Advanced Computation:
Computational Electromagnetics

2x2 Matrix Formulation

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Outline

- Derivation of 2×2 matrix equation
- Solution to 2×2 matrix equation
- Summary of formulations

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Derivation of 2×2 Matrix Equation

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Recall Derivation Up to 4×4

Start with Maxwell's equations from Lecture 2.

Assume LHI.

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = k_0 \mu_r \tilde{H}_x$$
$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = k_0 \mu_r \tilde{H}_y$$
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = k_0 \mu_r \tilde{H}_z$$

$$\frac{\partial \tilde{H}_z}{\partial y} - \frac{\partial \tilde{H}_y}{\partial z} = k_0 \varepsilon_r E_x$$
$$\frac{\partial \tilde{H}_x}{\partial z} - \frac{\partial \tilde{H}_z}{\partial x} = k_0 \varepsilon_r E_y$$
$$\frac{\partial \tilde{H}_y}{\partial x} - \frac{\partial \tilde{H}_x}{\partial y} = k_0 \varepsilon_r E_z$$

→

Assume device is infinite and uniform in x and y directions.

$$\frac{\partial}{\partial x} = jk_x \qquad \frac{\partial}{\partial y} = jk_y$$

$$jk_y E_z - \frac{dE_y}{dz} = k_0 \mu_r \tilde{H}_x$$
$$\frac{dE_x}{dz} - jk_x E_z = k_0 \mu_r \tilde{H}_y$$
$$jk_x E_y - jk_y E_x = k_0 \mu_r \tilde{H}_z$$

$$jk_y \tilde{H}_z - \frac{d\tilde{H}_y}{dz} = k_0 \varepsilon_r E_x$$
$$\frac{d\tilde{H}_x}{dz} - jk_x \tilde{H}_z = k_0 \varepsilon_r E_y$$
$$jk_x \tilde{H}_y - jk_y \tilde{H}_x = k_0 \varepsilon_r E_z$$

→

Normalize z and wave vectors k_x, k_y, and k_z.

$$z' = k_0 z$$
$$\tilde{k}_x = \frac{k_x}{k_0} \quad \tilde{k}_y = \frac{k_y}{k_0} \quad \tilde{k}_z = \frac{k_z}{k_0}$$

$$j\tilde{k}_y E_z - \frac{dE_y}{dz'} = \mu_r \tilde{H}_x$$
$$\frac{dE_x}{dz'} - j\tilde{k}_x E_z = \mu_r \tilde{H}_y$$
$$j\tilde{k}_x E_y - j\tilde{k}_y E_x = \mu_r \tilde{H}_z$$

$$j\tilde{k}_y \tilde{H}_z - \frac{d\tilde{H}_y}{dz'} = \varepsilon_r E_x$$
$$\frac{d\tilde{H}_x}{dz'} - j\tilde{k}_x \tilde{H}_z = \varepsilon_r E_y$$
$$j\tilde{k}_x \tilde{H}_y - j\tilde{k}_y \tilde{H}_x = \varepsilon_r E_z$$

→

Eliminate longitudinal components E_z and H_z by substitution.

$$\frac{dE_x}{dz'} = \frac{\tilde{k}_x \tilde{k}_y}{\varepsilon_r} \tilde{H}_x + \left(\mu_r - \frac{\tilde{k}_x^2}{\varepsilon_r} \right) \tilde{H}_y$$
$$\frac{dE_y}{dz'} = \left(\frac{\tilde{k}_y^2}{\varepsilon_r} - \mu_r \right) \tilde{H}_x - \frac{\tilde{k}_x \tilde{k}_y}{\varepsilon_r} \tilde{H}_y$$

$$\frac{d\tilde{H}_x}{dz'} = \frac{\tilde{k}_x \tilde{k}_y}{\mu_r} E_x + \left(\varepsilon_r - \frac{\tilde{k}_x^2}{\mu_r} \right) E_y$$
$$\frac{d\tilde{H}_y}{dz'} = \left(\frac{\tilde{k}_y^2}{\mu_r} - \varepsilon_r \right) E_x - \frac{\tilde{k}_x \tilde{k}_y}{\mu_r} E_y$$

