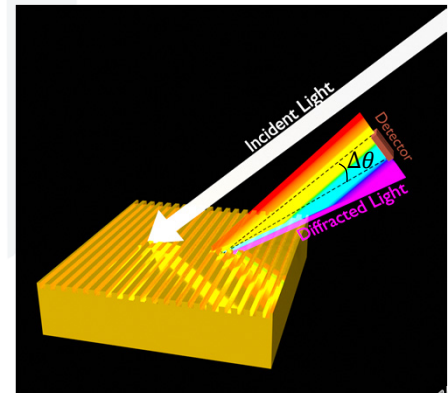
$$c_0 = f\lambda_0 \rightarrow 0 = f d\lambda_0 + \lambda_0 df \rightarrow \Delta f = \Delta\lambda_0 \frac{f}{\lambda_0} \rightarrow \Delta f = \Delta\lambda_0 \frac{c_0}{\lambda_0^2} = \Delta\lambda_0 \frac{f^2}{c_0}$$

Recall  $\Delta\lambda_0 = n\Lambda_x \Delta\theta \cos \theta_{\text{inc}}$ . This is substituted into the last equation above to get

$$\Delta f = \frac{n\Lambda_x \Delta\theta f^2}{c_0} \cos \theta_{\text{inc}}$$

Substitute in the numbers to get

$$\Delta f = \frac{(1.0)(2.12 \text{ cm}) \left( \frac{\pi}{180^\circ} 5^\circ \right) (10 \text{ GHz})^2}{3 \times 10^8 \frac{\text{m}}{\text{s}}} \cos(45^\circ) = \boxed{0.44 \text{ GHz}}$$



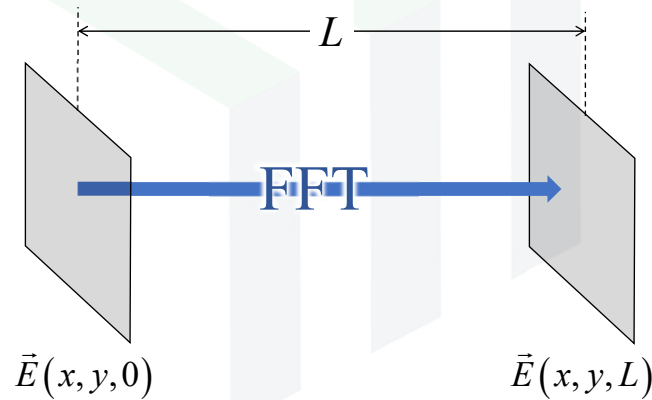
# Patterned Fanout Gratings

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## Near-Field to Far-Field

After propagating a long distance, the field within a plane tends toward the Fourier transform of the initial field.



