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Computational Science:
Computational Methods in Engineering

Finite-Difference Time-Domain in Electromagnetics

Outline

- Time-Domain Solution of Maxwell's Equations
- Sequence of Code Development for 1D FDTD
- Sources in FDTD
- FDTD Algorithm
- Sequence of Code Development for 2D FDTD

Time-Domain Solution of Maxwell's Equations

Time-Domain Solution of Maxwell's Equations

$$\nabla \times \vec{E}(t) = -\mu \frac{\partial \vec{H}(t)}{\partial t}$$

A circulating ***E*** field induces a change in the ***H*** field at the center of circulation in proportion to the permeability.

$$\nabla \times \vec{H}(t) = \varepsilon \frac{\partial \vec{E}(t)}{\partial t}$$

A circulating ***H*** field induces a change in the ***E*** field at the center of circulation in proportion to the permittivity.

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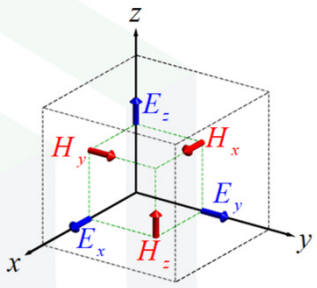
Fields are Staggered in Both Space and Time



$$\nabla\times\vec{E}(t)=-\mu\frac{\partial\vec{H}(t)}{\partial t}\quad\Rightarrow\quad\nabla\times\vec{E}\big|_t\cong-\mu\frac{\vec{H}\big|_{t+\Delta t/2}-\vec{H}\big|_{t-\Delta t/2}}{\Delta t}$$

$$\nabla\times\vec{H}(t)=\varepsilon\frac{\partial\vec{E}(t)}{\partial t}\quad\Rightarrow\quad\nabla\times\vec{H}\big|_{t+\Delta t/2}\cong\varepsilon\frac{\vec{E}\big|_{t+\Delta t}-\vec{E}\big|_t}{\Delta t}$$

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Update Equations

$$\nabla\times\vec{E}\big|_t=-\mu\frac{\vec{H}\big|_{t+\Delta t/2}-\vec{H}\big|_{t-\Delta t/2}}{\Delta t}\quad\Rightarrow\quad\vec{H}\big|_{t+\Delta t/2}=\vec{H}\big|_{t-\Delta t/2}-\frac{\Delta t}{\mu}\left(\nabla\times\vec{E}\big|_t\right)$$

$$\nabla\times\vec{H}\big|_{t+\Delta t/2}=\varepsilon\frac{\vec{E}\big|_{t+\Delta t}-\vec{E}\big|_t}{\Delta t}\quad\Rightarrow\quad\vec{E}\big|_{t+\Delta t}=\vec{E}\big|_t+\frac{\Delta t}{\varepsilon}\left(\nabla\times\vec{H}\big|_{t+\Delta t/2}\right)$$

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Courant Stability Condition

Due to how the update equations are formulated, a disturbance cannot travel more than one grid cell in one time step.

The time step must be made small enough that a physical wave cannot outpace a numerical wave.

$$\Delta t < \frac{1}{c_0\sqrt{\frac{1}{(\Delta x)^2}+\frac{1}{(\Delta y)^2}+\frac{1}{(\Delta z)^2}}} \approx \frac{n_{\min}\Delta_{\min}}{2c_0}$$

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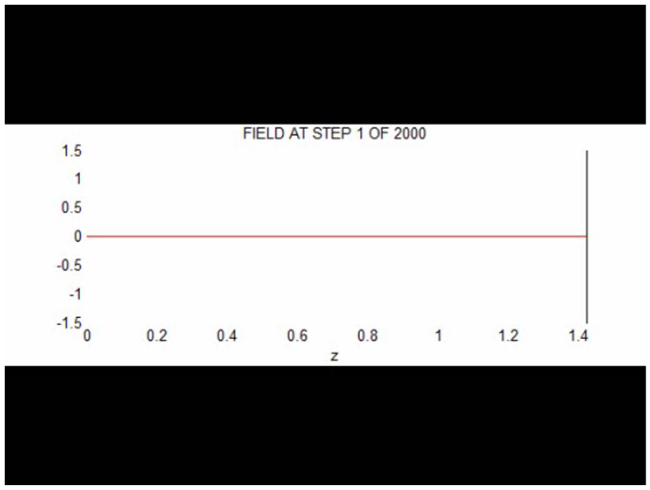
Sequence of Code Development for 1D FDTD

