Advanced Computation:
Computational Electromagnetics

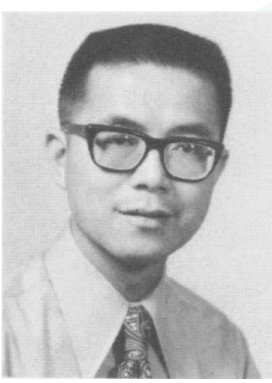
Introduction to the Yee Grid

Outline

- The Yee Grid Scheme
- Consequences of the Yee Grid

The Yee Grid Scheme

Kane S. Yee



Kane S. Yee

Kane S. Yee was born in Canton, China on March 26, 1934. He received a B.S.E.E., M.S.E.E and Ph.D. in Applied Mathematics from the University of California at Berkeley in 1957, 1958, and 1963, respectively, under the supervision of Bernard Friedman.

In 1966, Yee published a paper on the use of a finite difference staggered grids algorithm in the solution of Maxwell's equations. Yee was initially motivated by his self-studies in Fortran to develop the method. Appearing on IEEE Transactions on Antennas and Propagation, the article received little attention at the time of its release.

Between 1966 and 1984, Yee became a professor of electrical engineering and mathematics at the University of Florida and later at Kansas State University. He became a consultant to Lawrence Livermore National Laboratory in 1966, working on microwave vulnerability problems at the same institute from 1984 to 1987. In 1987, he became a research scientist at Lockheed Palo Alto Research Lab, working on computational electromagnetics problems and retiring in 1996.

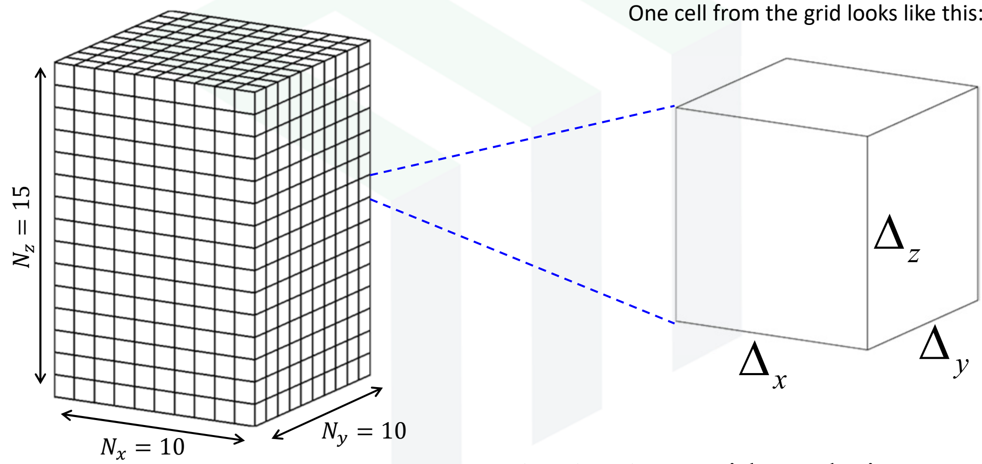
Notes Borrowed from: https://en.wikipedia.org/wiki/Kane_S._Yee

EMPossible

3D Grids

A three-dimensional grid looks like this:

One cell from the grid looks like this:



$$\Delta_x, \Delta_y, \Delta_z \equiv \text{grid resolution parameters}$$

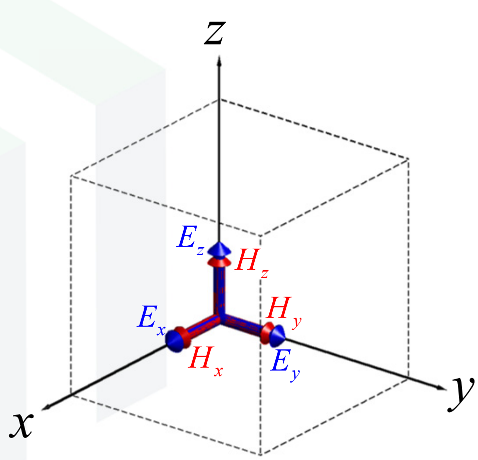
EMPossible

Slide 5

Collocated Grid

Within the grid cell, where should E_x, E_y, E_z, H_x, H_y, and H_z be placed?

