



Advanced Electromagnetics:
21st Century Electromagnetics

Nonlinear Materials



Lecture Outline

- Electric polarization
- Potential well description
- Nonsymmetric potential well
- Applications
- Notes

Electric Polarization of Nonlinear Materials

In general, the relation between the applied electric field \vec{E} and the electric polarization \vec{P} is nonlinear so it can be expressed as a polynomial.

$$P = \underbrace{\epsilon_0 \chi_e^{(1)} E}_{\text{Linear response}} + \underbrace{\epsilon_0 \chi_e^{(2)} E^2 + \epsilon_0 \chi_e^{(3)} E^3 + \dots}_{\text{Nonlinear response}}$$

These terms are usually ignored. They tend to only become significant when the electric field is very strong.

$\chi_e^{(2)}$ is pronounced "chi two"

$\chi_e^{(3)}$ is pronounced "chi three"

⋮

$\chi_e^{(n)} \equiv$ electric susceptibility

$\chi_e^{(1)}$ no units

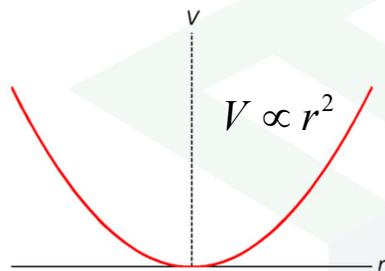
$\chi_e^{(2)}$ m/V

$\chi_e^{(3)}$ m²/V²

⋮

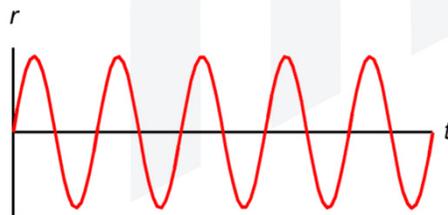
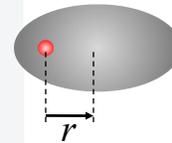
$\chi_e^{(n)}$ mⁿ⁻¹/Vⁿ⁻¹

"Potential Well" Description



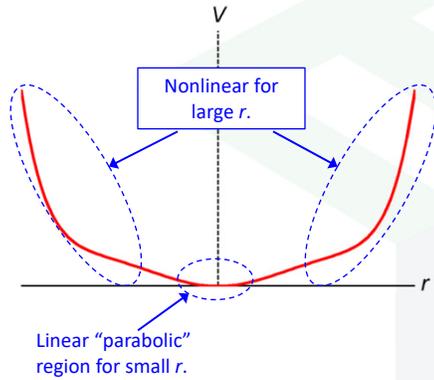
Linear materials have a parabolic potential well.
This leads to a resonator that oscillates as a perfect sinusoid.

V is the "push" required to displace the charge by a distance r .



Harmonic oscillation

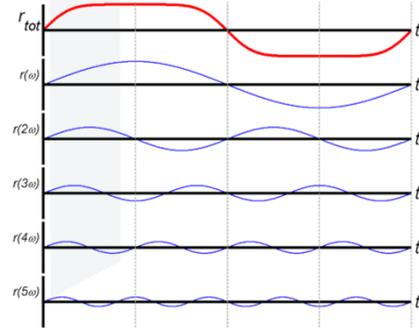
“Potential Well” for Nonlinear Materials



As bound charges are pushed very hard, the restoring force is no longer linear.

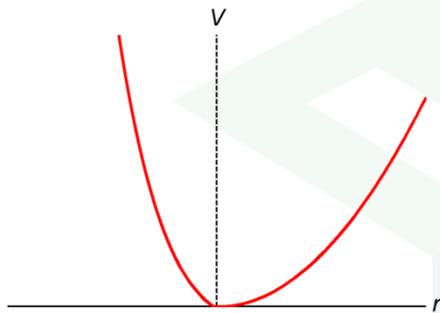
This results in **anharmonic oscillation**.

This type of resonance does not oscillate as a sinusoid. It can be seen as a series of sinusoids.



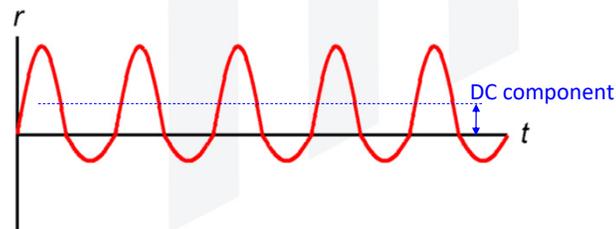
The oscillator is effectively resonating at multiple frequencies at the same time.

Nonsymmetric Potentials



This is a non-symmetrical potential and therefore exhibits a preferred direction for polarization.

The result is a rectified signal along with potentially other frequency content.



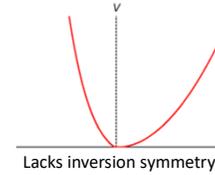
Applications of Nonlinear Materials

$\chi^{(1)}$ Materials

- Ordinary linear materials

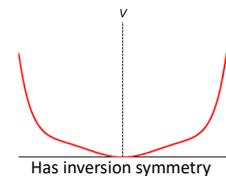
$\chi^{(2)}$ Materials

- *Rectification* – can create a DC potential from an optical wave
- *Frequency doubling (second harmonic generation)* – can make lasers at otherwise impossible wavelengths.
- *Parametric mixing* – can provide sum and difference frequencies
- *Pockel's effect* – can introduce birefringence from an applied electric field.



$\chi^{(3)}$ Materials

- *Kerr effect* – field dependent dielectric constant
- *Third harmonic generation* – can generate very short wavelength waves.
- *Raman scattering* – the mechanical vibration of a molecule can shift the frequency of the wave
- *Brillouin scattering* – dielectric response changes with applied pressure
- *Two photon absorption* – two photons are absorbed simultaneously where as a single photon would not be absorbed.



Notes About Nonlinear Materials

- All materials are nonlinear. Some just have stronger nonlinear behavior than others.
- It generally requires very intense electric fields to observe nonlinear properties.
- At radio frequencies, materials tend to breakdown before becoming nonlinear.
- Nonlinear properties are commonly exploited at optical frequencies.