Sequence of Code Development
Step 1 – Basic FDTD Algorithm



- Basic update equations

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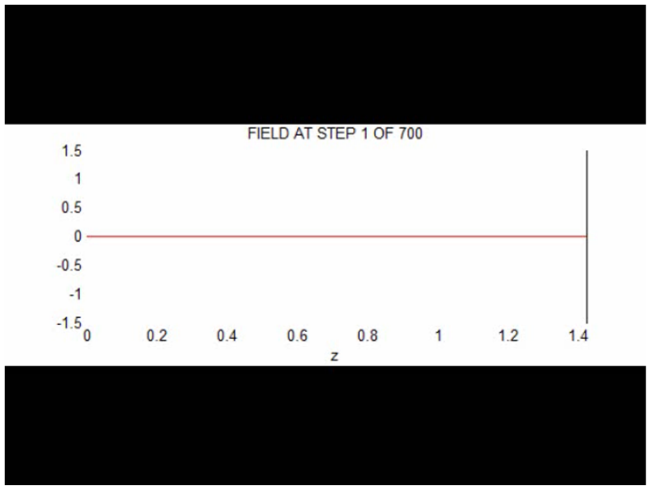
Sequence of Code Development
Step 2 – Add Simple Soft Source



- Basic update equations
- Add a soft source

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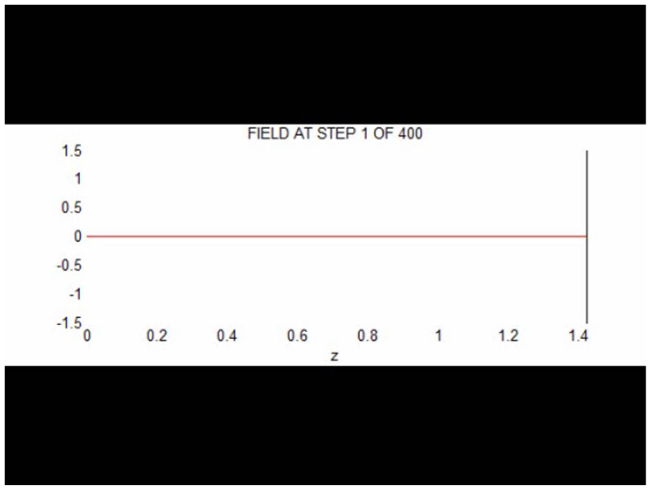
Sequence of Code Development
Step 3 – Add Absorbing Boundary



- Basic update equations
- Add a soft source
- Add perfect boundary condition

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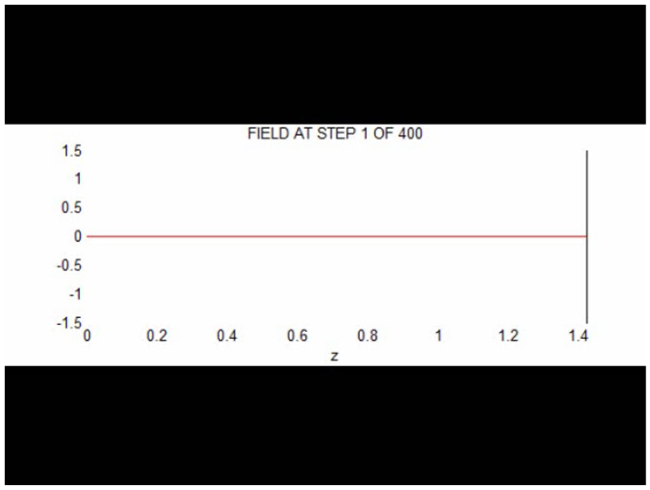
Sequence of Code Development
Step 4 – Add TF/SF Source