A straightforward approach would be to locate all of the field components at a common point within in a grid cell; perhaps at the origin.

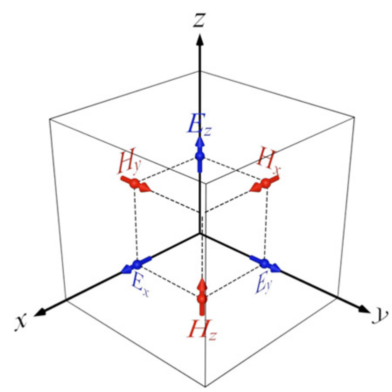


EMPossible

Slide 6

Yee Grid

Instead, the field components will be staggered within the grid cell.

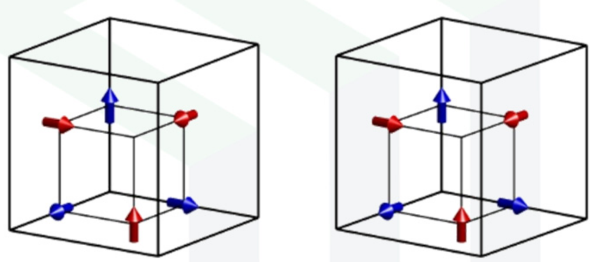


K. S. Yee, "Numerical solution of the initial boundary value problems involving Maxwell's equations in isotropic media," IEEE Trans. Microwave Theory and Techniques, vol. 44, pp. 61–69, 1966.

EMPossible

Slide 7

Stereo Image of Yee Cell



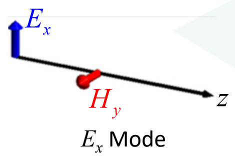
To view the Yee cell if full 3D, look past the image above so that they appear double. When the double images overlap so that you see three Yee cells, the middle image will be three-dimensional.

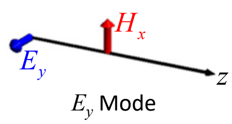
EMPossible

Slide 8

Yee Cell for 1D, 2D, and 3D Grids

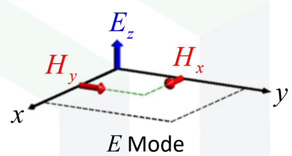
1D Yee Grids

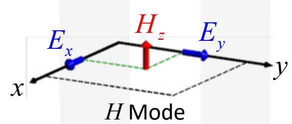




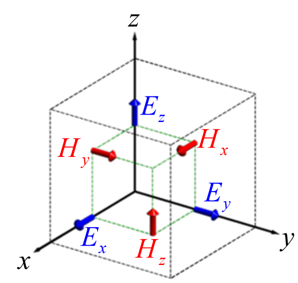
* These are the same for isotropic materials.

2D Yee Grids





3D Yee Grid



EMPossible

Slide 9

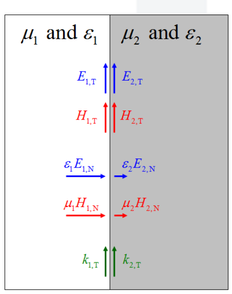
Reasons to Use the Yee Grid

1. Divergence-free

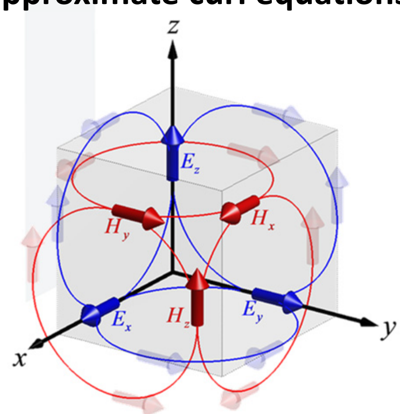
$$\nabla \bullet \left(\varepsilon \vec{E} \right) = 0$$

$$\nabla \bullet \left(\mu \vec{H} \right) = 0$$

2. Physical boundary conditions are naturally satisfied



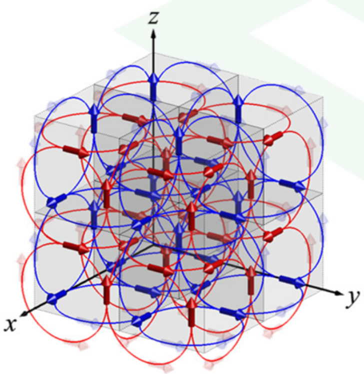
3. Elegant arrangement to approximate curl equations