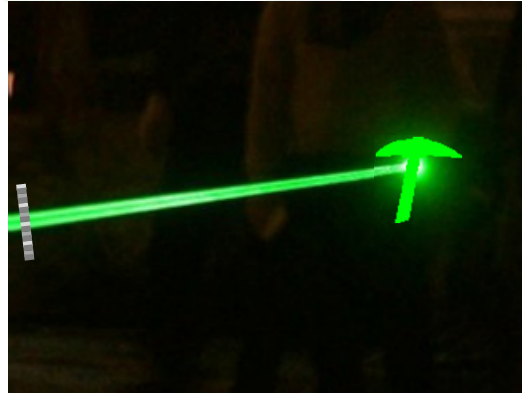
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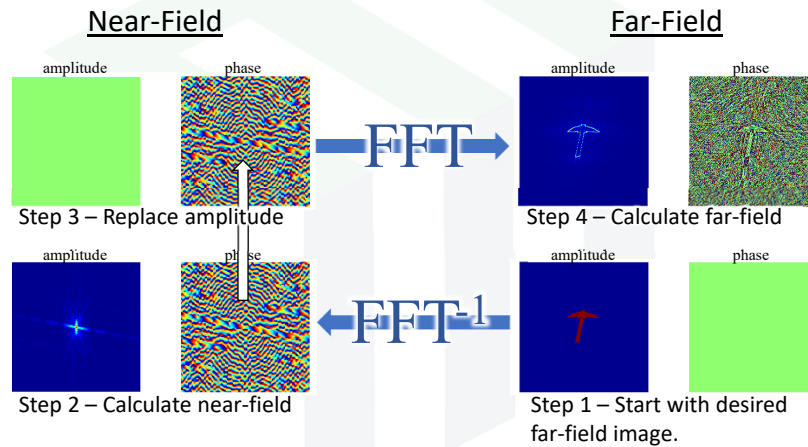
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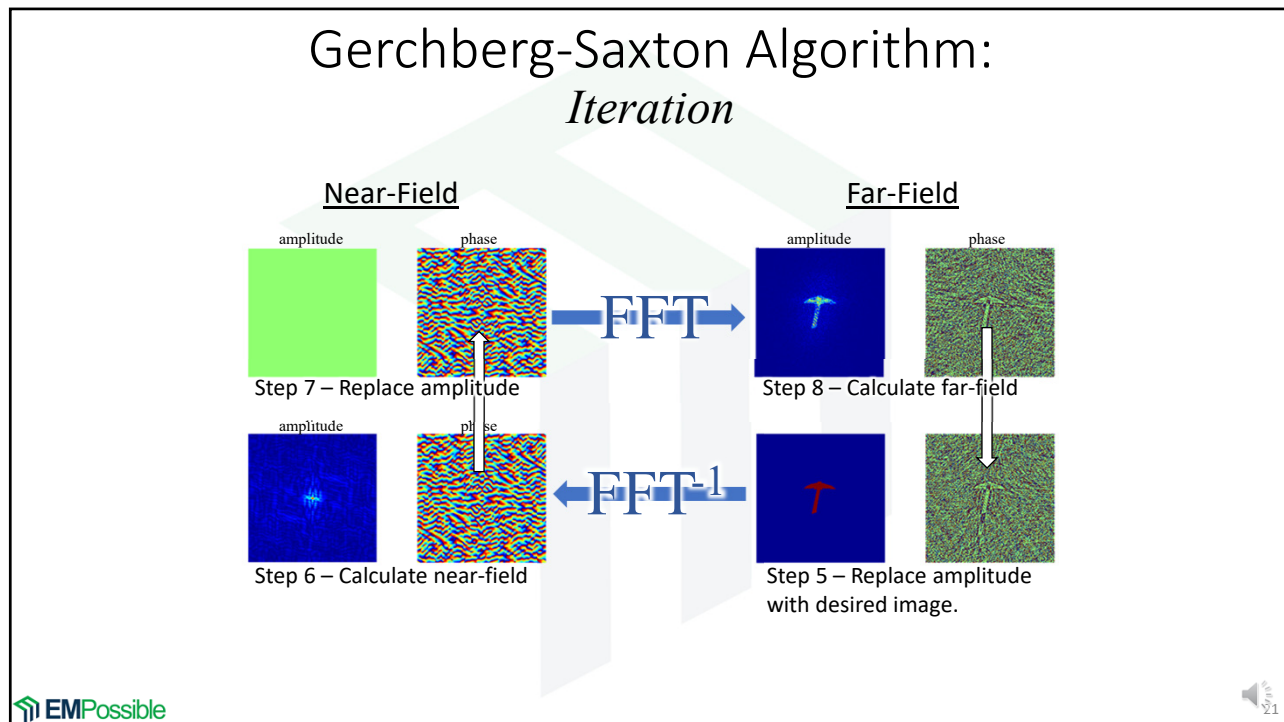
## What is a Patterned Fanout Grating?

A patterned fanout is formed whenever a diffraction grating forces the field to take on the profile of the inverse Fourier transform of an image. After propagating very far, the field takes on the profile of the image.

