Advanced Electromagnetics:
21st Century Electromagnetics

Special Cases of Drude Model

Lecture Outline

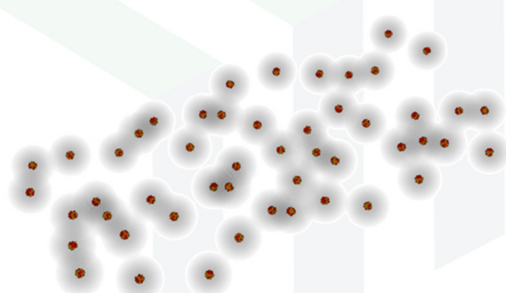
- Plasmons & the plasma frequency ω_p
- Low frequency properties of metals
- High frequency properties of metals

Plasmons & the Plasma Frequency ω_p

Slide 3

Metals

Metals are composed of an array of atoms that contain <u>many</u> electrons that are easily lost. When electrons are lost, the atoms become positive ions.



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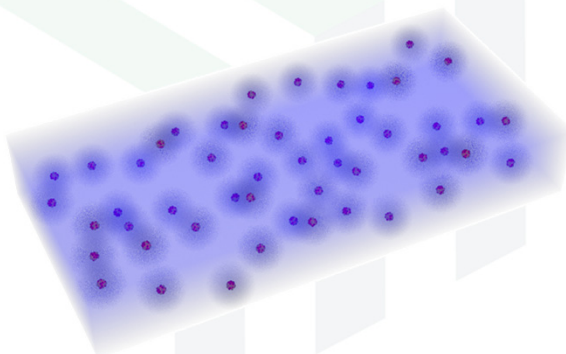
Slide 4

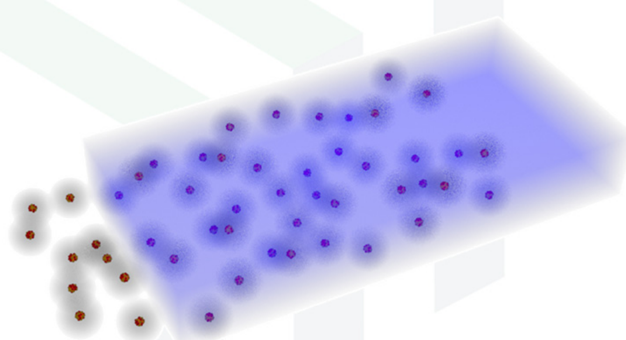
Conduction Electrons

When the electrons break free, they float around as a sea of free charges available for conduction. Overall charge is neutral.



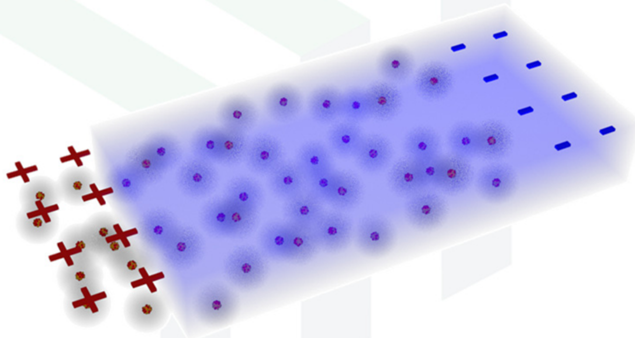
Electrons Shift Position

Suppose some of the free electrons experience a force that offsets them from their equilibrium position.



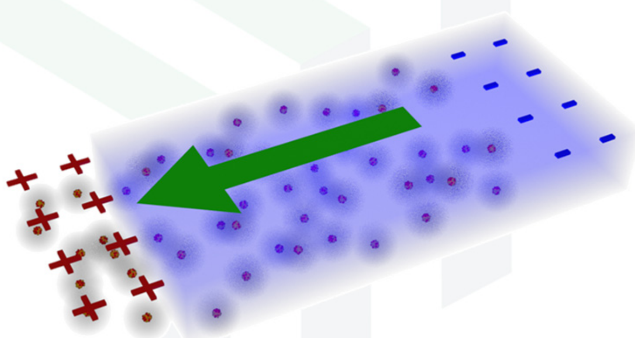
No Longer Charge Neutrality

When the sea of electrons shifts, this creates regions of positive and negative charge.