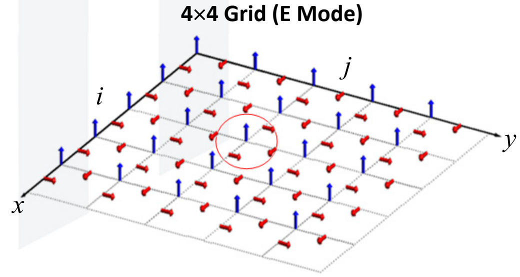


EMPossible

Slide 10

Visualizing Extended Yee Grids





EMPossible

Slide 11

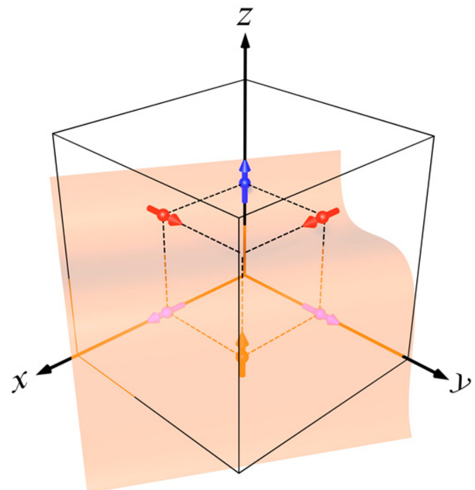
Consequences of the Yee Grid

Slide 12

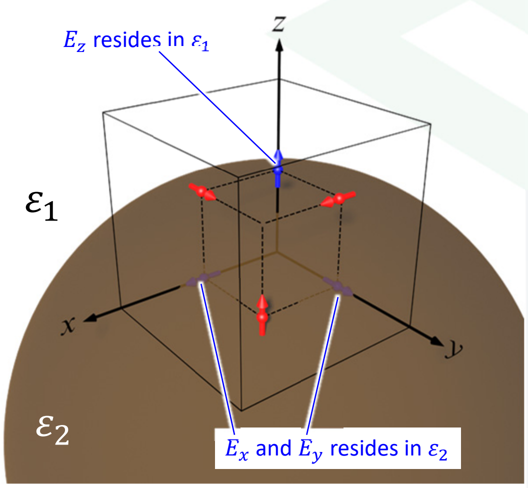
Field Components are at Physically Different Positions

Even though the field components reside within the same Yee cell, they are out of phase.

Any time that a wave is injected, extracted, or analyzed within a simulation, this must be accounted for.



Field Components are at Physically Different Positions



Even though the field components reside within the same Yee cell, they may reside in different media. This happens when an interface slices through the middle of a Yee cell.

This is the main reason why each field component is assigned its own material properties.

ε_{xx} assigned to E_x.
ε_{yy} assigned to E_y.
ε_{zz} assigned to E_z.
μ_{xx} assigned to H_x.
μ_{yy} assigned to H_y.
μ_{zz} assigned to H_z.

Dispersion on a Yee Grid

Recall the dispersion relation for an isotropic material with parameters μ_r and ε_r.

$$\left(\frac{\omega}{c_0}\right)^2 \mu_r \varepsilon_r = k_x^2 + k_y^2 + k_z^2$$

The equivalent dispersion relation on a frequency-domain Yee grid filled with μ_r and ε_r is

