Advanced Electromagnetics:
21st Century Electromagnetics

Anisotropic Materials

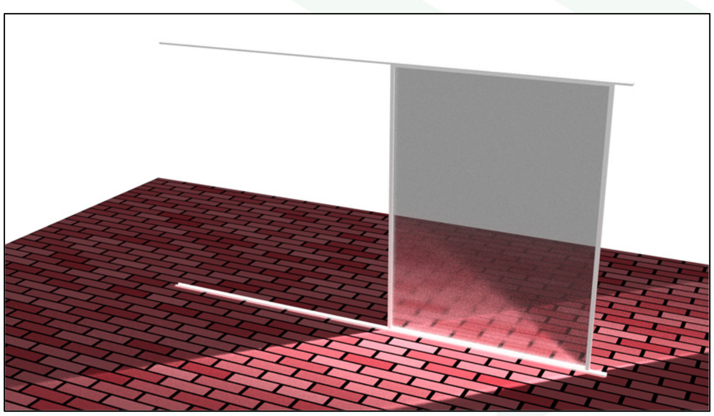
Lecture Outline

- Qualitative description of anisotropy
- Quantitative description of anisotropy

Analogy of Anisotropic Displacement

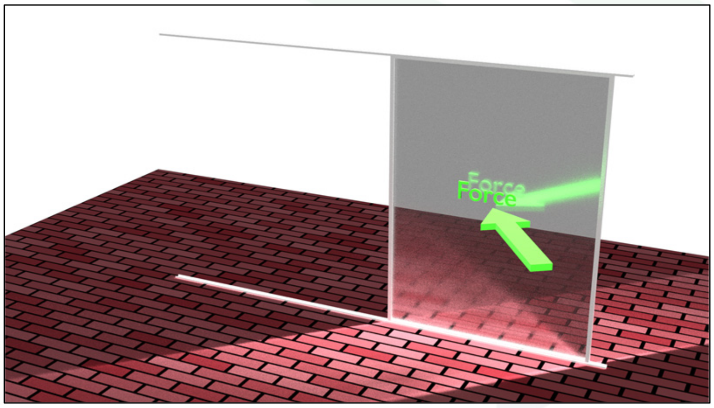
Sliding Glass Door Analogy for Anisotropy (1 of 4)

In some materials, charges are more easily displaced along certain directions than others. A good analogy is that of a sliding glass door.



Sliding Glass Door Analogy for Anisotropy (2 of 4)

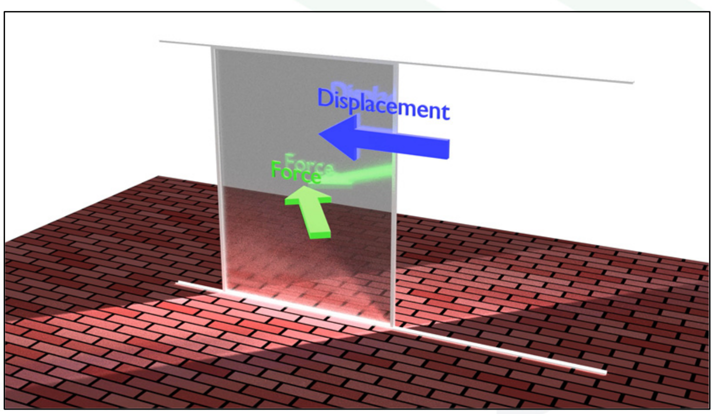
In some materials, charges are more easily displaced along certain directions than others. A good analogy is that of a sliding glass door.



A force is applied at some angle relative to the sliding door.

Sliding Glass Door Analogy for Anisotropy (3 of 4)

In some materials, charges are more easily displaced along certain directions than others. A good analogy is that of a sliding glass door.

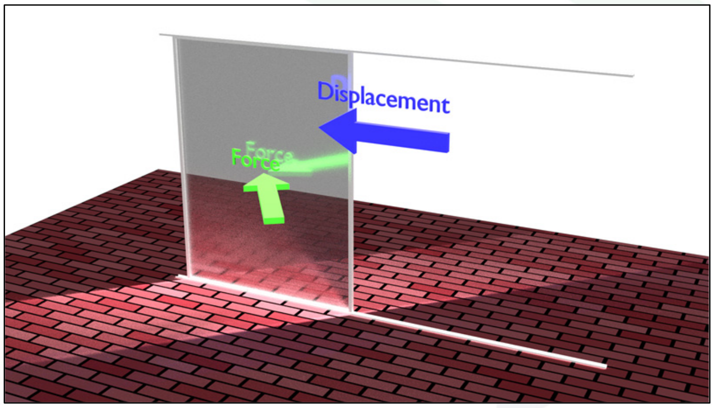


A force is applied at some angle relative to the sliding door.

The sliding door displaces in a direction different than the applied force.

Sliding Glass Door Analogy for Anisotropy (4 of 4)

In some materials, charges are more easily displaced along certain directions than others. A good analogy is that of a sliding glass door.



