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Electromagnetics:

Microwave Engineering

One-port Networks

Lecture Outline

- Why use Network Analysis?
- Equivalent Voltages and Currents
- The Concept of Impedance
- One Port Networks
- Foster’s Reactance Theorem
- Properties of $Z(\omega)$ and $\Gamma(\omega)$

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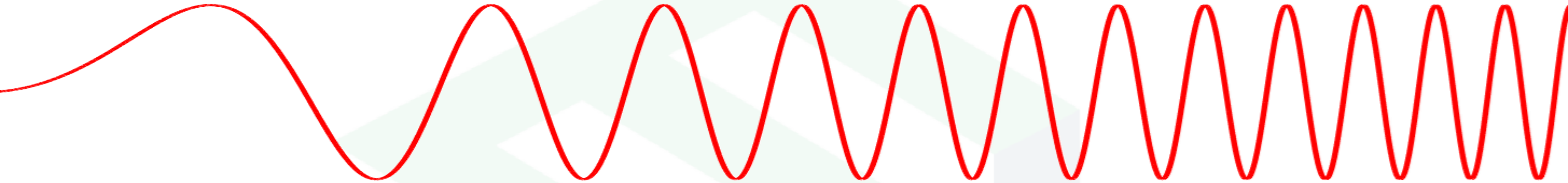
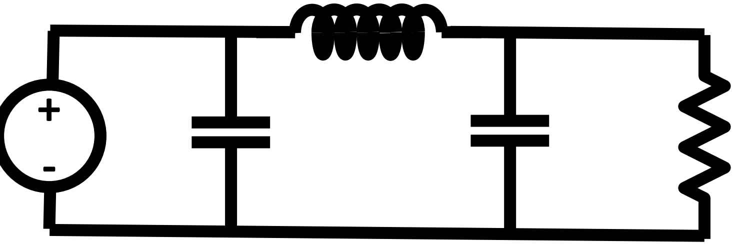
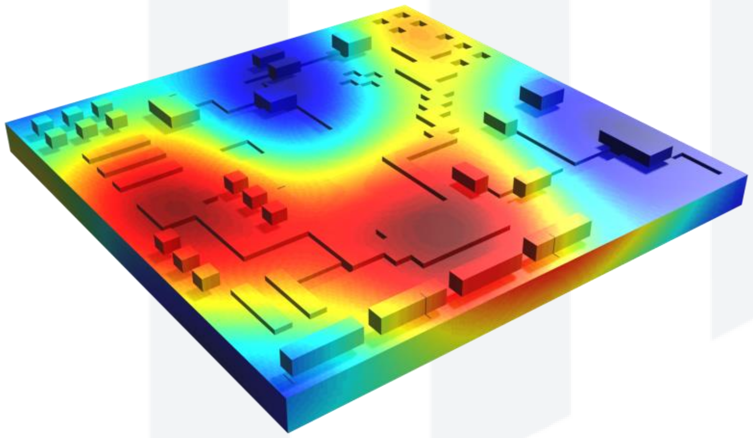
Why use Network Analysis?