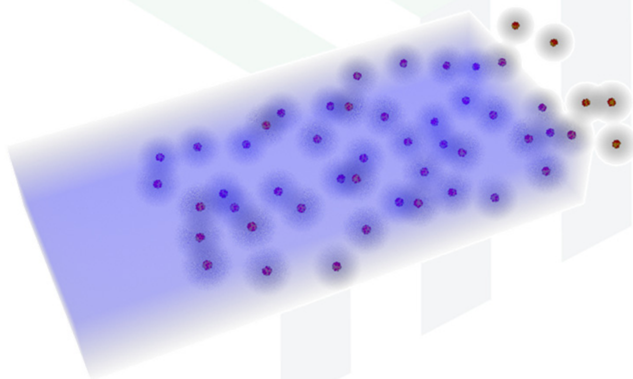
Force Induced Onto Free Electrons

A force is induced on the free electrons that acts to return them to their equilibrium position and restore charge neutrality everywhere.



Free Electrons Shift Position

The free electrons shift position and overshoot their equilibrium position because of inertia.



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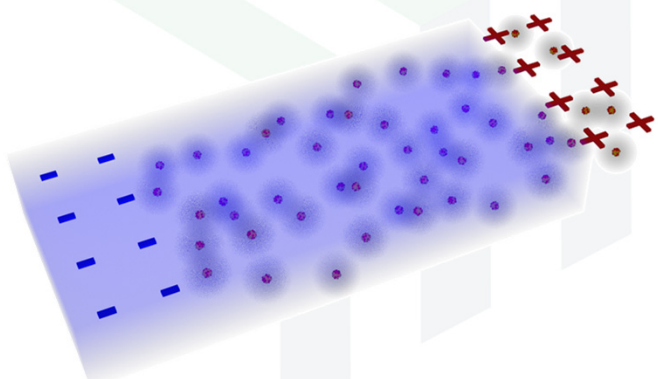
No Longer Charge Neutrality

This creates new regions of positive and negative charge.



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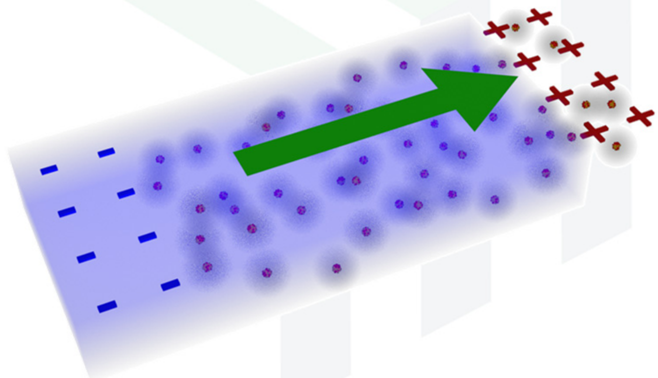
Another Force is Induced

Another force is induced in the opposite direction of the previous force so as to return the free electrons to their equilibrium position.



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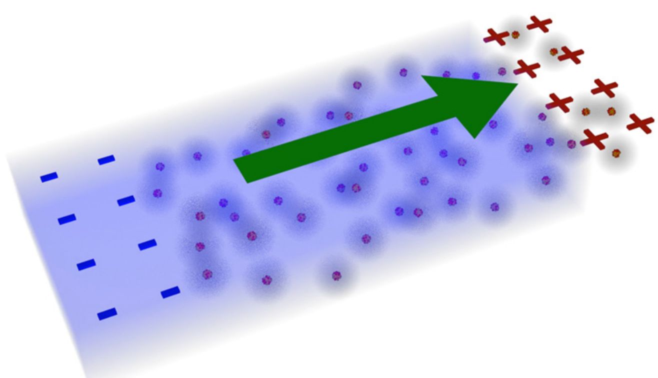
An Oscillation Arises

A resonance arises at the plasma frequency ω_p. This is a *plasmon*. A plasmon propagates through a plasma much like a sound wave propagates through air.



$$\omega_\mathrm{p} = \sqrt{\frac{Nq^2}{\varepsilon_0 m_\mathrm{e}}}$$

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Slide 12

Typical Plasma Frequencies

The plasma frequency for typical metals lies in the ultra-violet.

