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Advanced Electromagnetics:
21st Century Electromagnetics

Introduction to Electromagnetic Properties of Materials

Lecture Outline

- The fundamental parameters
- Classification of materials
- Materials Models

Fundamental Parameters

Equations Describing Classical Electromagnetics

Maxwell's Equations

Divergence Equations

$$\nabla \bullet \vec{D} = \rho_{\mathrm{v}}$$ Gauss' Law

$$\nabla \bullet \vec{B} = 0$$ Gauss' Law for Magnetic Fields

Curl Equations

$$\nabla \times \vec{E} = -j\omega\vec{B}$$ Faraday's Law

$$\nabla \times \vec{H} = \vec{J} + j\omega\vec{D}$$ Ampere's Circuit Law

Constitutive Relations

$$\vec{D} = \varepsilon\vec{E}$$ Electric Response

$$\vec{B} = \mu\vec{H}$$ Magnetic Response

$$\vec{J} = \sigma\vec{E}$$ Ohm's Law

Maxwell's equations describe what induces electromagnetic fields. They do not directly describe how fields interact with materials.

The constitutive relations describe how fields interact with materials. These equations contain the fundamental parameters μ, ε and σ.

Fundamental Vs. Intuitive Parameters

Fundamental Parameters

These parameters are fundamental to Maxwell’s equations, but it is not obvious how they will effect fields and waves.

Magnetic Permeability, μ
Electric Permittivity, ε
Electrical Conductivity, σ

Intuitive Parameters

These parameters isolate specific information about waves from the fundamental parameters.

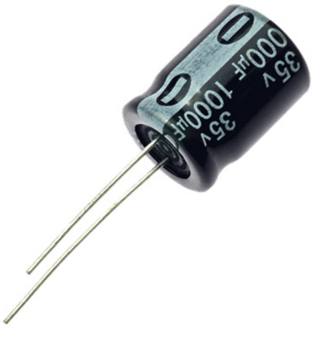
Refractive index, n
Impedance, η
Wavelength, λ
Velocity, v
Wave Number, k
Propagation Constant, γ
Attenuation Coefficient, α
Phase Constant, β

Permittivity, ε (F/m)

The permittivity ε is a measure of how well a medium stores electric energy.

It can be thought of as a measure of how much interaction an electric field has with the medium it resides in.

Permittivity is most closely related to capacitance.



$$\varepsilon = \varepsilon_0 \varepsilon_{\mathrm{r}}$$

$$\varepsilon_0 = 8.8541878176 \times 10^{-12} \ \ \mathrm{F/m}$$

$$\varepsilon_{\mathrm{r}} \geq 1 \ \ (\text{no units})$$

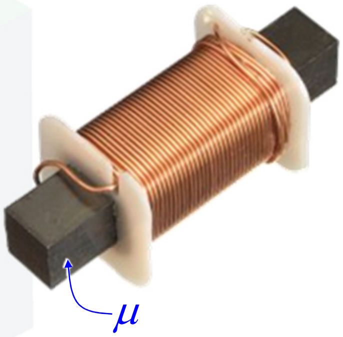
$\varepsilon \equiv$ permittivity
$\varepsilon_0 \equiv$ vacuum permittivity
$\varepsilon_{\mathrm{r}} \equiv$ relative permittivity (dielectric constant)

Permeability, μ (H/m)

The permeability μ is a measure of how well a medium stores magnetic energy.

It can be thought of as a measure of how much interaction a magnetic field has with the medium it resides in.

Permeability is most closely related to inductance.



$$\mu = \mu_0 \mu_r$$

$$\mu_0 = 1.2566370614 \times 10^{-6} \ \ \mathrm{H/m}$$

$$\mu_r \geq 1 \ (\text{no units})$$

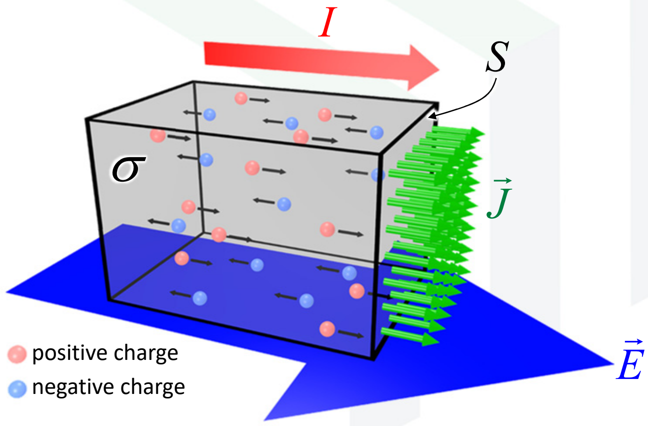
$\mu \equiv$ permeability

$\mu_0 \equiv$ vacuum permeability

$\mu_r \equiv$ relative permeability

Conductivity, σ (1/ Ω·m)

The conductivity describes the degree to which a material conducts electricity.



