



Advanced Electromagnetics:
21st Century Electromagnetics

Electromagnetic Waves in Periodic Structures



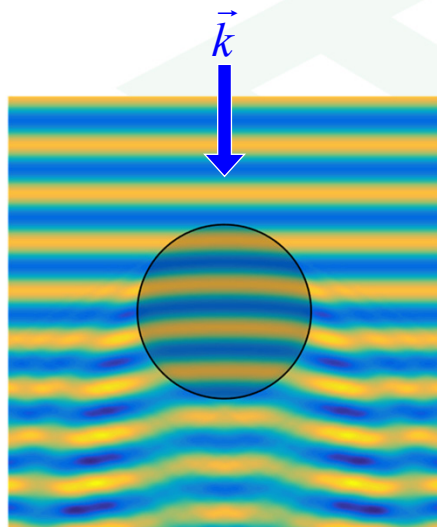
Lecture Outline

- Bloch waves
- Electromagnetic bands
- Isofrequency contours

Bloch Waves

Slide 3

Waves are Perturbed by Objects



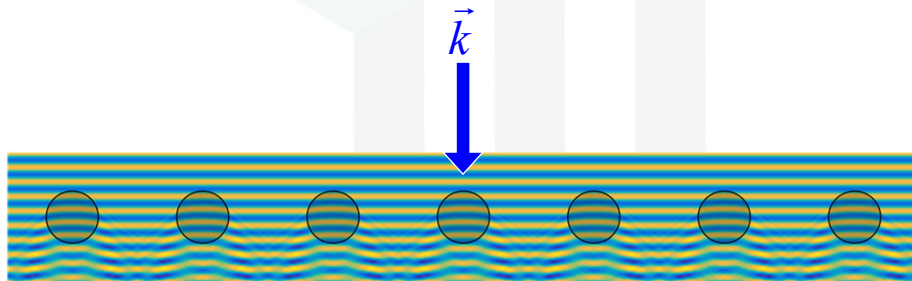
When a portion of a wave propagates through an object, that portion is delayed.

Thus, a wave gets perturbed by the object.

Slide 4

Fields in Periodic Structures

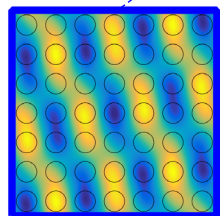
Waves in periodic structures take on the same symmetry and periodicity as their host.



The Bloch Theorem

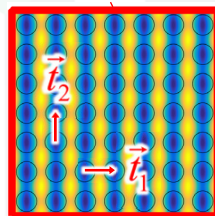
Waves inside of a periodic structure are analogous to plane waves, but they are modulated by an envelope function. It is the envelope function that takes on the same symmetry and periodicity as the structure.

$$\vec{E}(\vec{r}) = \vec{A}(\vec{r}) e^{j\vec{\beta} \cdot \vec{r}}$$

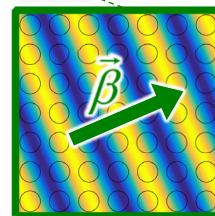


Overall field is the combination of the envelope and plane wave term.

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Envelope function has the same symmetry and periodicity as the periodic structure.

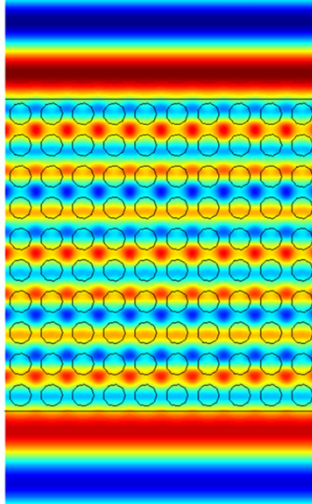


Plane-wave like phase "tilt" term.

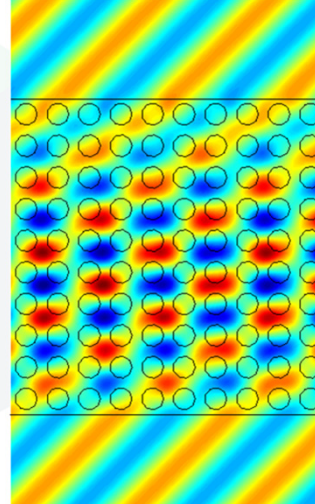
$\vec{\beta} \equiv$ Bloch wave vector

Example Waves in a Periodic Lattice

Wave normally incident onto a periodic structure.