- Basic update equations
- Add a soft source
- Add perfectly absorbing boundary condition
- Incorporate TF/SF "one-way" source

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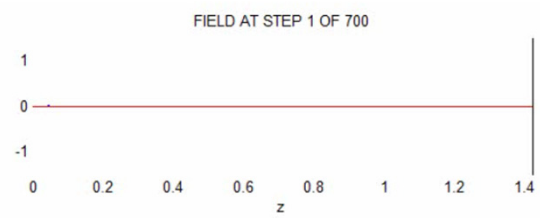
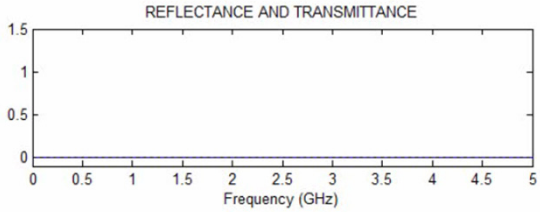
Sequence of Code Development
Step 5 – Move Source and Add T & R





- Basic update equations
- Add a soft source
- Add perfect boundary condition
- Incorporate TF/SF "one-way" source
- Move position of source
- Calculate transmittance and reflectance

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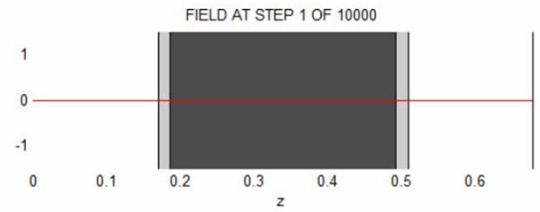
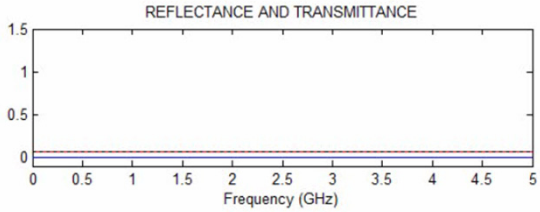
Sequence of Code Development
Step 6 – Add Device (Algorithm Done)





- Basic update equations
- Add a soft source
- Add perfect boundary condition
- Incorporate TF/SF "one-way" source
- Move position of source
- Calculate transmittance and reflectance
- Add a real device

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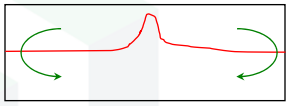
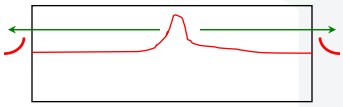
Summary of Code Development Sequence

