Nyquist Sampling Theorem

If a signal is periodic with period \( \tau \), the maximum possible sampling period \( \Delta t \) that can still resolve the signal is

\[
\Delta t \leq \frac{\tau}{2}
\]

Similarly, if a signal has a maximum frequency of \( f_{\text{max}} \), the minimum sampling frequency \( f_s \) to resolve the signal is

\[
f_s \geq 2f_{\text{max}}
\]
Frequency Limits

Given a discrete function \( f(n) \) sampled at intervals of \( \Delta t \):

(1) the maximum frequency \( f_{\text{max}} \) resolved by the FFT is determined by the sampling period \( \Delta t \).

\[
f_{\text{max}} = \frac{0.5}{\Delta t}
\]

(2) the frequency resolution \( \Delta f \) is determined by the number of samples \( N \).

\[
\Delta f = \frac{2 f_{\text{max}}}{N} = \frac{1}{N \cdot \Delta t}
\]

Frequency Axis (1 of 2)

The FFT calculates the complex amplitude of the frequencies over a range of frequencies from \(-f_{\text{max}}\) up to \(+f_{\text{max}}\).

Based on this, calculate the frequency axis \( \text{freq} \) according to

\[
\begin{align*}
\text{freq} &= \text{lin} \left( -f_{\text{max}}, f_{\text{max}}, N \right); \\
\text{df} &= \frac{1}{(N \times \Delta t)}; \\
f_{\text{max}} &= 0.5 / \Delta t; \quad \text{Note: Make } N \text{ odd!}
\end{align*}
\]
Frequency Axis (2 of 2)

- f(n), 101 points
- Frequency Shifted F(k), 101 points
- F(k) with Frequency Axis, 101 points

Padding the FFT to Plot the DTFT

- Discrete-Time Function, f(n)
- Discrete Fourier Transform, F(k)
- Discrete-Time Function with Padding, f(n)
- Discrete Fourier Transform, F(k)