

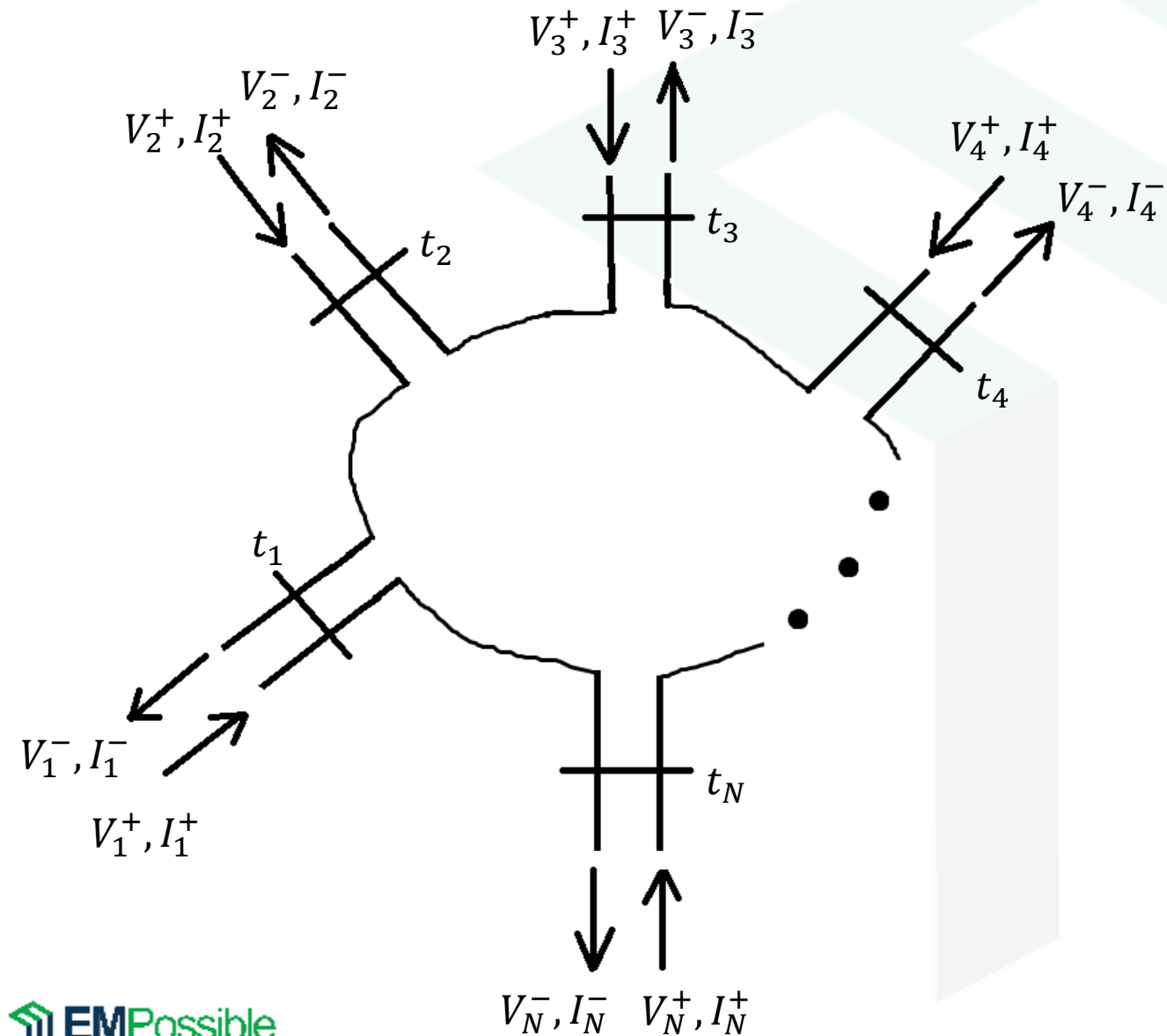


Electromagnetics:
Microwave Engineering

N-Port Networks



N-Port Networks

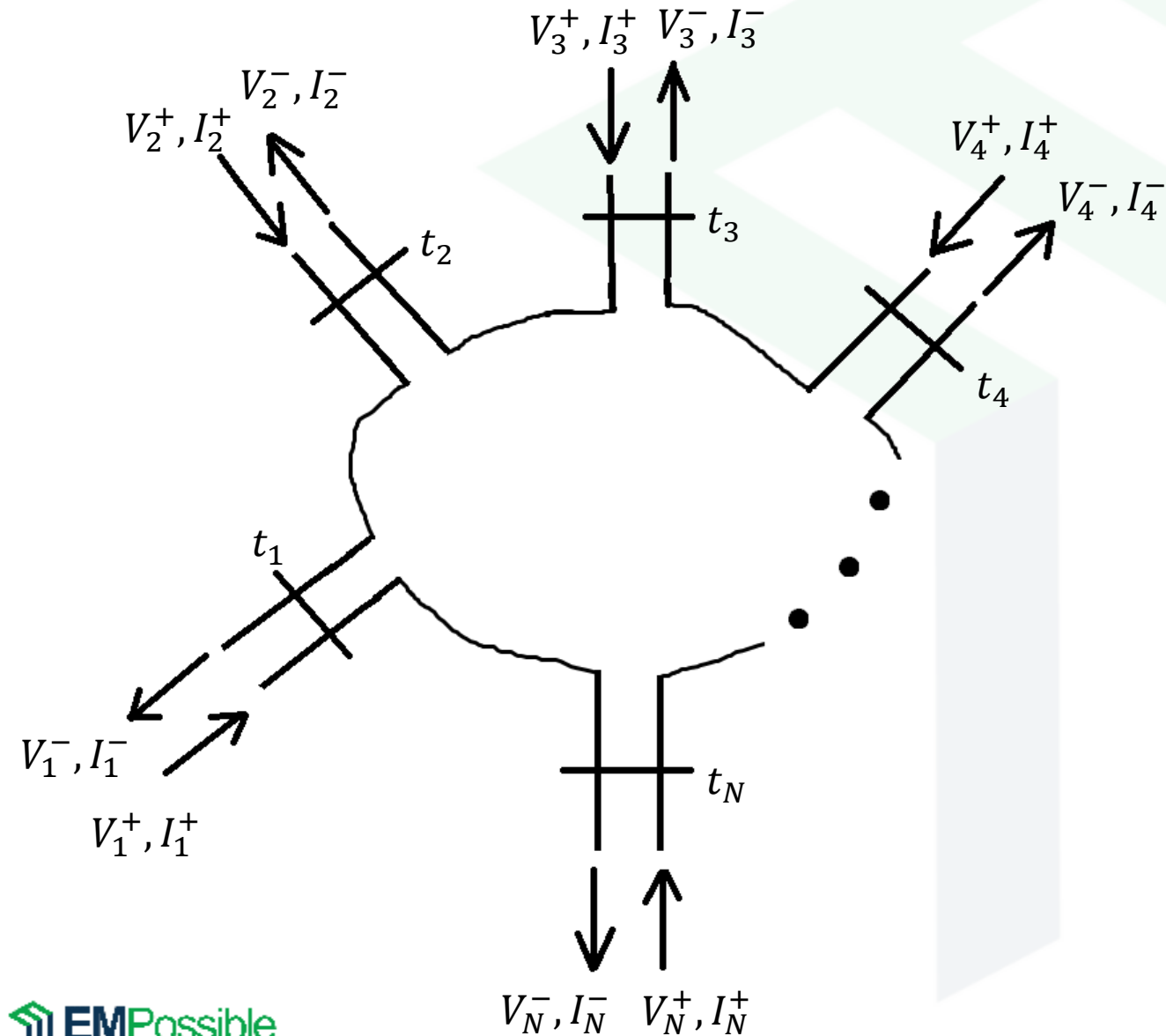


At each port, the total voltage and current is given by

$$V_n = V_n^+ + V_n^-$$

$$I_n = I_n^+ + I_n^-$$

Impedance and Admittance Matrices



V_n and I_n are related through impedance. In fact, all of the voltages and currents in any combination of ports are also related.

$$Z_{ij} = \frac{V_i}{I_j} \text{ when all other currents are zero.}$$

Impedance Matrices

V_n and I_n are related through impedance. In fact, all of the voltages and currents in any combination of ports are also related.

$$\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

$$[V] = [Z][I]$$

$$Z_{ij} = \left. \frac{V_i}{I_j} \right|_{\text{all other currents zero}}$$

Admittance Matrices

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y_{22} & \cdots & Y_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{N1} & Y_{N2} & \cdots & Y_{NN} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

$$[I] = [Y][V]$$

$$Y_{ij} = \left. \frac{I_i}{V_j} \right|_{\text{all other voltages zero}}$$

$$[Y] = [Z]^{-1}$$

$$[Z] = [Y]^{-1}$$



Network Classifications

Reciprocal

$$Z_{ij} = Z_{ji}$$

$$Y_{ij} = Y_{ji}$$

No non-reciprocal media like active devices, ferrites, plasmas, etc.

$$[Z] = Z^T$$

$$[Y] = Y^T$$

Lossless

$$\operatorname{Re}\{Z_{ij}\} = \operatorname{Re}\{Y_{ij}\} = 0$$

Z_{ij} and Y_{ij} are completely imaginary