Step 1 – Implement basic FDTD algorithm

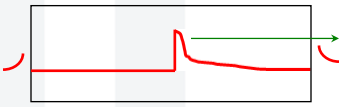
Step 2 – Add the source



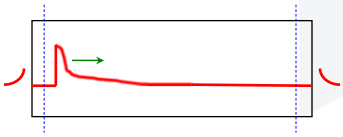
Step 3 – Add absorbing boundary



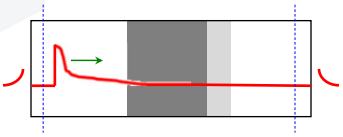
Step 4 – Add "one-way" source



Step 5 – Calculate transmittance and reflectance



Step 6 – Add a device



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Sources in FDTD

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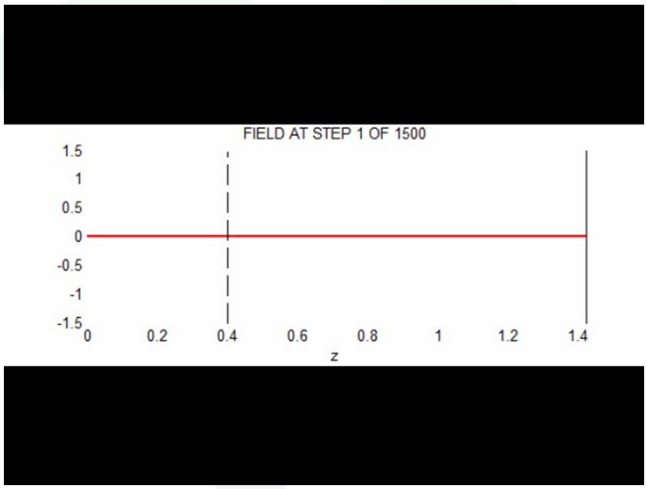
Movie of Simple Hard Source



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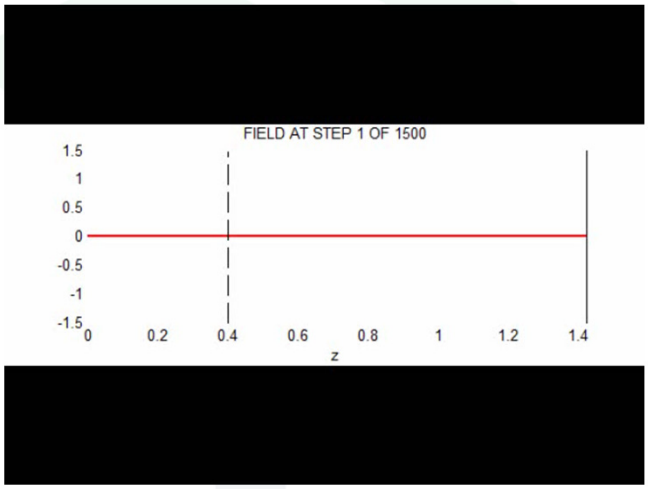
Movie of Simple Soft Source



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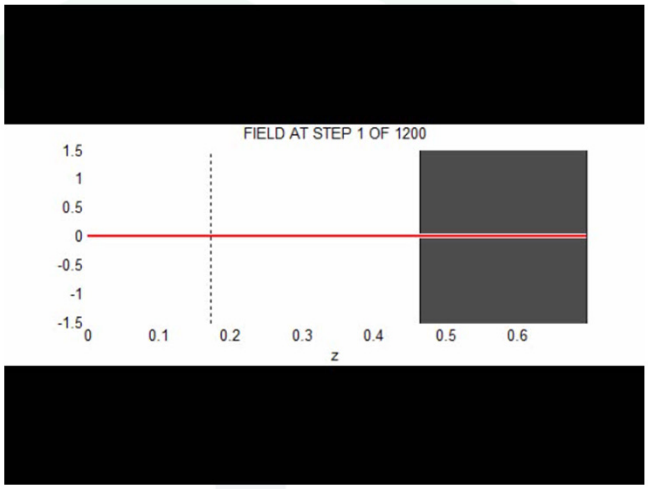
Movie of TF/SF Soft Source



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FDTD Algorithm

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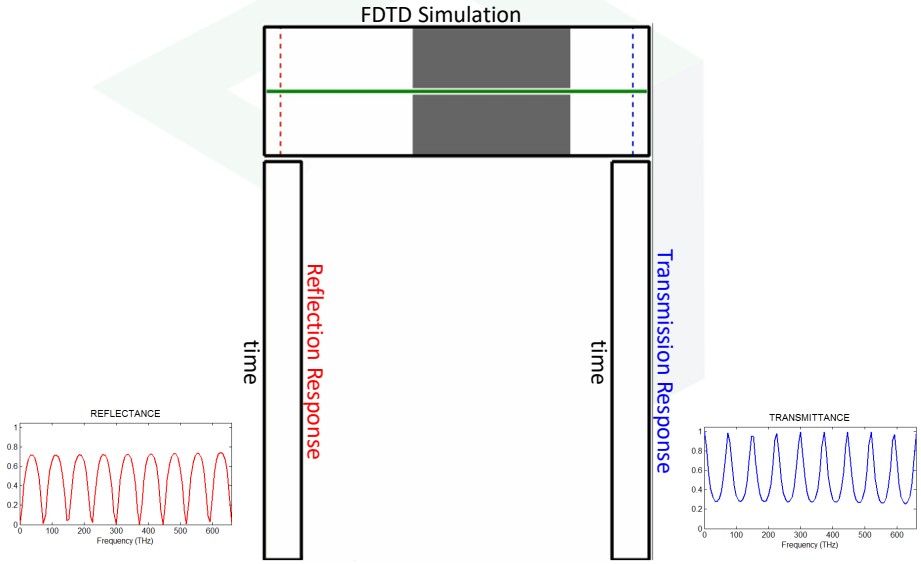
Calculating Transmission & Reflection