Resistivity

$$\rho \ (\Omega \cdot \mathrm{m}) = 1/\sigma$$

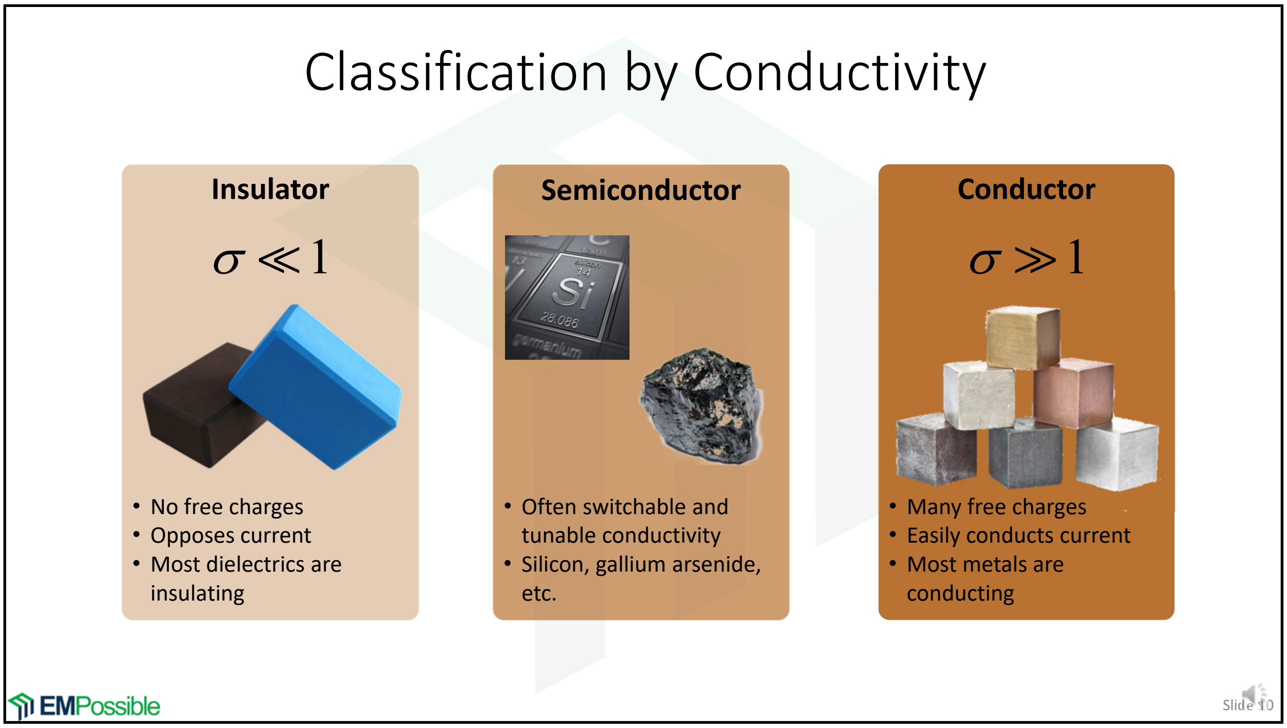
Ohm's Law for EM

$$\vec{J} = \sigma \vec{E}$$

Total Current

$$I = JS$$

Classification of Materials



Classification by Conductivity

Insulator

$\sigma \ll 1$

- No free charges
- Opposes current
- Most dielectrics are insulating

Semiconductor

- Often switchable and tunable conductivity
- Silicon, gallium arsenide, etc.

Conductor

$\sigma \gg 1$

- Many free charges
- Easily conducts current
- Most metals are conducting

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Classification by Linearity

Linear

$$D = \varepsilon_0 \varepsilon_r E$$

- Properties of the material do not depend on the strength of the field.
- Most of EM is taught assuming linear materials.