Wave incident at 45° onto the same periodic structure.



Mathematical Description of Periodicity

A structure is periodic if its material properties repeat. Given the lattice vectors, the periodicity is expressed as

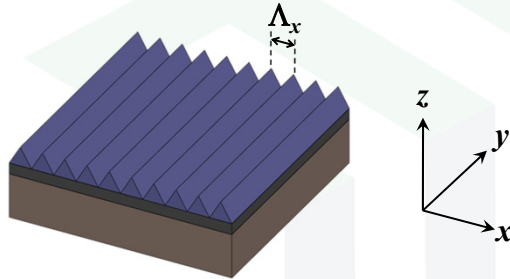
$$\varepsilon(\vec{r} + \vec{t}_{pqr}) = \varepsilon(\vec{r}) \quad \vec{t}_{pqr} = p\vec{t}_1 + q\vec{t}_2 + r\vec{t}_3$$

Recall that it is the amplitude of the Bloch wave that has the same periodicity as the structure the wave is in. Therefore,

$$A(\vec{r} + \vec{t}_{pqr}) = A(\vec{r}) \quad \vec{t}_{pqr} = p\vec{t}_1 + q\vec{t}_2 + r\vec{t}_3$$

Example – 1D Periodicity

Many devices are periodic along just one dimension.



For a device that is periodic only along one direction, these relations reduce to

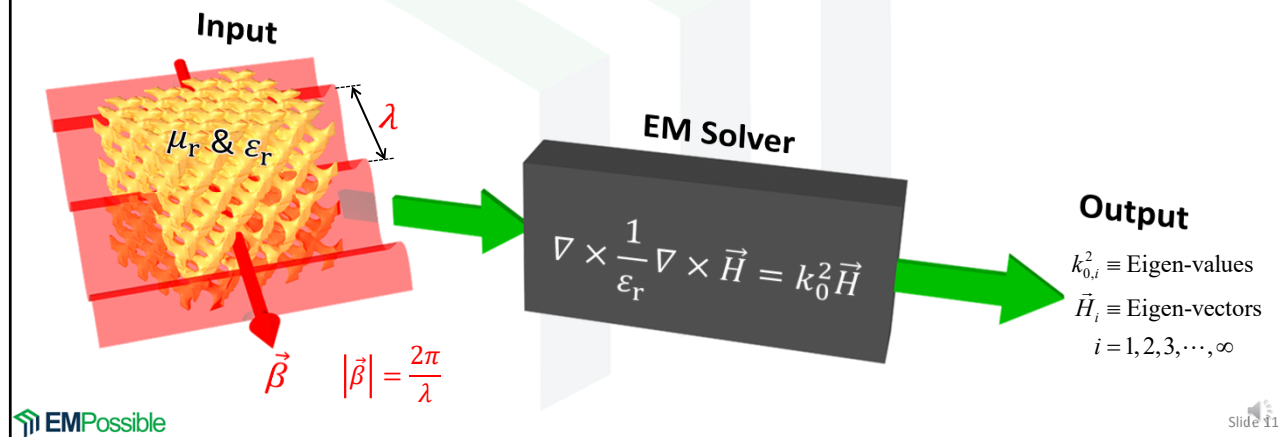
$$\begin{array}{ccc} \varepsilon(x + p\Lambda_x, y, z) = \varepsilon(x, y, z) & \xrightarrow{\text{more compact notation}} & \varepsilon(x + p\Lambda_x) = \varepsilon(x) \\ A(x + p\Lambda_x, y, z) = A(x, y, z) & & A(x + p\Lambda_x) = A(x) \end{array}$$