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Derivation of Two 2×2 Matrix Equations

Instead of one matrix equation, the four equations can be written as two separate matrix equations.

$$\frac{dE_x}{dz'} = \frac{\tilde{k}_x \tilde{k}_y}{\varepsilon_r}\tilde{H}_x + \left(\mu_r - \frac{\tilde{k}_x^2}{\varepsilon_r}\right)\tilde{H}_y$$
$$\frac{dE_y}{dz'} = \left(\frac{\tilde{k}_y^2}{\varepsilon_r} - \mu_r\right)\tilde{H}_x - \frac{\tilde{k}_x \tilde{k}_y}{\varepsilon_r}\tilde{H}_y$$

$$\Rightarrow \quad \frac{d}{dz'}\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \frac{1}{\varepsilon_r}\begin{bmatrix} \tilde{k}_x \tilde{k}_y & \mu_r\varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r\varepsilon_r & -\tilde{k}_x \tilde{k}_y \end{bmatrix}\begin{bmatrix} \tilde{H}_x \\ \tilde{H}_y \end{bmatrix}$$

$$\frac{d\tilde{H}_x}{dz'} = \frac{\tilde{k}_x \tilde{k}_y}{\mu_r}E_x + \left(\varepsilon_r - \frac{\tilde{k}_x^2}{\mu_r}\right)E_y$$
$$\frac{d\tilde{H}_y}{dz'} = \left(\frac{\tilde{k}_y^2}{\mu_r} - \varepsilon_r\right)E_x - \frac{\tilde{k}_x \tilde{k}_y}{\mu_r}E_y$$

$$\Rightarrow \quad \frac{d}{dz'}\begin{bmatrix} \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} = \frac{1}{\mu_r}\begin{bmatrix} \tilde{k}_x \tilde{k}_y & \mu_r\varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r\varepsilon_r & -\tilde{k}_x \tilde{k}_y \end{bmatrix}\begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

Note: These equations are valid regardless of the sign convention because there is always a k multiplying another k, erasing the sign.

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Standard "PQ" Form

The two matrix equations can be written more compactly by defining auxiliary **P** and **Q** matrices.

$$\frac{d}{dz'}\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \underbrace{\frac{1}{\varepsilon_r}\begin{bmatrix} \tilde{k}_x \tilde{k}_y & \mu_r\varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r\varepsilon_r & -\tilde{k}_x \tilde{k}_y \end{bmatrix}}\begin{bmatrix} \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} \qquad \frac{d}{dz'}\begin{bmatrix} \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} = \underbrace{\frac{1}{\mu_r}\begin{bmatrix} \tilde{k}_x \tilde{k}_y & \mu_r\varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r\varepsilon_r & -\tilde{k}_x \tilde{k}_y \end{bmatrix}}\begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\mathbf{P} = \frac{1}{\varepsilon_r}\begin{bmatrix} \tilde{k}_x \tilde{k}_y & \mu_r\varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r\varepsilon_r & -\tilde{k}_x \tilde{k}_y \end{bmatrix} \qquad \mathbf{Q} = \frac{1}{\mu_r}\begin{bmatrix} \tilde{k}_x \tilde{k}_y & \mu_r\varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r\varepsilon_r & -\tilde{k}_x \tilde{k}_y \end{bmatrix}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\frac{d}{dz'}\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \mathbf{P}\begin{bmatrix} \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} \qquad \frac{d}{dz'}\begin{bmatrix} \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} = \mathbf{Q}\begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

Note: This same "**PQ**" form will be seen again again for other methods like MoL, RCWA, and waveguide analysis. TMM, MoL, and RCWA are all implemented the same after **P** and **Q** are calculated.

