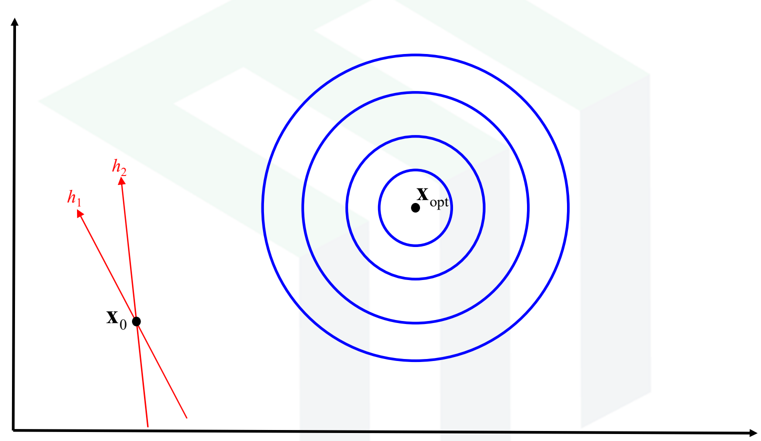
Computational Science:
Computational Methods in Engineering

Powell's Method

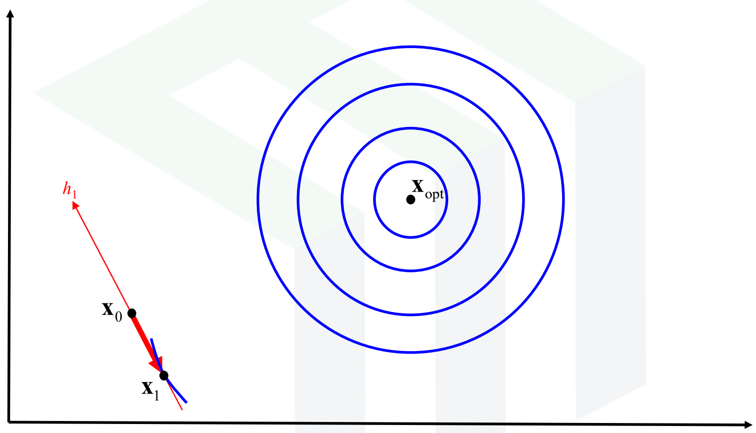
Outline

- Description of the Method
- Summary of the Method

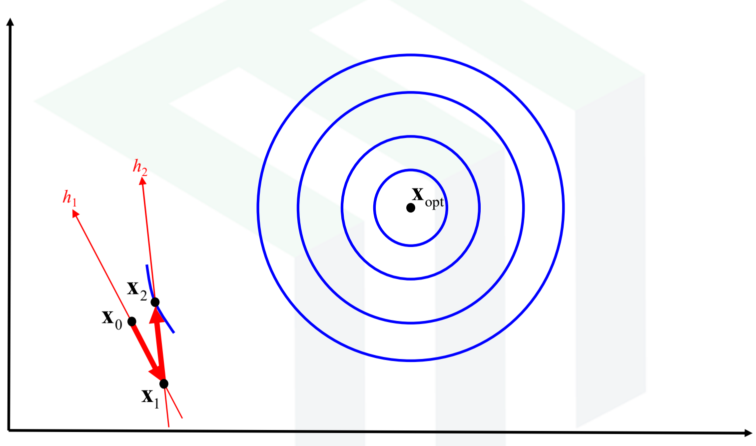
Description of the Method



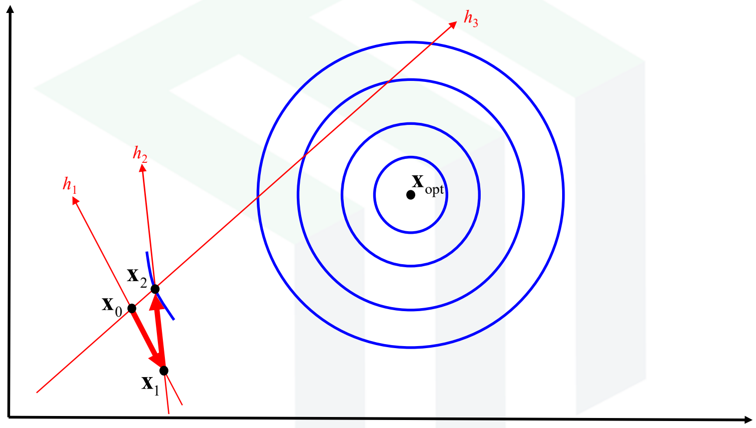
Pick a starting point $\mathbf{x}_0$ and two different starting directions h_1 and h_2.