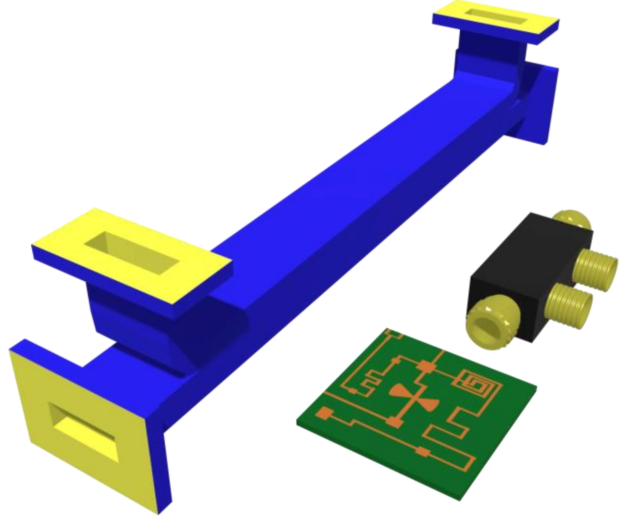
Low-Frequency vs. High-Frequency



Frequency







DC/Low-Frequency

High-Frequency/Microwaves

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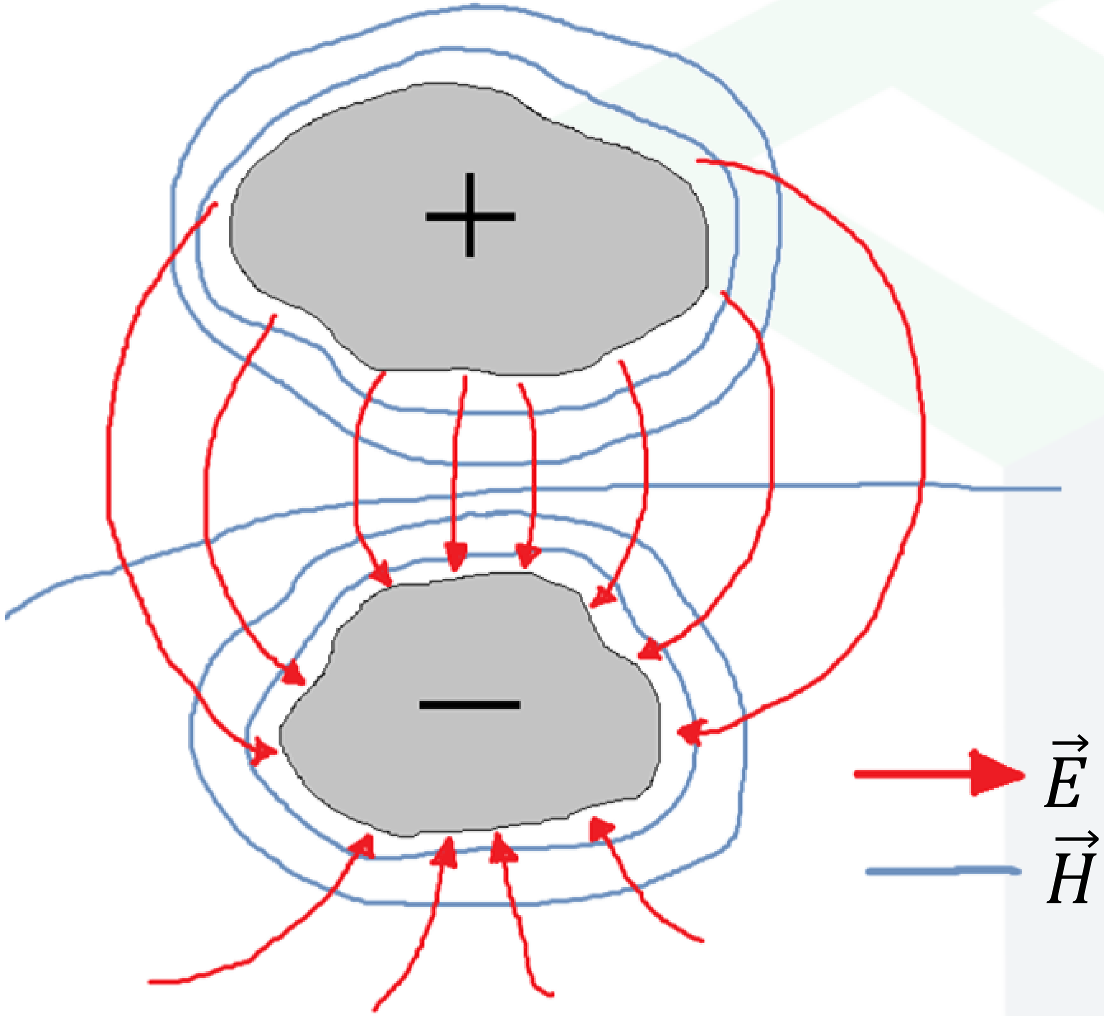
Equivalent Voltages and Currents