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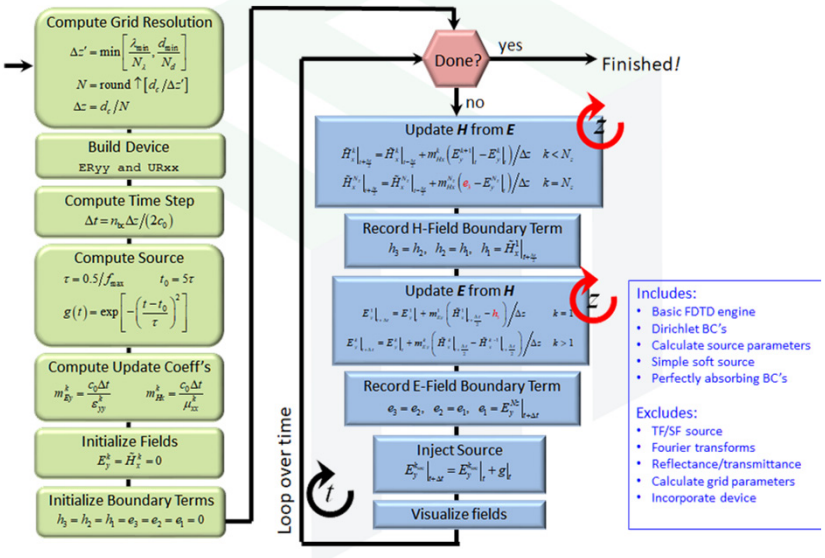
Block Diagram of 1D FDTD



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Animation of Numerical Dispersion

Simulation with near-zero numerical dispersion...

Simulation with strong numerical dispersion...

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Sequence of Code Development for 2D FDTD

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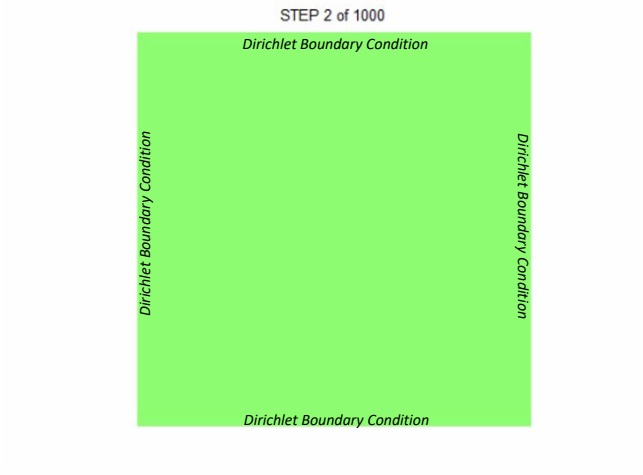
2D Code Development Sequence
Step 1 – Basic Update Equations