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Matrix Wave Equation

The equations to start with are

$$\frac{d}{dz'}\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \mathbf{P}\begin{bmatrix} \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} \quad \text{Eq. (1)} \qquad \frac{d}{dz'}\begin{bmatrix} \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} = \mathbf{Q}\begin{bmatrix} E_x \\ E_y \end{bmatrix} \quad \text{Eq. (2)}$$

To derive a matrix wave equation, first differentiate Eq. (1) with respect to z'.

$$\frac{d}{dz'} \cdot \frac{d}{dz'}\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \frac{d}{dz'} \cdot \mathbf{P}\begin{bmatrix} \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} \quad \rightarrow \quad \frac{d^2}{dz'^2}\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \mathbf{P}\frac{d}{dz'}\begin{bmatrix} \tilde{H}_x \\ \tilde{H}_y \end{bmatrix}$$

Second, substitute Eq. (2) into this result.

$$\frac{d^2}{dz'^2}\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \mathbf{P} \cdot \mathbf{Q}\begin{bmatrix} E_x \\ E_y \end{bmatrix} \quad \Rightarrow \quad \frac{d^2}{dz'^2}\begin{bmatrix} E_x \\ E_y \end{bmatrix} - \mathbf{\Omega}^2\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbf{\Omega}^2 = \mathbf{PQ}$$

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Solution to 2×2 Matrix Equation

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Numerical Solution (1 of 3)

The system of equations to be solved is

$$\frac{d^2}{dz'^2}\begin{bmatrix} E_x \\ E_y \end{bmatrix} - \mathbf{\Omega}^2 \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \mathbf{\Omega}^2 = \mathbf{PQ}$$

This has the general solution of

$$\begin{bmatrix} E_x(z') \\ E_y(z') \end{bmatrix} = e^{\mathbf{\Omega} z'}\mathbf{a}^+ + e^{-\mathbf{\Omega} z'}\mathbf{a}^-$$

$\mathbf{a}^+ \equiv$ proportionality constant of forward wave

$\mathbf{a}^- \equiv$ proportionality constant of backward wave

No mode sorting! ☺ Here, a second-order differential equation is solved so the modes are all propagating in a single direction. They must be explicitly written twice to account for forward and backward waves and thus they are automatically distinguished. In the 4×4 approach, a first-order differential equation was solved that lumped forward and backward modes together.

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Numerical Solution (2 of 3)

Recall that

$$f(\mathbf{A}) = \mathbf{V} \cdot f(\mathbf{D}) \cdot \mathbf{V}^{-1}$$

This relation is used to calculate the matrix exponentials.

$$e^{\mathbf{\Omega} z'} = \mathbf{W} e^{\mathbf{\lambda} z'} \mathbf{W}^{-1} \qquad e^{-\mathbf{\Omega} z'} = \mathbf{W} e^{-\mathbf{\lambda} z'} \mathbf{W}^{-1}$$

$\mathbf{W} \equiv$ Eigen-vector matrix of $\mathbf{\Omega}^2$

$\mathbf{\lambda}^2 \equiv$ Eigen-value matrix of $\mathbf{\Omega}^2$

$$e^{\mathbf{\lambda} z'} = \begin{bmatrix} e^{\sqrt{\lambda_1^2} z'} & & & \\ & e^{\sqrt{\lambda_2^2} z'} & & \\ & & \ddots & \\ & & & e^{\sqrt{\lambda_N^2} z'} \end{bmatrix} = \begin{bmatrix} e^{\lambda_1 z'} & & & \\ & e^{\lambda_2 z'} & & \\ & & \ddots & \\ & & & e^{\lambda_N z'} \end{bmatrix}$$

So the overall solution is can be written as

$$\begin{bmatrix} E_x(z') \\ E_y(z') \end{bmatrix} = \mathbf{W} e^{\mathbf{\lambda} z'} \mathbf{W}^{-1}\mathbf{a}^+ + \mathbf{W} e^{-\mathbf{\lambda} z'} \mathbf{W}^{-1}\mathbf{a}^-$$

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Numerical Solution (3 of 3)

The overall solution is

$$\begin{bmatrix} E_x(z') \\ E_y(z') \end{bmatrix} = \mathbf{W}e^{\lambda z'}\underbrace{\mathbf{W}^{-1}\mathbf{a}^+}_{\mathbf{c}^+} + \mathbf{W}e^{-\lambda z'}\underbrace{\mathbf{W}^{-1}\mathbf{a}^-}_{\mathbf{c}^-}$$

The column vectors $\mathbf{a}^+$ and $\mathbf{a}^-$ are proportionality constants that have not yet been determined.