E and V of an arbitrary TEM two-conductor line

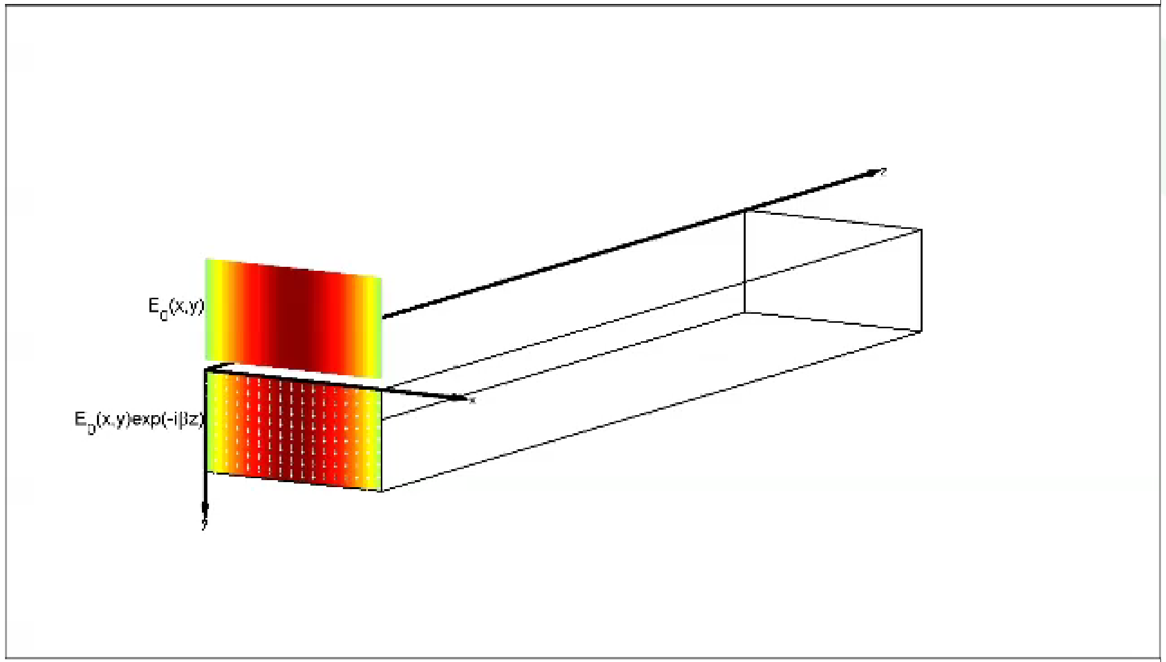


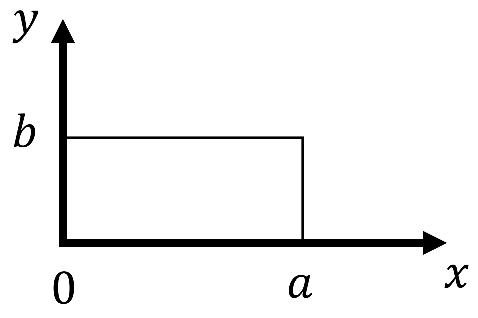
$$V = \int_{-}^{+} \vec{E} \bullet d\vec{l}$$

$$I = \oint_{C^+} \vec{H} \bullet d\vec{l}$$

$$Z_0 = \frac{V}{I}$$

E and V of a rectangular waveguide





$$E_y(x,y,z) = \frac{j\omega\mu a}{\pi} A\sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} = Ae_y(x,y)e^{-j\beta z}$$

$$H_x(x,y,z) = \frac{j\beta a}{\pi} A\sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} = Ah_x\,(x,y)e^{-j\beta z}$$

$$V = \int_{-}^{+} \vec{E} \cdot d\vec{l}$$

$$V = \frac{-j\omega\mu a}{\pi} A\sin\left(\frac{\pi x}{a}\right) e^{-j\beta z} \int_{y} dy$$