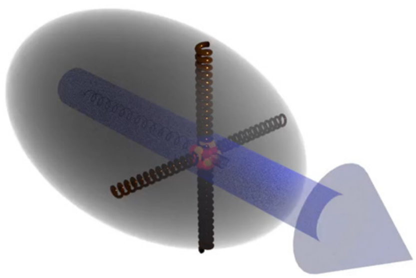
A force is applied at some angle relative to the sliding door.

The sliding door displaces in a direction different than the applied force.

This happens because the sliding door is more easily displaced along the direction of the rails.

Anisotropic Lorentz Model

Lorentz Model of Anisotropic Materials



Charges are more easily displaced in some directions than others. This leads to greater polarizability in some directions than others. This makes electric susceptibility a tensor quantity.

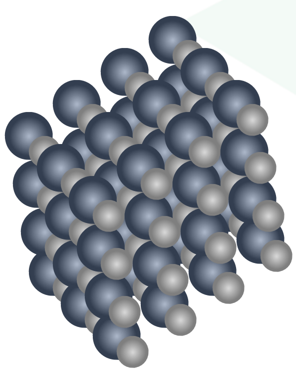
$$\vec{P}(\omega) = \varepsilon_0 \left[\chi_{\mathrm{e}}(\omega)\right] \vec{E}(\omega)$$

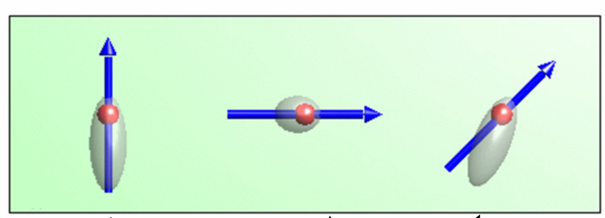
$$= \varepsilon_0 \begin{bmatrix} \chi_{\mathrm{e},xx} & \chi_{\mathrm{e},xy} & \chi_{\mathrm{e},xz} \\ \chi_{\mathrm{e},yx} & \chi_{\mathrm{e},yy} & \chi_{\mathrm{e},yz} \\ \chi_{\mathrm{e},zx} & \chi_{\mathrm{e},zy} & \chi_{\mathrm{e},zz} \end{bmatrix} \begin{bmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{bmatrix}$$

Isotropic materials are good for simple transmission of electromagnetic waves.

Anisotropic materials are good for controlling and manipulating the waves.

Atomic Scale Picture





High χ_e
Charges are easily displaced in the vertical direction.

Low χ_e
Charges are more difficult to displace in the horizontal direction.

In other directions, displacement is not aligned with the applied force.

Quantitative Description of Anisotropy

Tensor Relations for Anisotropic Materials

The material polarization is now expressed as:

$$\begin{bmatrix} P_x(\omega) \\ P_y(\omega) \\ P_z(\omega) \end{bmatrix} = \varepsilon_0 \begin{bmatrix} \chi_{xx}(\omega) & \chi_{xy}(\omega) & \chi_{xz}(\omega) \\ \chi_{yx}(\omega) & \chi_{yy}(\omega) & \chi_{yz}(\omega) \\ \chi_{zx}(\omega) & \chi_{zy}(\omega) & \chi_{zz}(\omega) \end{bmatrix} \begin{bmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{bmatrix}$$

Materials can become polarized in directions that are slightly different than the electric field!

This is most often treated through a dielectric tensor.

$$\left[\tilde{\varepsilon}_{\mathrm{r}}(\omega)\right] = \mathbf{I} + \left[\chi_e(\omega)\right]$$

The constitutive relation between $\vec{E}$ and $\vec{D}$ is then

$$\begin{bmatrix} D_x(\omega) \\ D_y(\omega) \\ D_z(\omega) \end{bmatrix} = \varepsilon_0 \begin{bmatrix} \tilde{\varepsilon}_{xx}(\omega) & \tilde{\varepsilon}_{xy}(\omega) & \tilde{\varepsilon}_{xz}(\omega) \\ \tilde{\varepsilon}_{yx}(\omega) & \tilde{\varepsilon}_{yy}(\omega) & \tilde{\varepsilon}_{yz}(\omega) \\ \tilde{\varepsilon}_{zx}(\omega) & \tilde{\varepsilon}_{zy}(\omega) & \tilde{\varepsilon}_{zz}(\omega) \end{bmatrix} \begin{bmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{bmatrix}$$

The dielectric tensor has Hermitian symmetry for the general lossy case.

$$\varepsilon_{ij} = \varepsilon_{ji}^*$$

This means the field components are no longer independent. ☹

But…it presents many new possibilities!!