The eigen-vector matrix $\mathbf{W}$ multiplies $\mathbf{a}^+$ and $\mathbf{a}^-$ to give another column vector of undetermined constants.

To simplify the math, combine these products into new column vectors labeled $\mathbf{c}^+$ and $\mathbf{c}^-$.

$$\begin{bmatrix} E_x(z') \\ E_y(z') \end{bmatrix} = \mathbf{W}e^{\lambda z'}\mathbf{c}^+ + \mathbf{W}e^{-\lambda z'}\mathbf{c}^-$$

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Solution for the Magnetic Field (1 of 2)

The magnetic field has a solution of the same form, but will have its own eigen-vector matrix $\mathbf{V}$ to describe its modes.

$$\begin{bmatrix} \tilde{H}_x(z') \\ \tilde{H}_y(z') \end{bmatrix} = \mathbf{V}e^{\lambda z'}\mathbf{c}^+ - \mathbf{V}e^{-\lambda z'}\mathbf{c}^-$$

Any sign can be chosen here because it can be accounted for in $\mathbf{c}^-$. A minus sign is chosen here so that both terms in the differentiated equation will be positive. You will see soon why this is desired.

Since the electric and magnetic fields are coupled and not independent, it should be possible to compute $\mathbf{V}$ from $\mathbf{W}$. First, differentiate the above solution with respect to z'.

$$\frac{d}{dz'}\begin{bmatrix} \tilde{H}_x(z') \\ \tilde{H}_y(z') \end{bmatrix} = \mathbf{V}\boldsymbol{\lambda}e^{\lambda z'}\mathbf{c}^+ + \mathbf{V}\boldsymbol{\lambda}e^{-\lambda z'}\mathbf{c}^-$$

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Solution for the Magnetic Field (2 of 2)

From the previous slide,

$$\frac{d}{dz'}\begin{bmatrix}\tilde{H}_x(z')\\ \tilde{H}_y(z')\end{bmatrix} = \mathbf{V}\boldsymbol{\lambda}e^{\lambda z'}\mathbf{c}^+ + \mathbf{V}\boldsymbol{\lambda}e^{-\lambda z'}\mathbf{c}^-$$

Recall

$$\frac{d}{dz'}\begin{bmatrix}\tilde{H}_x(z')\\ \tilde{H}_y(z')\end{bmatrix} = \mathbf{Q}\begin{bmatrix}E_x(z')\\ E_y(z')\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}E_x(z')\\ E_y(z')\end{bmatrix} = \mathbf{W}e^{\lambda z'}\mathbf{c}^+ + \mathbf{W}e^{-\lambda z'}\mathbf{c}^-$$

Combining these results leads to

$$\begin{aligned}\mathbf{V}\boldsymbol{\lambda}e^{\lambda z'}\mathbf{c}^+ + \mathbf{V}\boldsymbol{\lambda}e^{-\lambda z'}\mathbf{c}^- &= \mathbf{Q}\left(\mathbf{W}e^{\lambda z'}\mathbf{c}^+ + \mathbf{W}e^{-\lambda z'}\mathbf{c}^-\right)\\ &= \mathbf{QW}e^{\lambda z'}\mathbf{c}^+ + \mathbf{QW}e^{-\lambda z'}\mathbf{c}^-\end{aligned}$$

Comparing the terms on the left and right sides of this equation shows that

$$\mathbf{V}\boldsymbol{\lambda} = \mathbf{QW} \quad \rightarrow \quad \boxed{\mathbf{V} = \mathbf{QW}\boldsymbol{\lambda}^{-1}}$$