$$p = -\infty, \dots, -2, -1, 0, 1, 2, \dots, \infty$$

Electromagnetic Bands

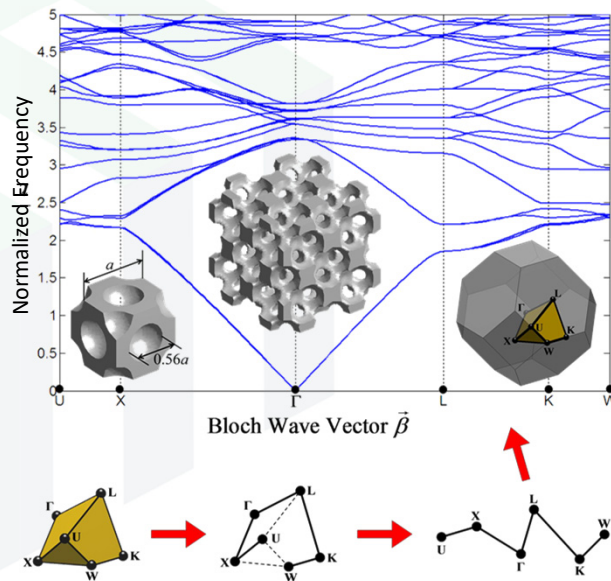
Band Diagrams (1 of 2)

Band diagrams are a compact, but incomplete, means of characterizing the electromagnetic properties of a periodic structure. It is essentially a map of the frequencies of the eigenmodes as a function of the Bloch wave vector $\vec{\beta}$.



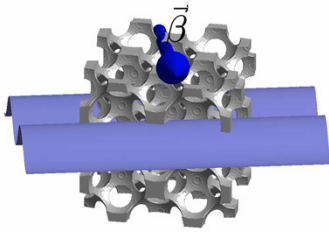
Band Diagrams (2 of 2)

To construct a band diagram, we make small steps around the perimeter of the irreducible Brillouin zone (IBZ) and compute the eigen-values at each step. When we plot all these eigen-values as a function of β , the points line up to form continuous “bands.”