The concept of Impedance

The different types of impedance

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

Intrinsic Impedance of the medium

$$Z_w = \frac{E_t}{H_t} = \frac{1}{Y_w}$$

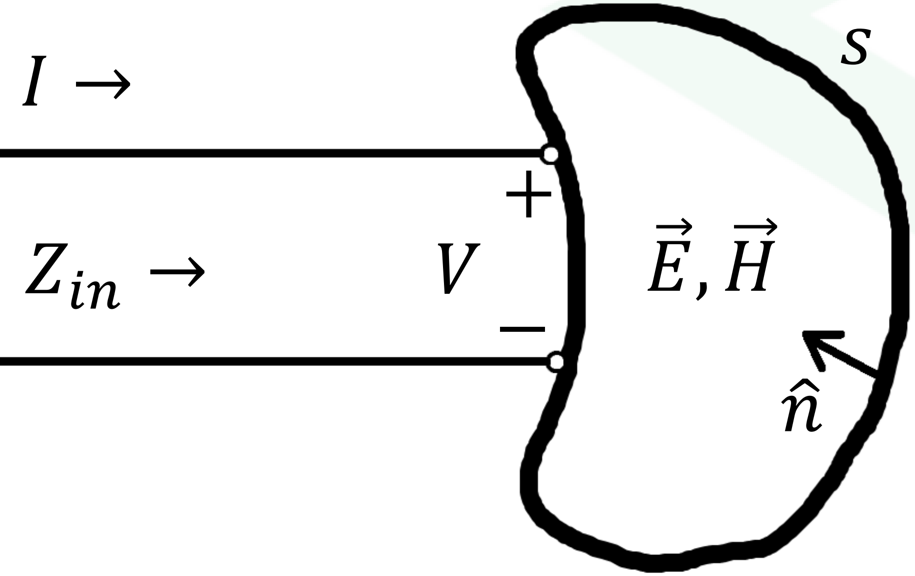
Wave Impedance

$$Z_0 = \frac{1}{Y_0} = \frac{V^+}{I^+}$$

Characteristic Impedance

One-Port Networks

Impedance Properties of One-Port Networks



Recall Poynting's Theorem

$$P_s = \frac{1}{2}\oint\!\!\!\oint_S (\vec{E} \times \vec{H})\bullet d\vec{s} + P_\sigma + 2j\omega(W_m - W_e)$$

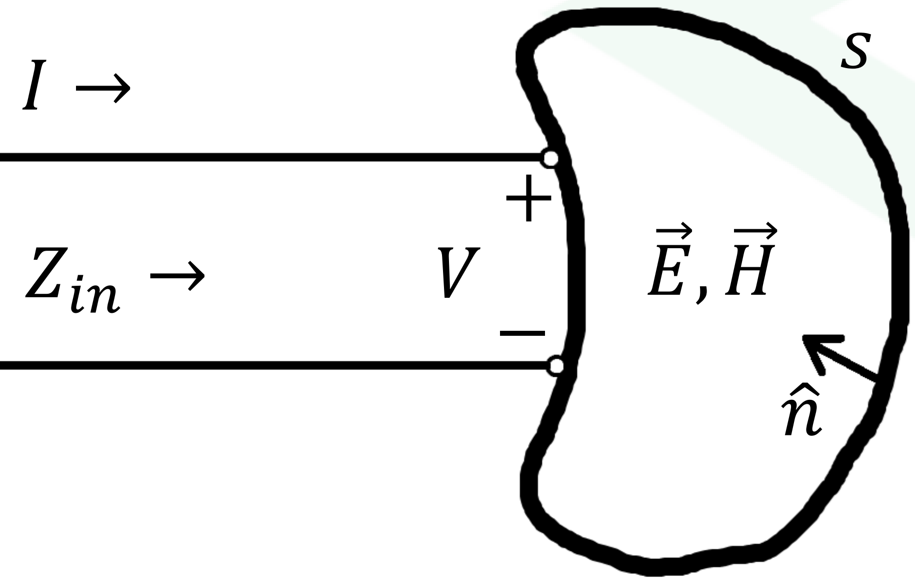
$P_s \equiv$ Power supplied through port

$\frac{1}{2}\oint\!\!\!\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{s} \equiv$ Power transmitted through surface

$P_\sigma \equiv$ Power dissipated by ohmic losses

$2j\omega(W_m - W_e) \equiv$ Total stored energy (magnetic and electric)

Impedance Properties of One-Port Networks



The complex power dissipated (or stored) by a complex load is

$$P(\omega) = V(\omega)I^*(\omega) = [Z(\omega)I(\omega)]I^*(\omega) = Z|I(\omega)^2|$$

We can separate the impedance into resistance, inductance and capacitance as follows:

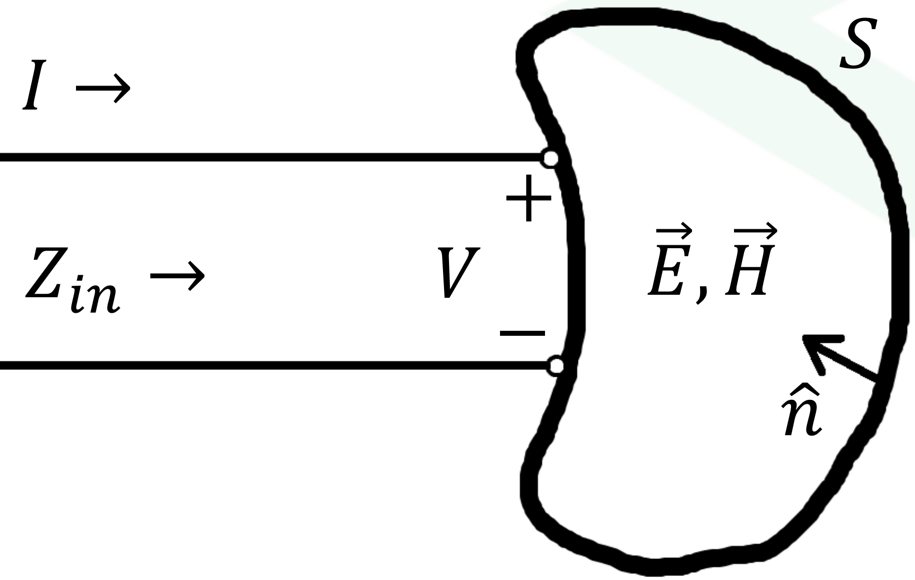
$$Z(\omega) = R + j\omega L - j\frac{1}{\omega C}$$