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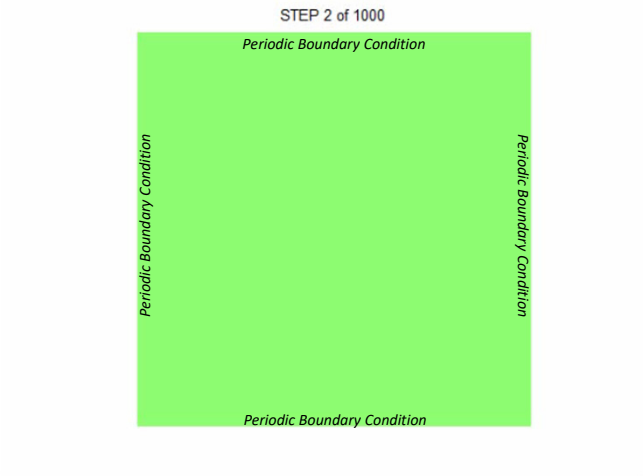
2D Code Development Sequence
Step 2 – Periodic Boundary Conditions



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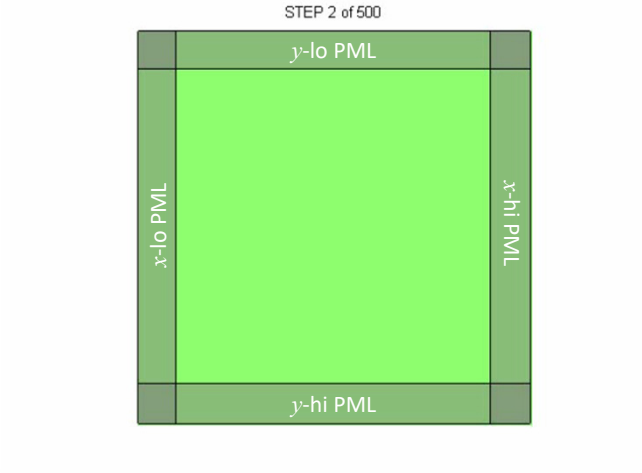
2D Code Development Sequence
Step 2 – Perfectly Matched Layer



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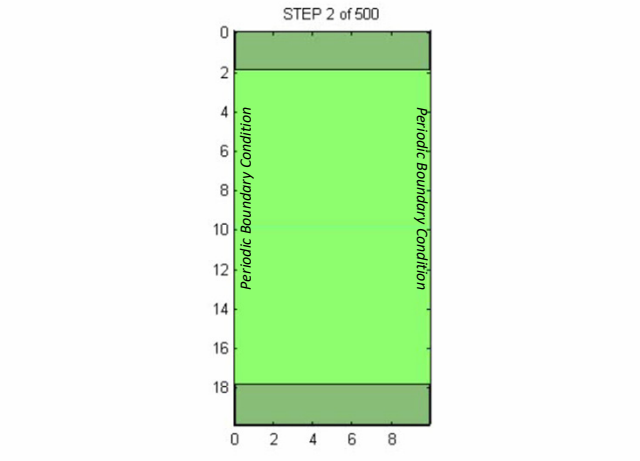
2D Code Development Sequence
Step 4 – TF/SF Source



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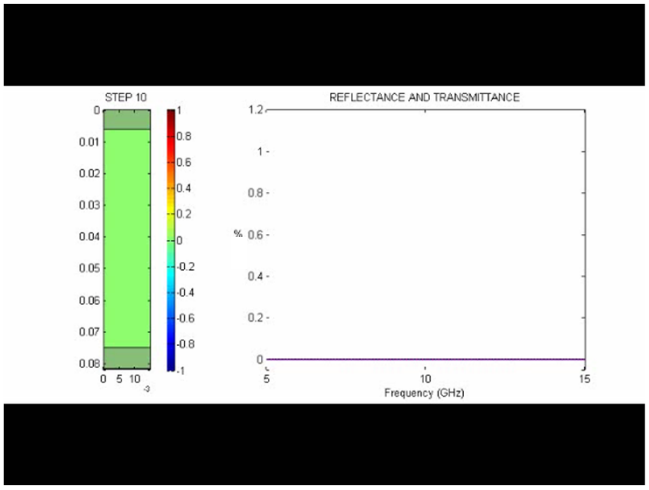
2D Code Development Sequence
Step 5 – Transmission & Reflection