Staring at $\mathbf{x}_0$, perform a 1D optimization along h_1 to find extremum $\mathbf{x}_1$.



Staring at $\mathbf{x}_1$, perform a 1D optimization along h_2 to find extremum $\mathbf{x}_2$.



Define h_3 to be in the direction connecting $\mathbf{x}_0$ to $\mathbf{x}_2$.

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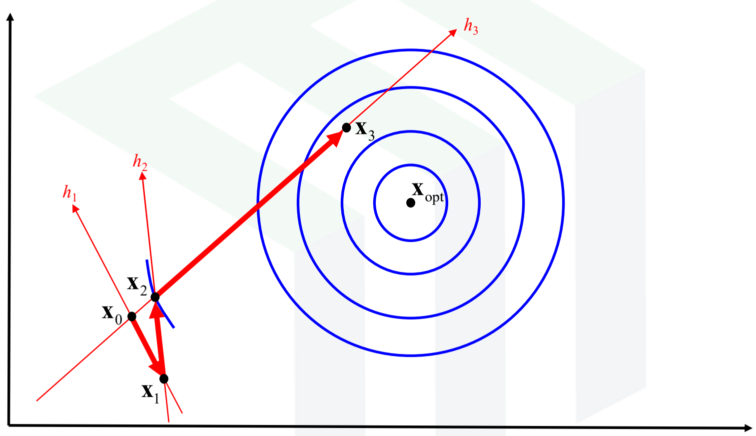

Staring at $\mathbf{x}_2$, perform a 1D optimization along h_3 to find extremum $\mathbf{x}_3$.

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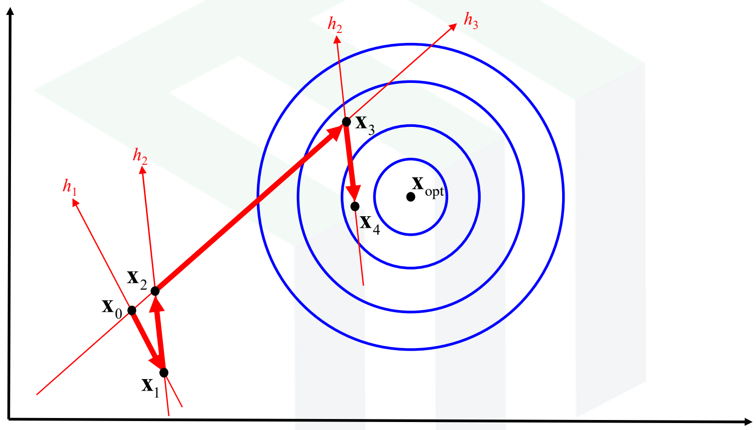
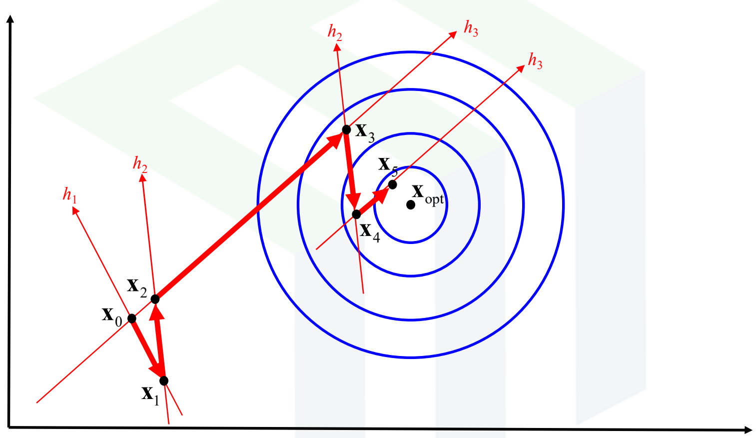
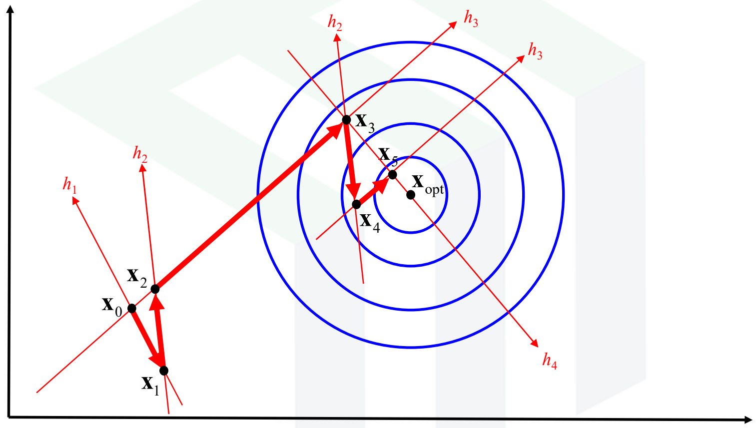