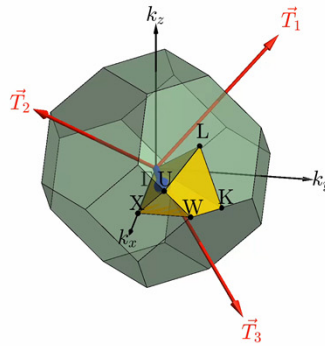


Animation of the Construction of a Band Diagram

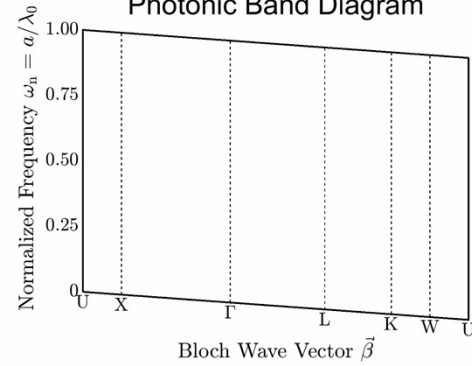
Lattice + Bloch Wave



Brillouin Zone

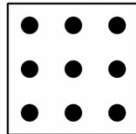


Photonic Band Diagram

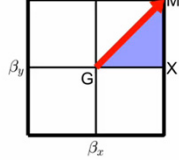


2D Animation of Bands and Bloch Waves

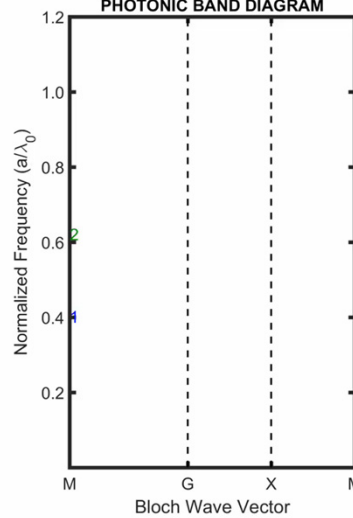
3x3 Lattice



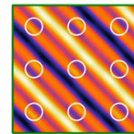
Brillouin Zone



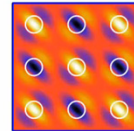
PHOTONIC BAND DIAGRAM



Bloch Mode 2



Bloch Mode 1

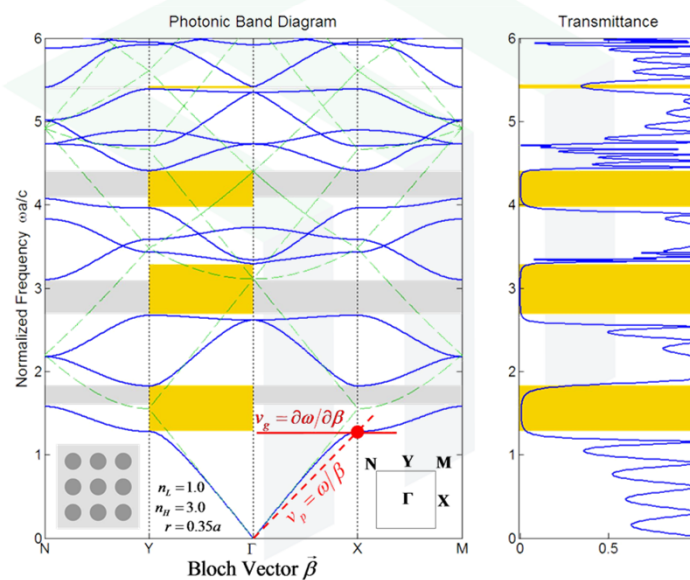


Reading Band Diagrams

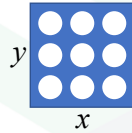
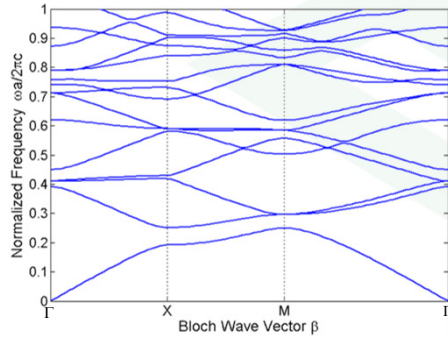
At least five electromagnetic properties can be estimated from a band diagram.

- **Band gaps**
 - Absence of any bands within a range of frequencies indicates a band gap.
 - A COMPLETE BAND GAP is one that exists over all possible Bloch wave vectors.
- **Transmission/reflection spectra**
 - Band gaps lead to suppressed transmission and enhanced reflection
- **Phase velocity**
 - The slope of the line connecting Γ to the point on the band corresponds to phase velocity.
 - From this, we get the effective phase refractive index.
- **Group velocity**
 - The slope of the band at the point of interest corresponds to the group velocity.
 - From this, we get the effective group refractive index.
- **Dispersion**
 - Any time the band deviates from the "light line" there is dispersion.
 - The phase and group velocity are the same except when there is dispersion.

Reading Band Diagrams

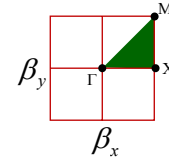


The Band Diagram is Missing Information



Direct lattice: This example is an array of air holes in a dielectric medium with $n = 3.0$.

Reciprocal lattice: Construct the band diagram by marching around the perimeter of the irreducible Brillouin zone.



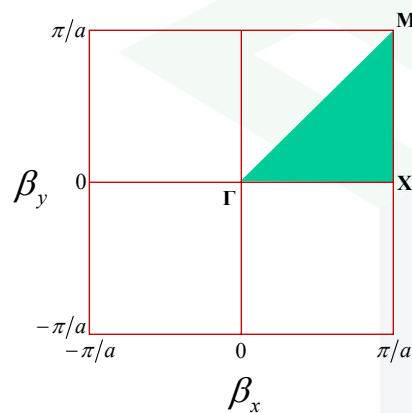
The band extremes “almost” always occur at the key points of symmetry.

But information is missing from inside the Brillouin zone.

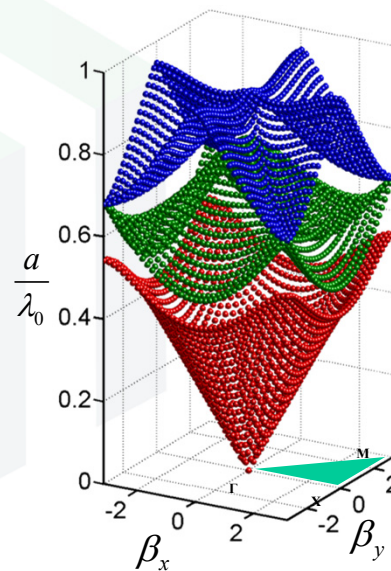
This information is often very important!