The power stored in a purely inductive load and purely capacitive load is:

$$P_L(\omega) = j\omega \mathrm{L} \cdot |I(\omega)|^2 \qquad P_C(\omega) = -j\frac{1}{\omega C} \cdot |I(\omega)|^2$$

$$\mathrm{W} = W_m - W_e$$

Wave to Circuit Picture



The field in the terminal can be written as

$$\vec{E}(x,y,z) = V(z)\vec{A}(x,y)e^{-j\beta z}$$
$$\vec{H}(x,y,z) = I(z)\vec{B}(x,y)e^{-j\beta z}$$

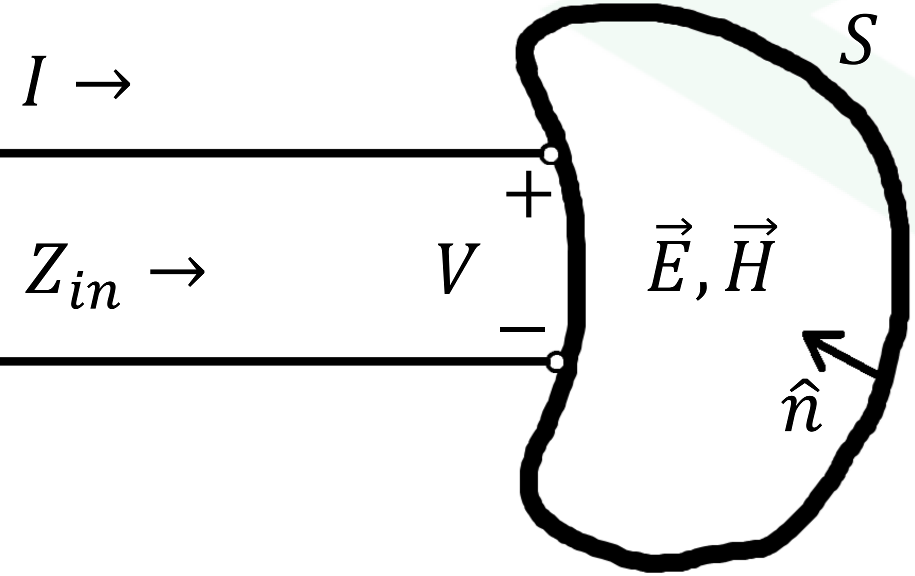
The amplitude functions A and B are normalized such that at the interface

$$\iint_S \left(\vec{A} \times \vec{B}\right) \bullet d\vec{s} = 1$$

In terms of waves, we can substitute for E and H into the integration

$$P(z) = \frac{1}{2} \iint_S \left(\vec{E} \times \vec{H}\right) \bullet d\vec{s}$$

Wave to Circuit Picture



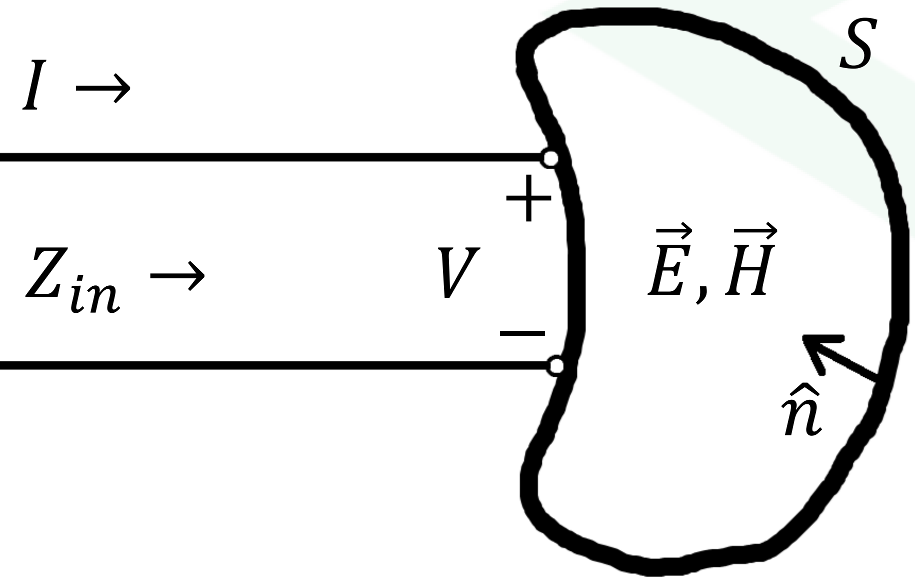
$$P(z) = \iint_S \left(\vec{E} \times \vec{H}\right) \bullet d\vec{s}$$