Staring at $\mathbf{x}_3$, perform a 1D optimization along h_2 to find extremum $\mathbf{x}_4$.



Staring at $\mathbf{x}_4$, perform a 1D optimization along h_3 to find extremum $\mathbf{x}_5$.



Define h_4 to be in the direction connecting $\mathbf{x}_3$ to $\mathbf{x}_5$.

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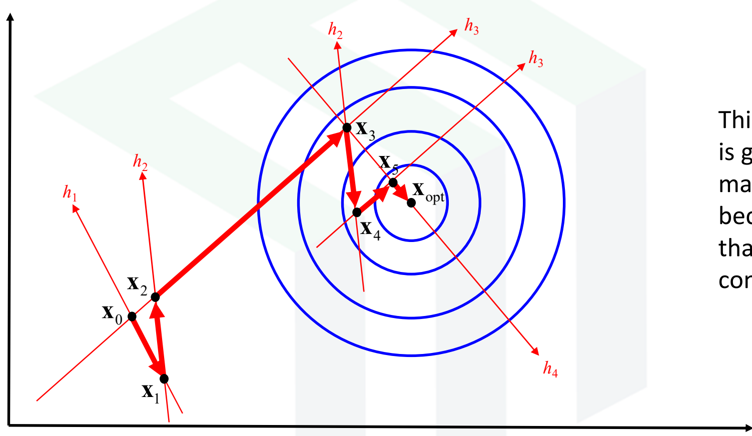

This last 1D optimization is guaranteed to find the maximum of a quadratic because Powell showed that h_3 and h_4 are both conjugate directions.

Staring at $\mathbf{x}_5$, perform a 1D optimization along h_4 to find extremum $\mathbf{x}_{opt}$.

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Method Summary

Algorithm Summary

1. Pick a starting point $\mathbf{x}_0$ and two different starting directions h_1 and h_2.
2. Staring at $\mathbf{x}_0$, perform a 1D optimization along h_1 to find extremum $\mathbf{x}_1$.
3. Staring at $\mathbf{x}_1$, perform a 1D optimization along h_2 to find extremum $\mathbf{x}_2$.
4. Define h_3 to be in the direction connecting $\mathbf{x}_0$ to $\mathbf{x}_2$.
5. Staring at $\mathbf{x}_2$, perform a 1D optimization along h_3 to find extremum $\mathbf{x}_3$.
6. Staring at $\mathbf{x}_3$, perform a 1D optimization along h_2 to find extremum $\mathbf{x}_4$.
7. Staring at $\mathbf{x}_4$, perform a 1D optimization along h_3 to find extremum $\mathbf{x}_5$.
8. Define h_4 to be in the direction connecting $\mathbf{x}_3$ to $\mathbf{x}_5$.
9. Staring at $\mathbf{x}_5$, perform a 1D optimization along h_4 to find extremum $\mathbf{x}_{\text{opt}}$.

Convergence

Powell's method is quadratically convergent and extremely efficient.

If iterated, it will converge in a finite number of iterations if the function is quadratic.

Most functions are nearly quadratic near their extrema.