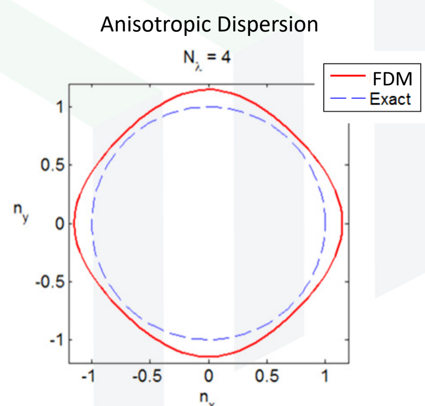
$$\left(\frac{\omega}{v}\right)^2 \mu_r \varepsilon_r = \left[\frac{2}{\Delta_x}\sin\left(\frac{k_x \Delta_x}{2}\right)\right]^2 + \left[\frac{2}{\Delta_y}\sin\left(\frac{k_y \Delta_y}{2}\right)\right]^2 + \left[\frac{2}{\Delta_z}\sin\left(\frac{k_z \Delta_z}{2}\right)\right]^2$$

In this equation, the speed of light c_0 is written as v because the velocity is different due to the numerical dispersion of the grid.

EMPossible

Drawback of Structured Grids

Structured grids exhibit highly anisotropic dispersion.



EMPossible

Compensation Factor γ (1 of 2)

The numerical dispersion equation is solved for velocity v.

$$v = \omega\sqrt{\mu_r\varepsilon_r}\left\{\left[\frac{2}{\Delta_x}\sin\left(\frac{k_x\Delta_x}{2}\right)\right]^2 + \left[\frac{2}{\Delta_y}\sin\left(\frac{k_y\Delta_y}{2}\right)\right]^2 + \left[\frac{2}{\Delta_z}\sin\left(\frac{k_z\Delta_z}{2}\right)\right]^2\right\}^{-\frac{1}{2}}$$

In the absence of numerical dispersion, v should be exactly the speed of light c_0. Due to the Yee grid, waves slow down by a factor γ.

$$v = c_0/\gamma$$

This factor γ can be calculated by combining the above equations.

$$\gamma = \frac{c_0}{\omega\sqrt{\mu_r\varepsilon_r}}\sqrt{\left[\frac{2}{\Delta_x}\sin\left(\frac{k_x\Delta_x}{2}\right)\right]^2 + \left[\frac{2}{\Delta_y}\sin\left(\frac{k_y\Delta_y}{2}\right)\right]^2 + \left[\frac{2}{\Delta_z}\sin\left(\frac{k_z\Delta_z}{2}\right)\right]^2}$$

EMPossible

17

Compensation Factor γ (2 of 2)

Here is a simpler and more useful expression for γ.

$$\gamma = \frac{1}{k_0 n}\sqrt{\left[\frac{2}{\Delta_x}\sin\left(\frac{k_x\Delta_x}{2}\right)\right]^2 + \left[\frac{2}{\Delta_y}\sin\left(\frac{k_y\Delta_y}{2}\right)\right]^2 + \left[\frac{2}{\Delta_z}\sin\left(\frac{k_z\Delta_z}{2}\right)\right]^2}$$

$$k_0 = \frac{2\pi}{\lambda_0}$$

$$n = \sqrt{\mu_r\varepsilon_r}$$

EMPossible

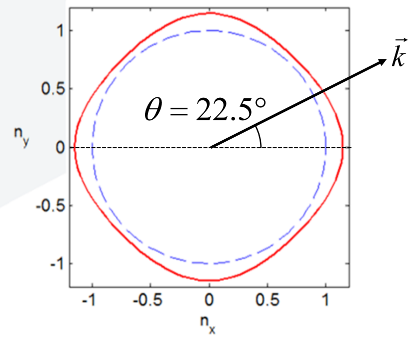
18

Compensating for Numerical Dispersion

Given that the wave slows down by factor γ in the direction of $\vec{k}$, it follows that numerical dispersion can be mitigated by artificially "speeding up" the wave.

This can be accomplished by decreasing the values of μ_r and ε_r across the entire grid by the factor γ.

$$\mu_r' = \mu_r / \gamma \qquad \varepsilon_r' = \varepsilon_r / \gamma$$



Notes About This Mitigation Technique:

1. Dispersion can only be completely eliminated in one direction $\vec{k}$.
2. Dispersion can only be mitigated for one choice of material values μ_r and ε_r.
3. It is best to choose average or dominant values for μ_r and ε_r.
4. Choose $\theta = 22.5°$ if nothing else is known.

EMPossible