$$P(z) = \frac{1}{2} \iint_S \left[V(z)\vec{A}(x,y)e^{-j\beta z} \times I^*(z)\vec{B}(x,y)e^{-j\beta z}\right] \bullet d\vec{s}$$

$$= \frac{1}{2} V(z) I^*(z) \iint_S \left[\vec{A}(x,y) \times \vec{B}(x,y)\right] \bullet d\vec{s}$$

$$= \frac{1}{2} V(z) I^*(z)$$

Input Impedance



$Z = R + jX$ (impedance) = (resistance) + j(reactance)

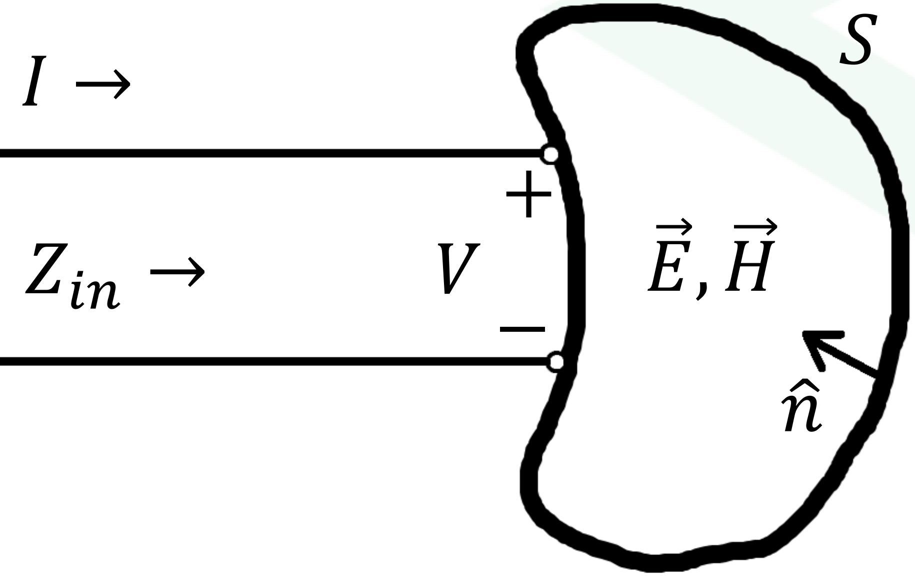
$Y = G + jB$ (admittance) = (conductance) + j(susceptance)

The input impedance is defined as

$$Z_{in} = \frac{V}{I} = \frac{V \cdot I^*}{I \cdot I^*} = \frac{VI^*}{|I|^2} = \frac{2P}{|I|^2} = \frac{2P_\sigma + 4j\omega(W_m - W_e)}{|I|^2}$$

$$Z_{in} = R + jX = \frac{2P_\sigma}{|I|^2} + j\frac{4\omega(W_m - W_e)}{|I|^2}$$

