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Computational Science:
Computational Methods in Engineering

Numerical Differentiation Using the FFT

Property of the Fourier Transform and the DFT

If the derivative of a function is Fourier transformed, the following property applies.

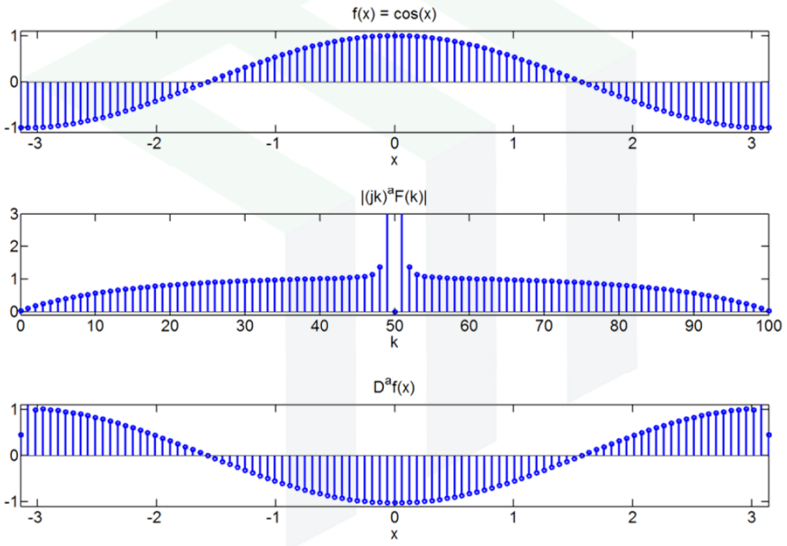
$$\text{FT}\left\{\frac{d^a}{dt^a} f(t)\right\} = (j\omega)^a F(\omega)$$

There is a similar property for the DFT

$$\text{DFT}\left\{\frac{d^a}{dt^a} f(n)\right\} = (jk)^a F(k)$$

```
% CALCULATE DERIVATIVE OF ORDER a
a  = 2;
k  = (2*pi/L)*[-floor(N/2):floor(N/2)];
F  = fftshift(fft(f));
F  = ((1i*k).^a).*F;
fd = ifft(ifftshift(F));
```

Visualizing the Calculation ($a = 2$)



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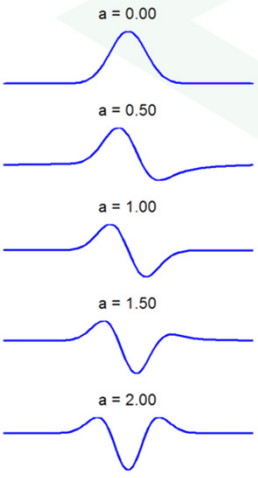
Fractional Derivatives

This can be used to calculate fractional derivatives. Just let a be a fractional number!




```
% FUNCTION
N = 101;
x = linspace(-1,1,N);
f = exp(-(x/0.2).^2);

% CREATE ARRAY OF DERIVATIVE ORDERS
A = [0:0.5:2];
NA = length(A);

% SHOW FRACTIONAL DERIVATIVES
L = max(x) - min(x);
for na = 1 : NA

    % Calculate Derivative Using FFT
    k  = (2*pi/L)*[-floor(N/2):floor(N/2)];
    F  = fftshift(fft(f));
    F  = ((1i*k).^a).*F;
    fd = real(ifft(ifftshift(F)));

    % Show Derivative
    subplot(NA,1,na);
    plot(x,fd,'-b');
    title(['a = ' num2str(a,'%4.2f')]);
    axis tight off;

end
```

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