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Principle Axes $\hat{a}$, $\hat{b}$ and $\hat{c}$

It is always possible to choose a coordinate system such that the dielectric tensor becomes diagonal.

$$\begin{matrix} \hat{x} & & \hat{a} \\ \hat{y} & \Rightarrow & \hat{b} \\ \hat{z} & & \hat{c} \end{matrix}$$

$\vec{a}$, $\vec{b}$, and $\vec{c}$ are called the *Principal Axes* of the crystal.
$\vec{a}$, $\vec{b}$, and $\vec{c}$ are not necessarily perpendicular to each other.

$$\begin{bmatrix} D_a(\omega) \\ D_b(\omega) \\ D_c(\omega) \end{bmatrix} = \varepsilon_0 \begin{bmatrix} \tilde{\varepsilon}_a(\omega) & 0 & 0 \\ 0 & \tilde{\varepsilon}_b(\omega) & 0 \\ 0 & 0 & \tilde{\varepsilon}_c(\omega) \end{bmatrix} \begin{bmatrix} E_a(\omega) \\ E_b(\omega) \\ E_c(\omega) \end{bmatrix}$$

Alternative Description: There are only three degrees of freedom for 3D tensors. Numbers can only occur in the off-diagonal elements when the tensor is rotated.

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Maxwell's Equations with Anisotropic Materials

Maxwell's equations remain unchanged.

$$\nabla \times \vec{E} = -j\omega\vec{B} \qquad \nabla \bullet \vec{D} = 0$$

$$\nabla \times \vec{H} = j\omega\vec{D} \qquad \nabla \bullet \vec{B} = 0$$

The constitutive relations now include tensors

$$\vec{D} = [\varepsilon]\vec{E}$$

$$\vec{B} = [\mu]\vec{H}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

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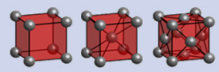
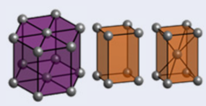
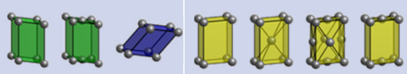
Symmetry and Anisotropy

Anisotropy	Crystal Symmetry	Dielectric Properties
Isotropic	Cubic 	$\begin{bmatrix} \tilde{\varepsilon} & 0 & 0 \\ 0 & \tilde{\varepsilon} & 0 \\ 0 & 0 & \tilde{\varepsilon} \end{bmatrix}$
Uniaxial	Hexagonal Tetragonal 	$\begin{bmatrix} \tilde{\varepsilon}_o & 0 & 0 \\ 0 & \tilde{\varepsilon}_o & 0 \\ 0 & 0 & \tilde{\varepsilon}_e \end{bmatrix}$ $\tilde{\varepsilon}_e > \tilde{\varepsilon}_o$ positive birefringence $\tilde{\varepsilon}_e < \tilde{\varepsilon}_o$ negative birefringence
Biaxial	Monoclinic Triclinic Orthorhombic 	$\begin{bmatrix} \tilde{\varepsilon}_a & 0 & 0 \\ 0 & \tilde{\varepsilon}_b & 0 \\ 0 & 0 & \tilde{\varepsilon}_c \end{bmatrix}$ $\tilde{\varepsilon}_a < \tilde{\varepsilon}_b < \tilde{\varepsilon}_c$

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Examples of Anisotropic Materials

Refractive indices of some common uniaxial crystals at $\lambda_0 = 593$ nm.

Crystal	Chemical structure	Symmetry class	type	n_o	n_e
Ice	H_2O	trigonal	positive	1.309	1.313
Quartz	SiO_2	trigonal	positive	1.544	1.553
Beryl	$Be_3Al_2(SiO_3)_6$	hexagonal	negative	1.581	1.575
Sodium nitrate	$NaNO_3$	trigonal	negative	1.584	1.336
Calcite	$CaCO_3$	trigonal	negative	1.658	1.486
Tourmaline	complex silicate	trigonal	negative	1.669	1.638
Sapphire	Al_2O_3	trigonal	negative	1.768	1.760
Zircon	$ZrSiO_4$	tetragonal	positive	1.923	1.968
Rutile	TiO_2	tetragonal	positive	2.616	2.903

Fox, Mark. "Optical properties of solids." Oxford (2002): 1269-1270.

Table 4.2. Refractive Indices of Some Typical Crystals

Isotropic	CdTe		2.69	
	NaCl		1.544	
	Diamond		2.417	
	Fluorite		1.392	
	GaAs		3.40	
Uniaxial:		n_o		n_e
positive	Ice	1.309		1.310
	Quartz	1.544		1.553
	BeO	1.717		1.732
	Zircon	1.923		1.968
	Rutile	2.616		2.903
	ZnS	2.354		2.358
negative	$(NH_4)H_2PO_4$(ADP)	1.522		1.478
	Beryl	1.598		1.590
	KH_2PO_4(KDP)	1.507		1.467
	$NaNO_3$	1.587		1.336
	Calcite	1.658		1.486
	Tourmaline	1.638		1.618
	$LiNbO_3$	2.300		2.208
	$BaTiO_3$	2.416		2.364
	Proustite	3.019		2.739
Biaxial		n_x	n_y	n_z
	Gypsum	1.520	1.523	1.530
	Feldspar	1.522	1.526	1.530
	Mica	1.552	1.582	1.588
	Topaz	1.619	1.620	1.627
	$NaNO_2$	1.344	1.411	1.651
	SbSI	2.7	3.2	3.8
	$YAlO_3$	1.923	1.938	1.947

Yariv, Amnon, and Pochi Yeh. Optical waves in crystals. Vol. 5. New York: Wiley, 1984.

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