Metal	Symbol	Plasma Wavelength	Plasma Frequency
Aluminum	Al	82.78 nm	3624 THz
Chromium	Cr	115.35 nm	2601 THz
Copper	Cu	114.50 nm	2620 THz
Gold	Au	137.32 nm	2185 THz
Nickel	Ni	77.89 nm	3852 THz
Silver	Ag	137.62 nm	2180 THz

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13

Low Frequency Properties of Metals $(\omega \ll \omega_{\mathrm{p}})$

Slide 14

Low Frequency Permittivity $\tilde{\varepsilon}_{\mathrm{r}}$

Radio frequency and microwave devices use frequencies much lower than where most metals have their plasma frequency ω_{p}. Typical conditions are

$$\omega \ll \omega_{\mathrm{p}}$$

Under these conditions, with $\sigma_0 = \varepsilon_0 \omega_{\mathrm{p}}^2 \tau$ the Drude model reduces to

$$\tilde{\varepsilon}_{\mathrm{r}}'(\omega) = 1 - \frac{\omega_{\mathrm{p}}^2 \tau^2}{1+\omega^2\tau^2} \cong 1 - \frac{\omega_{\mathrm{p}}^2 \tau^2}{1+\cancel{\omega^2\tau^2}} \cong 1 - \omega_{\mathrm{p}}^2 \tau^2$$

$$\tilde{\varepsilon}_{\mathrm{r}}''(\omega) = \frac{\omega_{\mathrm{p}}^2 \tau / \omega}{1+\omega^2\tau^2} \cong \frac{\omega_{\mathrm{p}}^2 \tau / \omega}{1+\cancel{\omega^2\tau^2}} \cong \frac{\sigma_0}{\omega\varepsilon_0}$$

$$\tilde{\varepsilon}_{\mathrm{r}}(\omega) \cong \left(1-\omega_{\mathrm{p}}^2\tau^2\right) + j\frac{\sigma_0}{\omega\varepsilon_0}$$

or

$$\varepsilon_{\mathrm{r}} \cong 1-\omega_{\mathrm{p}}^2\tau^2 \;\; \& \;\; \sigma \cong \sigma_0$$

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Low Frequency Refractive Index $\tilde{n}$

The complex refractive index is related to the low frequency permittivity through

$$\tilde{n}^2(\omega) = \left[n_{\mathrm{o}}(\omega) + j\kappa(\omega)\right]^2 = \left(1-\omega_{\mathrm{p}}^2\tau^2\right) + j\frac{\sigma_0}{\omega\varepsilon_0}$$

After some algebra, the complex refractive index is

$$n_{\mathrm{o}}(\omega) = \kappa(\omega) = \sqrt{\frac{\sigma_0}{2\omega\varepsilon_0}} \qquad \tilde{n}(\omega) = (1+j)\sqrt{\frac{\sigma_0}{2\omega\varepsilon_0}}$$

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Low Frequency Absorption Coefficient

The low frequency absorption coefficient is

$$\alpha(\omega) = k_0 \kappa(\omega)$$ By definition

$$\cong k_0 \sqrt{\frac{\sigma_0}{2\omega\varepsilon_0}} = \sqrt{k_0^2 \frac{\sigma_0}{2\omega\varepsilon_0}}$$ Recall at low frequencies that $\kappa = \sqrt{\sigma_0 / 2\omega\varepsilon_0}$.

$$\cong \sqrt{\omega^2 \mu_0 \varepsilon_0 \frac{\sigma_0}{2\omega\varepsilon_0}}$$ Recall that $k_0^2 = \omega^2 \mu_0 \varepsilon_0$.

$$\cong \sqrt{\frac{\omega\mu_0\sigma_0}{2}}$$ Simplify

$$\alpha(\omega) \cong \sqrt{\frac{\omega\mu_0\sigma_0}{2}}$$

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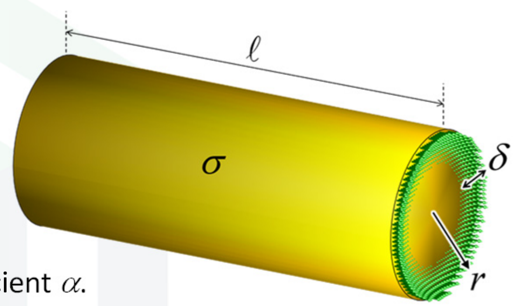
Low Frequency Skin Depth δ

Now that the complex refractive index $\tilde{n}$ is known, how quickly a wave attenuates due to loss can be determined.