Input Impedance



$$R = \frac{2P_\sigma}{|I|^2}$$

Dissipated power

$$X = \frac{4\omega(W_m - W_e)}{|I|^2}$$

Energy stored

For capacitive loads → $W_m < W_e \rightarrow X < 0$

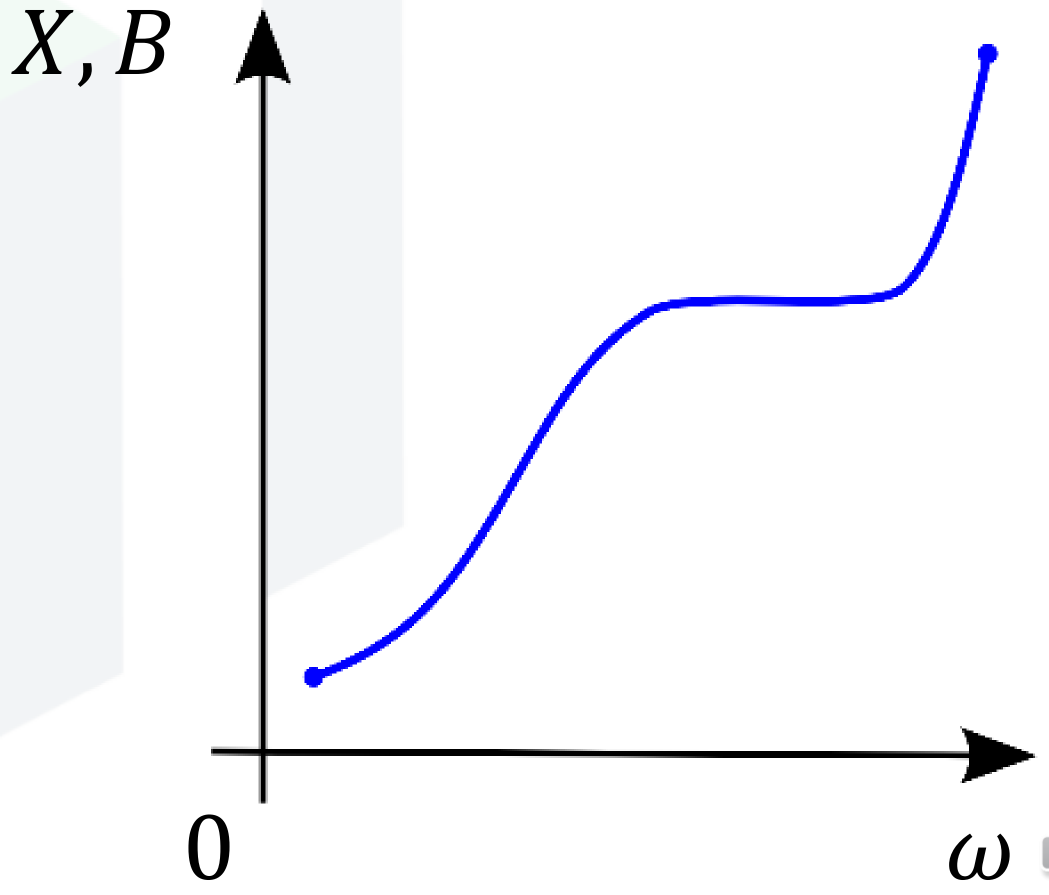
For inductive loads → $W_m > W_e \rightarrow X > 0$

Foster's Reactance Theorem

Foster's Reactance Theorem

The reactance (and susceptance) of a passive and lossless one port network always monotonically increases with frequency

$$\frac{\partial X}{\partial \omega} > 0 \quad \text{and} \quad \frac{\partial B}{\partial \omega} > 0$$



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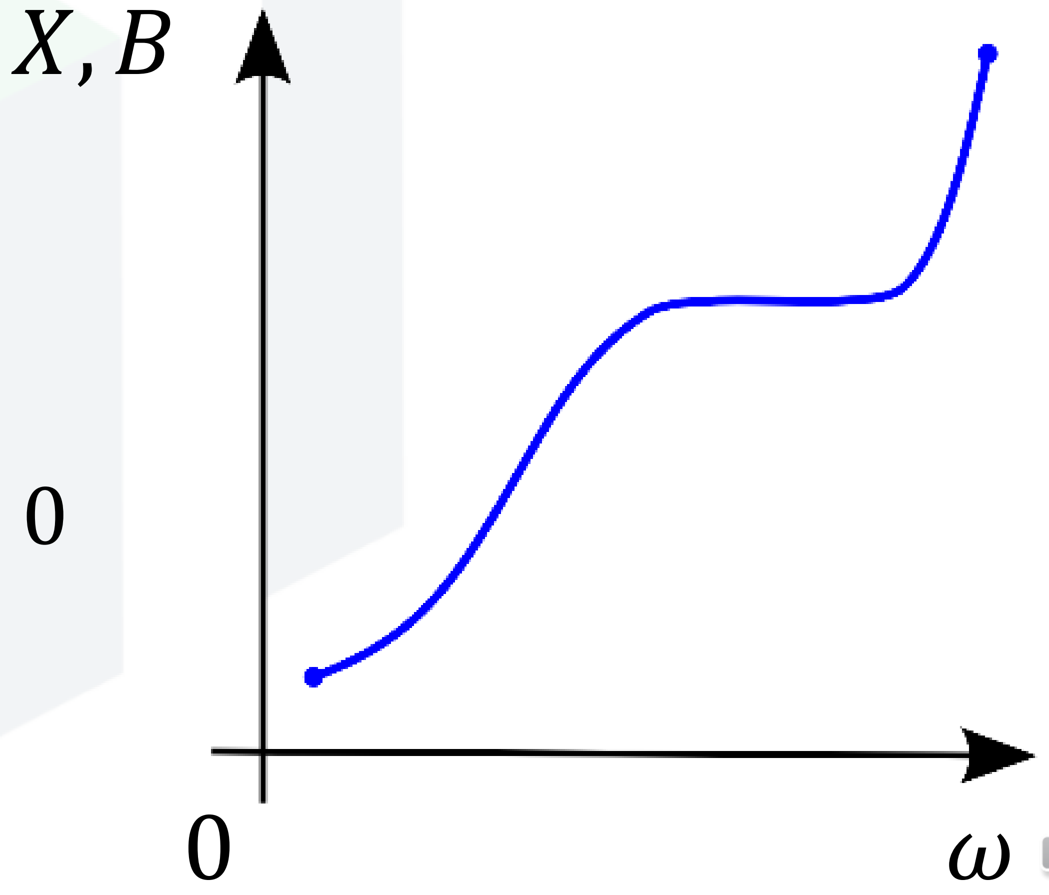
Proof of Theorem

