EMPossible

Electromagnetics:

Microwave Engineering

Two-Port Networks

Lecture Outline

- Two-Port Networks
- Matrix and Circuit Conversions

Two-Port Networks

Two-Port Network Matrices

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

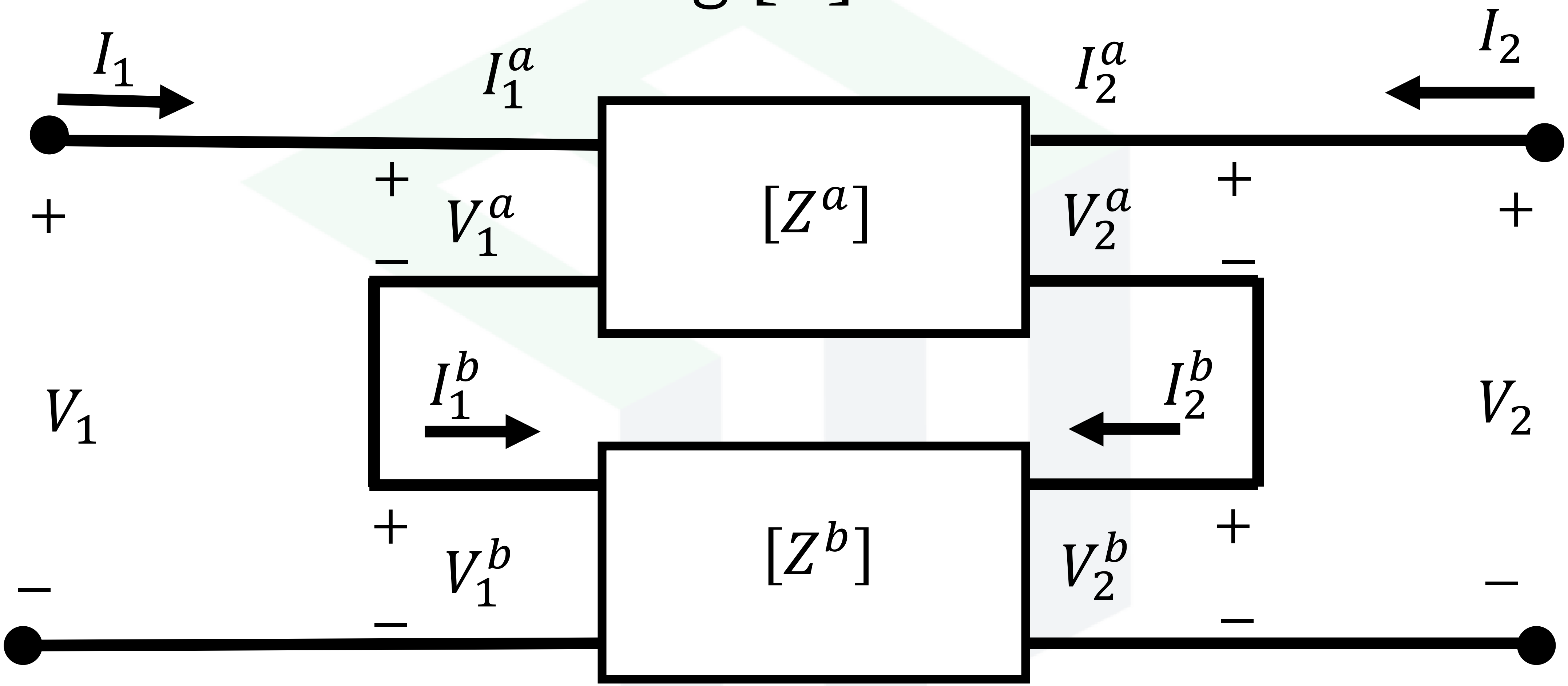
$$[S] = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

$$[T] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

We can convert between any two-port network parameters by using conversion tables.

Cascading [Z] Matrices



$$[Z] = [Z_a] + [Z_b] = \begin{bmatrix} Z_{11}^a + Z_{11}^b & Z_{12}^a + Z_{12}^b \\ Z_{21}^a + Z_{21}^b & Z_{22}^a + Z_{22}^b \end{bmatrix}$$