Skin depth δ is defined as the distance a wave travels where its amplitude decays by $1/e$ from its starting amplitude. This is simply the reciprocal of the absorption coefficient α.

$$\delta(\omega) = \frac{1}{\alpha(\omega)} \cong \sqrt{\frac{2}{\omega\mu_0\sigma_0}}$$

Higher frequencies experience greater loss and decay faster. For this reason, metallic structures often perform better at lower frequencies.

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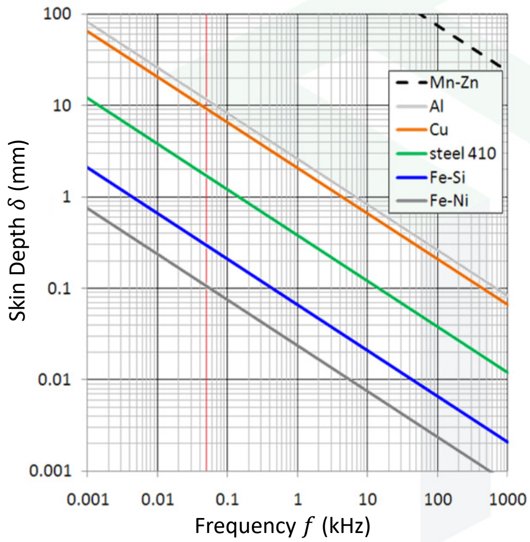
Skin Depth of Different Materials



https://en.wikipedia.org/wiki/Skin_effect#/media/File:Skin_depth_by_Zureks.png

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High Frequency Properties of Metals $(\omega \gg \omega_p)$

High Frequency Permittivity $\tilde{\varepsilon}_{\mathrm{r}}$

Typical conditions at high frequency are

$$\omega \gg \omega_{\mathrm{p}}$$

Under these conditions, with $\sigma_0 = \varepsilon_0 \omega_{\mathrm{p}}^2 \tau$ the Drude model reduces to

$$\tilde{\varepsilon}_{\mathrm{r}}'(\omega) = 1 - \frac{\omega_{\mathrm{p}}^2 \tau^2}{1 + \omega^2 \tau^2} \cong 1 - \frac{\omega_{\mathrm{p}}^2 \tau^2}{\cancel{1} + \omega^2 \tau^2} \cong 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2}$$

$$\tilde{\varepsilon}_{\mathrm{r}}''(\omega) = \frac{\omega_{\mathrm{p}}^2 \tau / \omega}{1 + \omega^2 \tau^2} \cong \frac{\omega_{\mathrm{p}}^2 \tau}{\cancel{\omega} + \omega^3 \tau^2} \cong \frac{\omega_{\mathrm{p}}^2}{\omega^3 \tau}$$

$$\tilde{\varepsilon}_{\mathrm{r}}(\omega) \cong 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2} + j \frac{\omega_{\mathrm{p}}^2}{\omega^3 \tau}$$

or

$$\varepsilon_{\mathrm{r}} \cong 1 - \frac{\omega_{\mathrm{p}}^2}{\omega^2} \quad \& \quad \sigma \cong \frac{\varepsilon_0 \omega_{\mathrm{p}}^2}{\omega^2 \tau}$$

High Frequency Refractive Index $\tilde{n}$

From the high frequency permittivity, it follows that

$$n_{\mathrm{o}}(\omega) \cong \sqrt{1 + \left(\frac{\omega_{\mathrm{p}}^2}{2\omega^3 \tau} \right)^2} \qquad \kappa(\omega) \cong \frac{\omega_{\mathrm{p}}^2}{2\omega^3 \tau}$$

It follows that the attenuation coefficient $\alpha(\omega)$ is

$$\alpha(\omega) = k_0 \kappa(\omega) \cong \frac{\omega_{\mathrm{p}}^2}{2 c_0 \omega^2 \tau} \cong \frac{1}{2 c_0 \tau} \frac{\lambda_0^2}{\lambda_{\mathrm{p}}^2}$$