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Combined Solution for $\vec{E}$ and $\vec{\tilde{H}}$

Electric Field Solution

$$\begin{bmatrix}E_x(z')\\ E_y(z')\end{bmatrix} = \mathbf{W}e^{\lambda z'}\mathbf{c}^+ + \mathbf{W}e^{-\lambda z'}\mathbf{c}^-$$

$\mathbf{c}^+ \equiv$ amplitude coefficients of forward wave

$\mathbf{c}^- \equiv$ amplitude coefficients of backward wave

$\mathbf{W} \equiv$ eigen-vector matrix

$\boldsymbol{\lambda} \equiv$ diagonal eigen-value matrix

Magnetic Field Solution

$$\begin{bmatrix}\tilde{H}_x(z')\\ \tilde{H}_y(z')\end{bmatrix} = \mathbf{V}e^{\lambda z'}\mathbf{c}^+ - \mathbf{V}e^{-\lambda z'}\mathbf{c}^- \qquad \mathbf{V} = \mathbf{QW}\boldsymbol{\lambda}^{-1}$$

Combined Solution

$$\boldsymbol{\psi}(z') = \begin{bmatrix}E_x(z')\\ E_y(z')\\ \tilde{H}_x(z')\\ \tilde{H}_y(z')\end{bmatrix} = \begin{bmatrix}\mathbf{W} & \mathbf{W}\\ \mathbf{V} & -\mathbf{V}\end{bmatrix}\begin{bmatrix}e^{\lambda z'} & \mathbf{0}\\ \mathbf{0} & e^{-\lambda z'}\end{bmatrix}\begin{bmatrix}\mathbf{c}^+\\ \mathbf{c}^-\end{bmatrix}$$

Does this equation look familiar?

This is the same equation obtained for the 4×4 approach after the modes were sorted.

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Summary of Formulations

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Two Paths to Combined Solution

4×4 Matrix

$$\frac{\partial}{\partial z'}\begin{bmatrix} E_x \\ E_y \\ \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} = \begin{bmatrix} -j\left(\tilde{k}_y \frac{\mu_{yz}}{\mu_{zz}} + \tilde{k}_x \frac{\varepsilon_{zx}}{\varepsilon_{zz}}\right) & j\tilde{k}_x\left(\frac{\mu_{yz}}{\mu_{zz}} - \frac{\varepsilon_{zy}}{\varepsilon_{zz}}\right) & \left(\frac{\tilde{k}_x\tilde{k}_y}{\varepsilon_{zz}} + \mu_{yx} - \frac{\mu_{yz}\mu_{zx}}{\mu_{zz}}\right) & \left(-\tilde{k}\frac{\hat{k}_x^2}{\varepsilon_{zz}} + \mu_{yy} - \frac{\mu_{yz}\mu_{zy}}{\mu_{zz}}\right) \\ j\tilde{k}_y\left(\frac{\mu_{xz}}{\mu_{zz}} - \frac{\varepsilon_{zx}}{\varepsilon_{zz}}\right) & -j\left(\tilde{k}_x \frac{\mu_{xz}}{\mu_{zz}} + \tilde{k}_y \frac{\varepsilon_{zy}}{\varepsilon_{zz}}\right) & \left(\frac{\tilde{k}_y^2}{\varepsilon_{zz}} - \mu_{xx} + \frac{\mu_{xz}\mu_{zx}}{\mu_{zz}}\right) & \left(-\frac{\tilde{k}_x\tilde{k}_y}{\varepsilon_{zz}} - \mu_{xy} + \frac{\mu_{xz}\mu_{zy}}{\mu_{zz}}\right) \\ \left(\frac{\tilde{k}_x\tilde{k}_y}{\mu_{zz}} + \varepsilon_{yx} - \frac{\varepsilon_{yz}\varepsilon_{zx}}{\varepsilon_{zz}}\right) & \left(-\frac{\tilde{k}_x^2}{\mu_{zz}} + \varepsilon_{yy} - \frac{\varepsilon_{yz}\varepsilon_{zy}}{\varepsilon_{zz}}\right) & -j\left(\tilde{k}_y \frac{\varepsilon_{yz}}{\varepsilon_{zz}} + \tilde{k}_x \frac{\mu_{zx}}{\mu_{zz}}\right) & j\tilde{k}_x\left(\frac{\varepsilon_{yz}}{\varepsilon_{zz}} - \frac{\mu_{zy}}{\mu_{zz}}\right) \\ \left(\frac{\tilde{k}_y^2}{\mu_{zz}} - \varepsilon_{xx} + \frac{\varepsilon_{xz}\varepsilon_{zx}}{\varepsilon_{zz}}\right) & \left(-\frac{\tilde{k}_x\tilde{k}_y}{\mu_{zz}} - \varepsilon_{xy} + \frac{\varepsilon_{xz}\varepsilon_{zy}}{\varepsilon_{zz}}\right) & j\tilde{k}_y\left(\frac{\varepsilon_{xz}}{\varepsilon_{zz}} - \frac{\mu_{zx}}{\mu_{zz}}\right) & -j\left(\tilde{k}_x \frac{\varepsilon_{xz}}{\varepsilon_{zz}} + \tilde{k}_y \frac{\mu_{zy}}{\mu_{zz}}\right) \end{bmatrix}\begin{bmatrix} E_x \\ E_y \\ \tilde{H}_x \\ \tilde{H}_y \end{bmatrix}$$