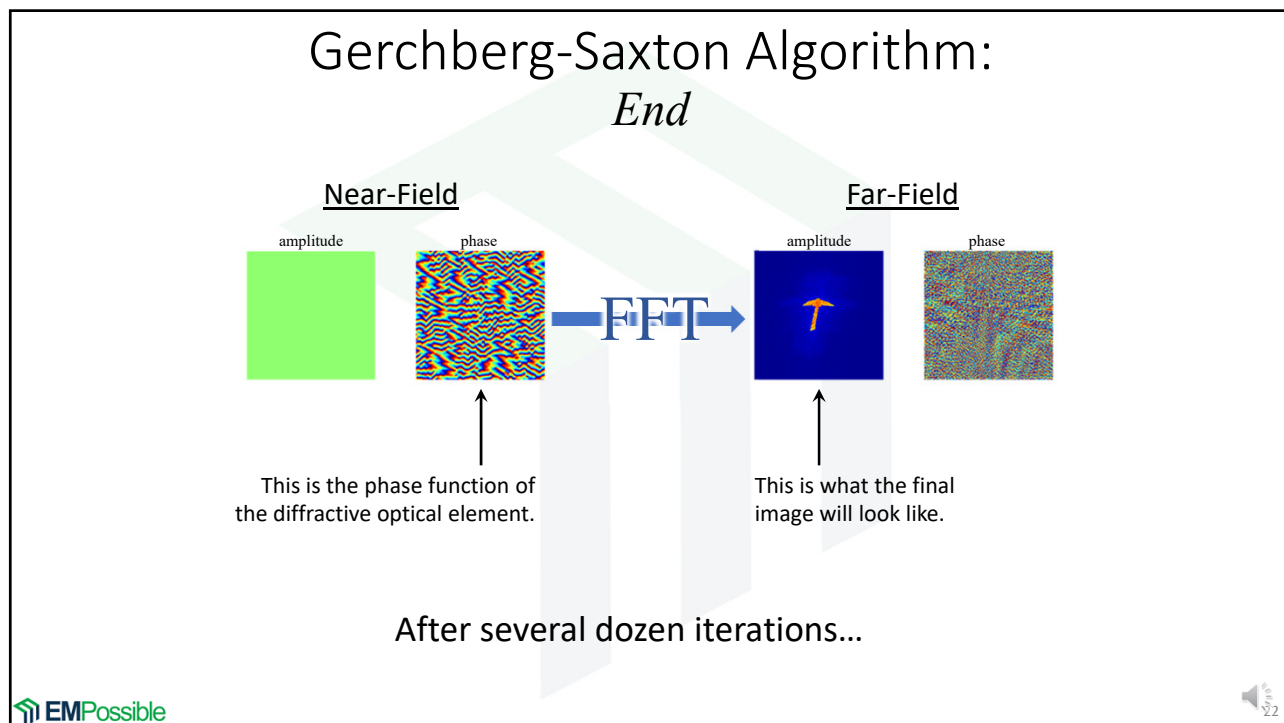


## Gerchberg-Saxton Algorithm: *Initialization*



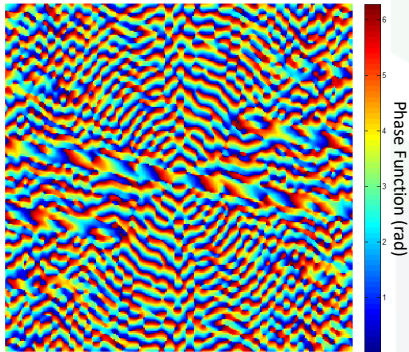


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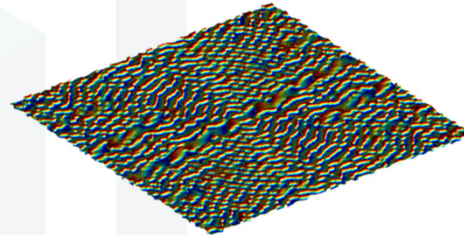


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## The Final Fanout Grating



Phase function as calculated by the Gerchbert-Saxton algorithm.

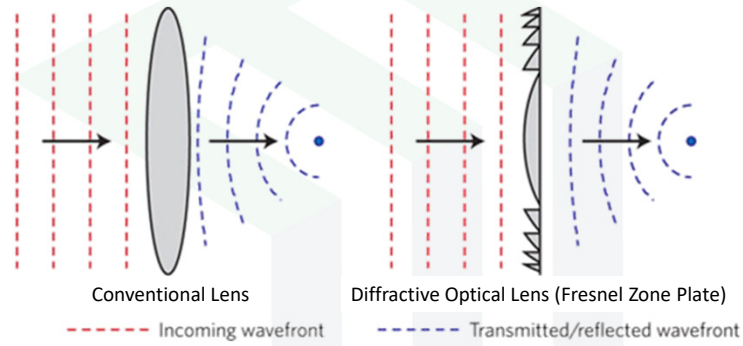


A surface relief pattern is etched into glass to induce the phase function onto the beam of light.

This can also be accomplished with an amplitude mask fabricated in a high-resolution laser printer.

# Diffraction Optical Elements

## What is a Diffractive Optical Element



If the device is only required to operate over a narrow band, devices can be “flattened.”

The flattened device is called a diffractive optical element (DOE).