The Complete Band Diagram

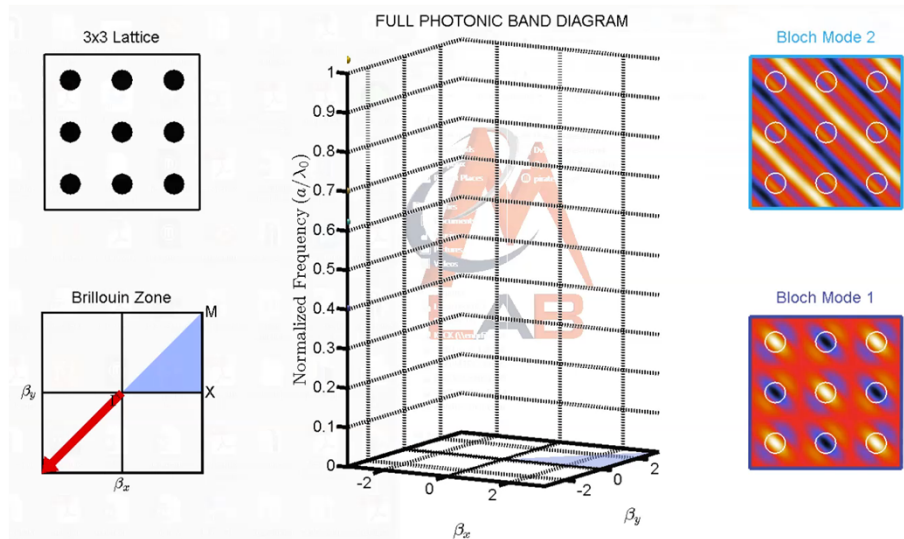
The Full Brillouin Zone...



There is an infinite set of eigen-frequencies associated with each point in the Brillouin zone. These form “sheets” as shown at right.

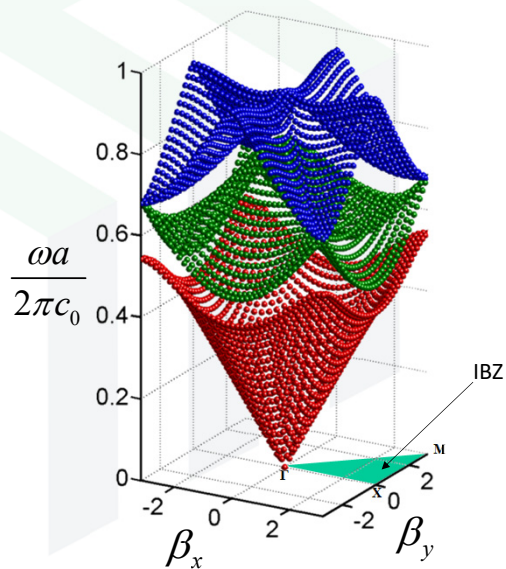


Animation of Complete Photonic Band Diagram



Relation Between the Full Band Diagram and the Band Diagram (1 of 4)

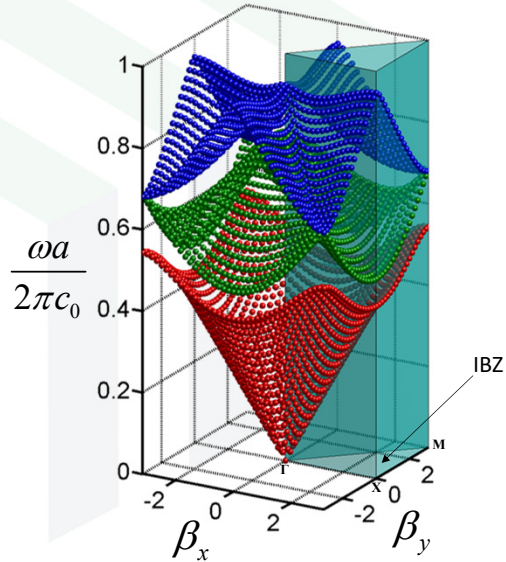
Start with the full band diagram.



Relation Between the Full Band Diagram and the Band Diagram (2 of 4)

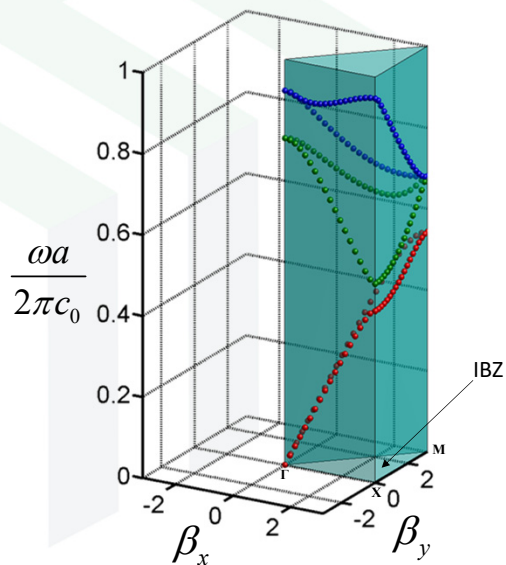
Raise walls around the perimeter of the irreducible Brillouin zone.