Nonlinear

$$D = \varepsilon_0 \varepsilon_r E + \varepsilon_0 \chi_e^{(2)} E^2 + \varepsilon_0 \chi_e^{(3)} E^3 + \cdots$$

- Properties depend on the intensity of the field.
- Lots of interesting properties arise.
- Rectification, frequency doubling, mixing, etc.

Classification by Anisotropy

Isotropic

$$\vec{D} = \varepsilon \vec{E}$$
$$\vec{B} = \mu \vec{H}$$
$$\vec{J} = \sigma \vec{E}$$

- Properties are independent of the direction of the fields
- Most of EM is taught assuming isotropic materials.

Anisotropic

$$\vec{D} = [\varepsilon] \vec{E}$$
$$\vec{B} = [\mu] \vec{H}$$
$$\vec{J} = [\sigma] \vec{E}$$

- Properties depend on the direction of the fields.
- Lots of interesting properties arise.
- Provide many ways to control and manipulate electromagnetic waves.

Classes of Anisotropic Materials

Ordinary Anisotropic Materials

$$\varepsilon_{mn} = \varepsilon_{nm}$$

$$[\varepsilon] = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix}$$

Isotropic

$$[\varepsilon] = \begin{bmatrix} \varepsilon_o & 0 & 0 \\ 0 & \varepsilon_o & 0 \\ 0 & 0 & \varepsilon_e \end{bmatrix}$$

Uniaxial
Negative Uniaxial: $\varepsilon_e < \varepsilon_o$
Positive Uniaxial: $\varepsilon_e > \varepsilon_o$

$$[\varepsilon] = \begin{bmatrix} \varepsilon_a & 0 & 0 \\ 0 & \varepsilon_b & 0 \\ 0 & 0 & \varepsilon_c \end{bmatrix}$$

Biaxial

- Symmetric tensors
- Natural modes are linearly polarized

Gyrotropic Anisotropic Materials

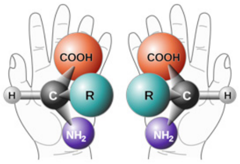
$$\varepsilon_{mn} = -j\varepsilon_{nm} \qquad m \neq n$$

$$[\varepsilon] = \begin{bmatrix} \varepsilon_a & -j\varepsilon_b & 0 \\ j\varepsilon_b & \varepsilon_a & 0 \\ 0 & 0 & \varepsilon_c \end{bmatrix}$$

gyroelectric

$$[\mu] = \begin{bmatrix} \mu_a & -j\mu_b & 0 \\ j\mu_b & \mu_a & 0 \\ 0 & 0 & \mu_c \end{bmatrix}$$

gyromagnetic



- Antisymmetric tensors
- Natural modes are circularly polarized
- Rotates linear polarization (Faraday rotation)
- Sugar and quartz

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Bi-Isotropic and Bi-Anisotropic

In “bi” materials, the constitutive relations couple both electric and magnetic fields.

Bi-Isotropic

$$\vec{D} = \varepsilon\vec{E} + \xi\vec{H}$$
$$\vec{B} = \mu\vec{H} + \zeta\vec{E}$$

$\varepsilon \equiv$ permittivity
$\mu \equiv$ permeability
$\xi, \zeta \equiv$ magnetoelectric coupling coefficients

Classification of Bi-Isotropic Media

$$\vec{D} = \varepsilon\vec{E} + (\chi - j\kappa)\sqrt{\mu\varepsilon}\,\vec{H} \qquad \vec{B} = \mu\vec{H} + (\chi + j\kappa)\sqrt{\mu\varepsilon}\,\vec{E}$$

	Nonchiral ($\kappa = 0$)	Chiral ($\kappa \neq 0$)
Reciprocal ($\chi = 0$)	Simple isotropic medium	Pasteur medium
Nonreciprocal ($\chi \neq 0$)	Tellegen medium	General bi-isotropic medium

$\chi \equiv$ reciprocity parameter
$\kappa \equiv$ chirality parameter

Bi-Anisotropic

$$\vec{D} = [\varepsilon]\vec{E} + [\xi]\vec{H}$$
$$\vec{B} = [\mu]\vec{H} + [\zeta]\vec{E}$$

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