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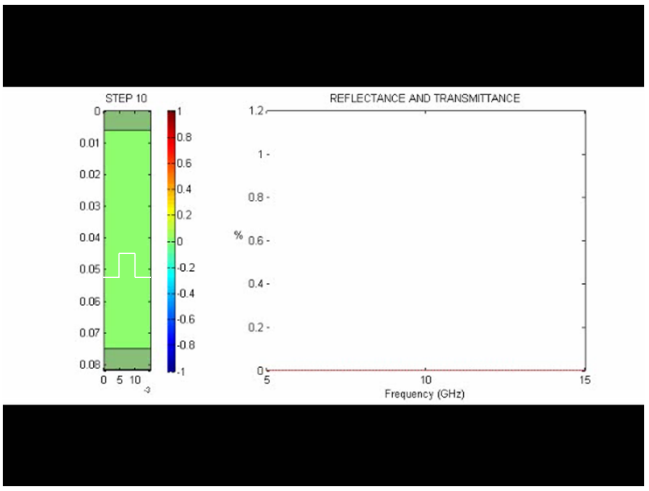
2D Code Development Sequence
Step 6 – Simulate Device



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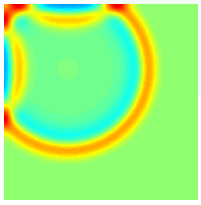
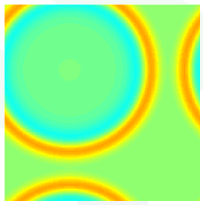
Summary of 2D Code Development Sequence

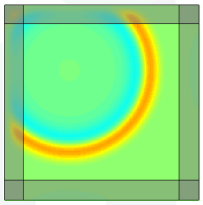
Step 1 – Basic Update + Dirichlet



Step 2 – Basic Update + Periodic BC

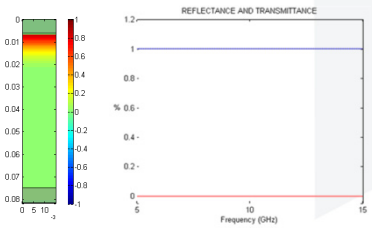


Step 3 – Add PML

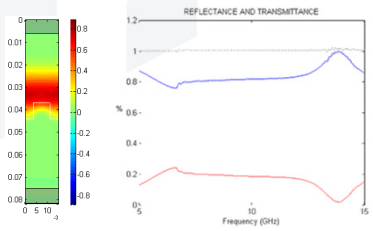


Step 4 – TF/SF

Step 5 – Calculate Response



Step 6 – Add a Device and Benchmark



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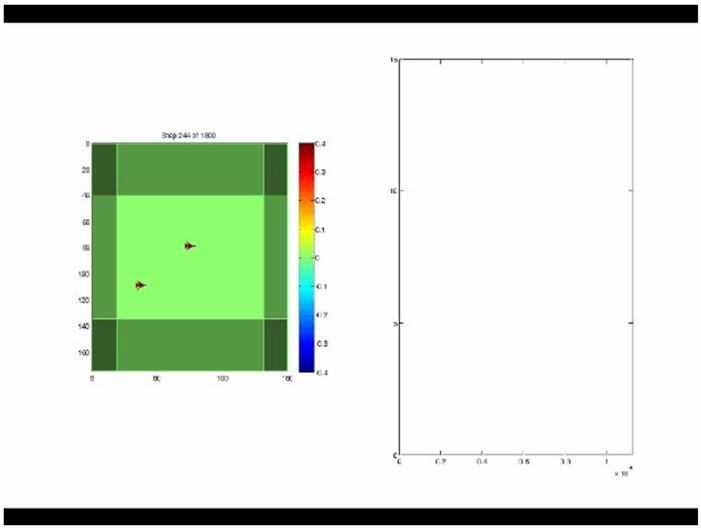
Real FDTD Simulation



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