These walls slice through the bands.



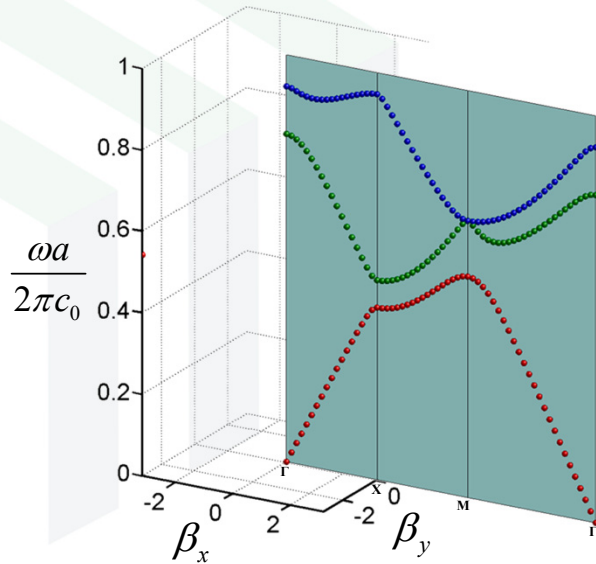
Relation Between the Full Band Diagram and the Band Diagram (3 of 4)

Focus only on the intersections of the walls and the bands.



Relation Between the Full Band Diagram and the Band Diagram (4 of 4)

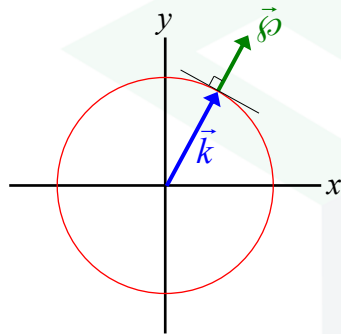
Unfold the walls to reveal the ordinary band diagram.



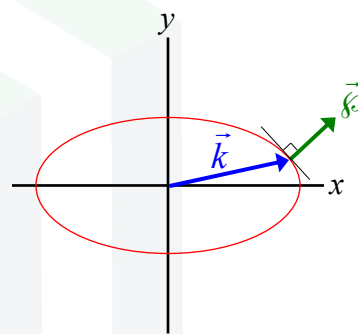
Isofrequency Contours (IFCs)