Sort Eigen-Modes

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_E^+ & \mathbf{W}_E^- \\ \mathbf{W}_H^+ & \mathbf{W}_H^- \end{bmatrix} \begin{matrix} E_x \\ E_y \\ \tilde{H}_x \\ \tilde{H}_y \end{matrix}$$

$$e^{\lambda z'} = \begin{bmatrix} e^{\lambda^+ z'} & \mathbf{0} \\ \mathbf{0} & e^{-\lambda^- z'} \end{bmatrix}$$

Anisotropic

Maxwell's Equations

$$\nabla \times \vec{E} = k_0 [\mu_r] \vec{\tilde{H}}$$

$$\nabla \times \vec{\tilde{H}} = k_0 [\varepsilon_r] \vec{E}$$

Field Solution

$$\boldsymbol{\psi}(z') = \begin{bmatrix} \mathbf{W}_E^+ & \mathbf{W}_E^- \\ \mathbf{V}_H^+ & \mathbf{V}_H^- \end{bmatrix}\begin{bmatrix} e^{\lambda^+ z'} & \mathbf{0} \\ \mathbf{0} & e^{-\lambda^- z'} \end{bmatrix}\begin{bmatrix} \mathbf{c}^+ \\ \mathbf{c}^- \end{bmatrix}$$

$$\boldsymbol{\psi}(z') = \begin{bmatrix} \mathbf{W} & \mathbf{W} \\ \mathbf{V} & -\mathbf{V} \end{bmatrix}\begin{bmatrix} e^{\lambda z'} & \mathbf{0} \\ \mathbf{0} & e^{-\lambda z'} \end{bmatrix}\begin{bmatrix} \mathbf{c}^+ \\ \mathbf{c}^- \end{bmatrix}$$

Isotropic or diagonally anisotropic

$$\mathbf{P} = \frac{1}{\varepsilon_r}\begin{bmatrix} \tilde{k}_x\tilde{k}_y & \mu_r\varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r\varepsilon_r & -\tilde{k}_x\tilde{k}_y \end{bmatrix}$$

$$\mathbf{Q} = \frac{1}{\mu_r}\begin{bmatrix} \tilde{k}_x\tilde{k}_y & \mu_r\varepsilon_r - \tilde{k}_x^2 \\ \tilde{k}_y^2 - \mu_r\varepsilon_r & -\tilde{k}_x\tilde{k}_y \end{bmatrix}$$

PQ Method

No sorting! ☺

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