



Computational Science:
Computational Methods in Engineering

Calculating Fourier Series Coefficients



The Fourier Series

The Fourier series is the Fourier transform of a sampled signal.

One-Dimensional Standard Fourier Series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nt) + b_n \sin(nt)]$$

$$a_n = \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} f(t) \cos(nt) dt \quad n \geq 0$$

$$b_n = \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} f(t) \sin(nt) dt \quad n \geq 1$$

One-Dimensional Complex Fourier Series

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{-j \frac{2\pi kt}{\tau}} \quad c_k = \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} f(t) e^{-j \frac{2\pi kt}{\tau}} dt$$

Calculating the Coefficients Using FFT

One-Dimensional Standard Fourier Series

```
% CALCULATE FOURIER SERIES COEFFICIENTS
F = fft(f)/N;
a0 = F(1);
an = -2*real(F(2:M));
bn = -2*imag(F(2:M));
```

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nt) + b_n \sin(nt)]$$

$$a_n = \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} f(t) \cos(nt) dt \quad n \geq 0$$

$$b_n = \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} f(t) \sin(nt) dt \quad n \geq 1$$

One-Dimensional Complex Fourier Series

```
% CALCULATE COMPLEX FOURIER SERIES COEFFICIENTS
F = fftshift(fft(f))/N;
m0 = 1 + floor(N/2);
ck = F(m0-M:m0+M);
```

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{-j\frac{2\pi kt}{\tau}}$$

$$c_k = \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} f(t) e^{-j\frac{2\pi kt}{\tau}} dt$$