Recall Phase Vs. Power Flow

Isotropic Materials



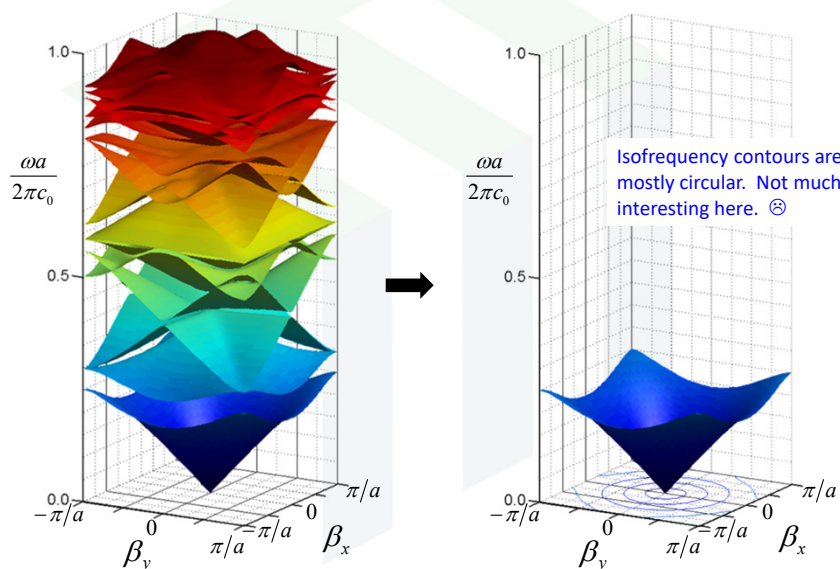
Anisotropic Materials



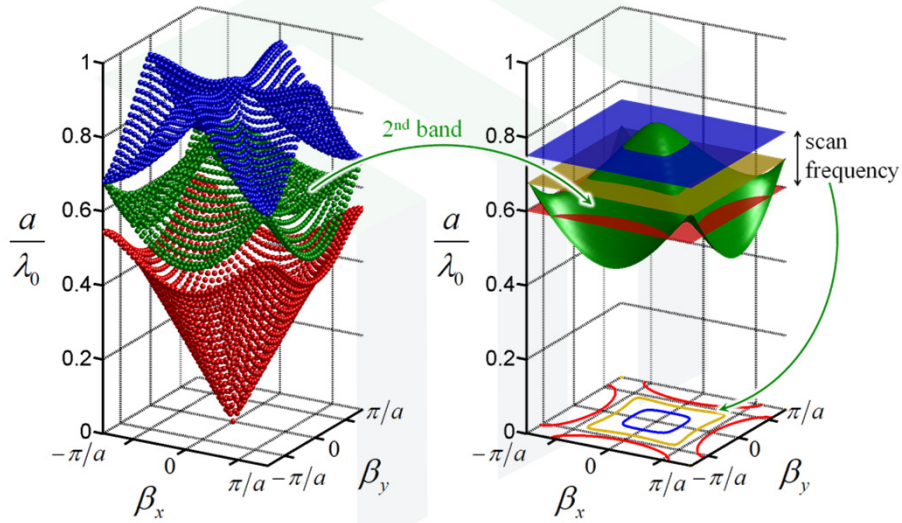
Phase propagates in the direction of \vec{k} . Therefore, the refractive index derived from $|\vec{k}|$ is best described as the *phase refractive index*. Velocity here is the phase velocity.

Power propagates in the direction of the Poynting vector \vec{S} , which is always normal to the surface of the index ellipsoid. From this, we can define a *group velocity* and a *group refractive index*.

Isofrequency Contours From First-Order Band

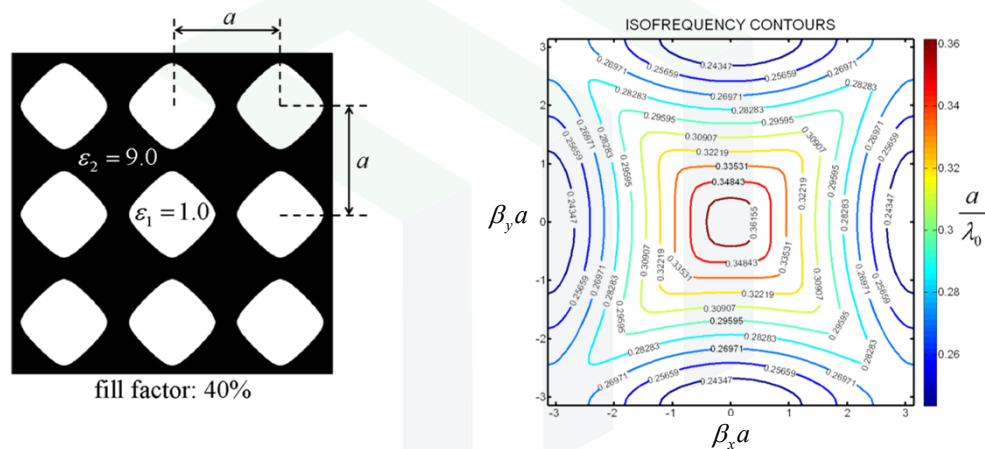


Isofrequency From Second-Order Band



Index ellipsoids are "isofrequency contours" in k -space.

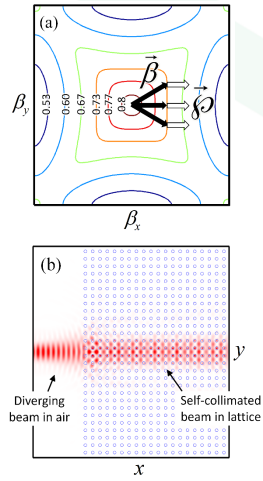
Standard View of Isofrequency Contours



Index ellipsoids in periodic structures are very interesting and useful because they can be things other than ellipsoids. They tend to resemble the shape of the Brillouin zone.

Example Applications

Self-Collimation



Negative Refraction