Suppose we have a load with inductance (stored magnetic energy) and capacitance (stored electric energy). The reactance is given by

$$jX(\omega) = j\omega L + \frac{1}{j\omega C}$$

$$X(\omega) = \omega L - \frac{1}{\omega C}$$

$$\frac{\partial X}{\partial \omega} = \frac{\partial}{\partial \omega}\left[\omega L - \frac{1}{\omega C}\right] = L + \frac{1}{\omega^2 C} > 0$$



Properties of $Z(\omega)$ and $\Gamma(\omega)$

Even and odd properties of $Z(\omega)$ and $\Gamma(\omega)$

Consider driving a one-port network with input impedance $Z(\omega)$ which responds with a current $I(\omega)$

$$V(\omega) = Z(\omega)I(\omega)$$

The Fourier Transform of a real-valued function must have Hermitian symmetry. Since the time-domain voltage and current must be real, then V and I have Hermitian symmetry.

$$v(t) = v^*(t),$$

$$\int_{-\infty}^{\infty} V(\omega)e^{j\omega t}d\omega = \int_{-\infty}^{\infty} V^*(\omega)e^{-j\omega t}d\omega = \int_{-\infty}^{\infty} V^*(-\omega)e^{j\omega t}d\omega$$

Even and odd properties of $Z(\omega)$ and $\Gamma(\omega)$

Which means that $V(\omega)$ and $I(\omega)$ satisfy

$$V(-\omega) = V^*(\omega) \qquad\qquad I(-\omega) = I^*(\omega)$$

Applying these to the impedance shows that impedance also must have Hermitian symmetry.

$$Z(\omega) = \frac{V(\omega)}{I(\omega)} \qquad\qquad Z^*(\omega) = \frac{V^*(\omega)}{I^*(\omega)} = \frac{V(-\omega)}{I(-\omega)} = Z(-\omega)$$

Even and odd properties of $Z(\omega)$ and $\Gamma(\omega)$

Now for $R(\omega)$ and $X(\omega)$ we get

$$Z^*(\omega) = Z(-\omega)$$

$$R(\omega) - jX(\omega) = R(-\omega) + jX(-\omega)$$

$R(\omega) = R(-\omega)$	even symmetry
$-X(\omega) = X(-\omega)$	odd symmetry

Even and odd properties of $Z(\omega)$ and $\Gamma(\omega)$

Applying these to the reflection coefficient shows that it also has Hermitian symmetry.

$$\Gamma(\omega) = \frac{Z(\omega) - Z_0}{Z(\omega) + Z_0} = \frac{R(\omega) + jX(\omega) - Z_0}{R(\omega) + jX(\omega) + Z_0}$$

$$\Gamma(-\omega) = \frac{R(-\omega) + jX(-\omega) - Z_0}{R(-\omega) + jX(-\omega) + Z_0} = \frac{R(\omega) - jX(\omega) - Z_0}{R(\omega) - jX(\omega) + Z_0} = \Gamma^*(\omega)$$

$$|\Gamma(\omega)|^2 = \Gamma(\omega)^*\Gamma(\omega) = \Gamma(\omega)\Gamma(-\omega) = |\Gamma(-\omega)|^2$$

Which means that $|\Gamma(\omega)|^2$ and $|\Gamma(\omega)|$ are even functions.