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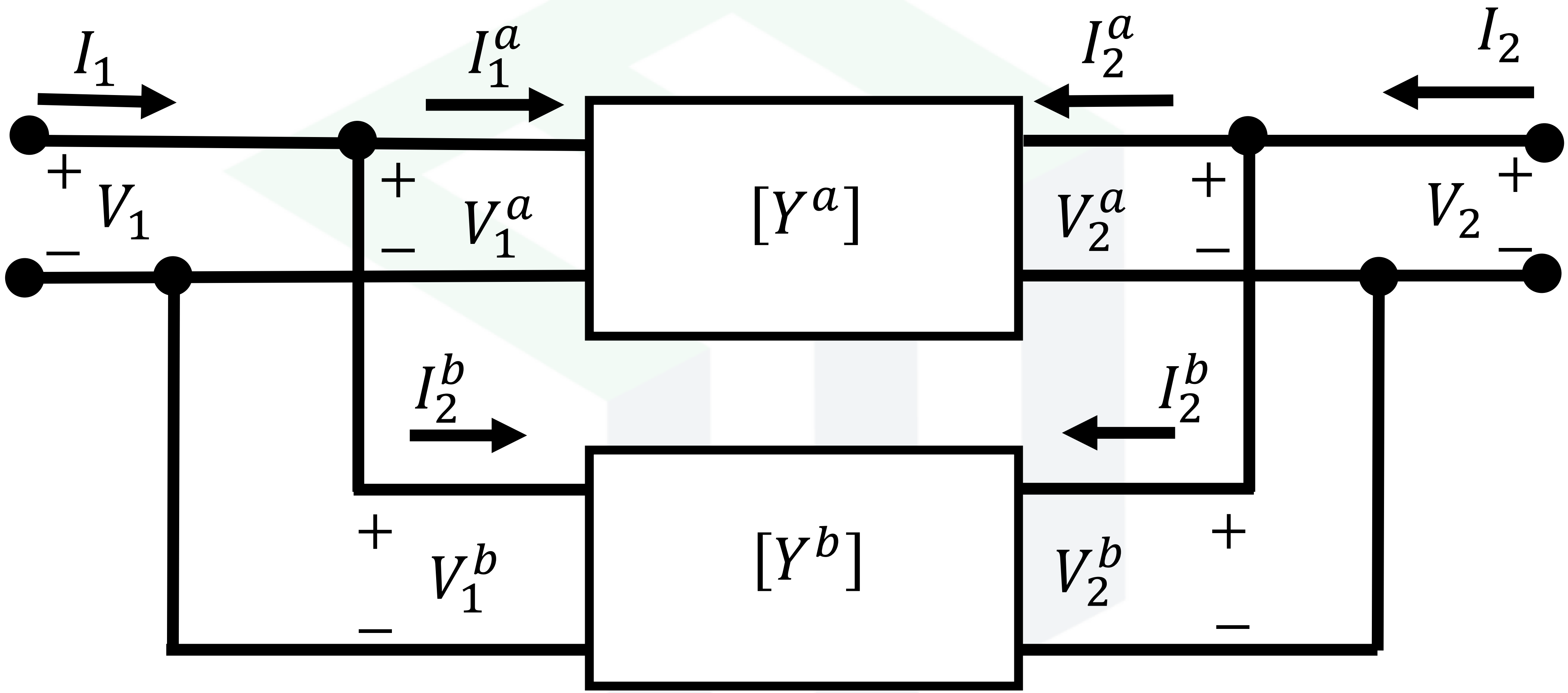
Cascading $[Y]$ Matrices



$$[Y] = [Y_a] + [Y_b] = \begin{bmatrix} Y_{11}^a + Y_{11}^b & Y_{12}^a + Y_{12}^b \\ Y_{21}^a + Y_{21}^b & Y_{22}^a + Y_{22}^b \end{bmatrix}$$

Cascading $[S]$ Matrices

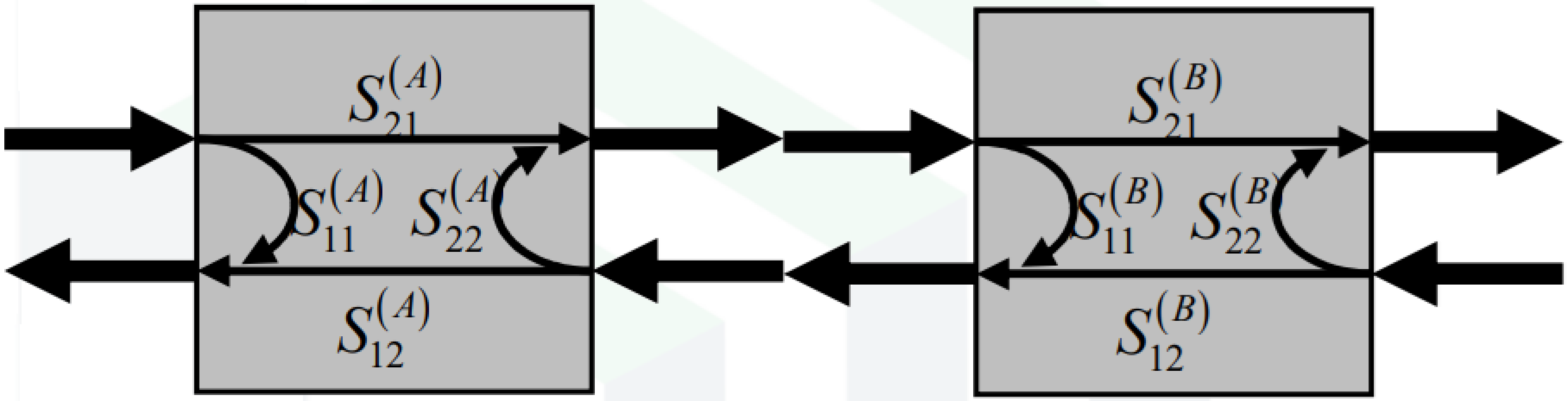
$$\begin{bmatrix} V_1^- \\ V_2^+ \end{bmatrix} = \begin{bmatrix} S_{11}^a & S_{12}^a \\ S_{21}^a & S_{22}^a \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^- \end{bmatrix}$$

$$\begin{bmatrix} V_2^- \\ V_3^+ \end{bmatrix} = \begin{bmatrix} S_{11}^b & S_{12}^b \\ S_{21}^b & S_{22}^b \end{bmatrix} \begin{bmatrix} V_2^+ \\ V_3^- \end{bmatrix}$$

↓

$$\begin{bmatrix} V_1^- \\ V_3^+ \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_3^- \end{bmatrix}$$

Redheffer Star Product



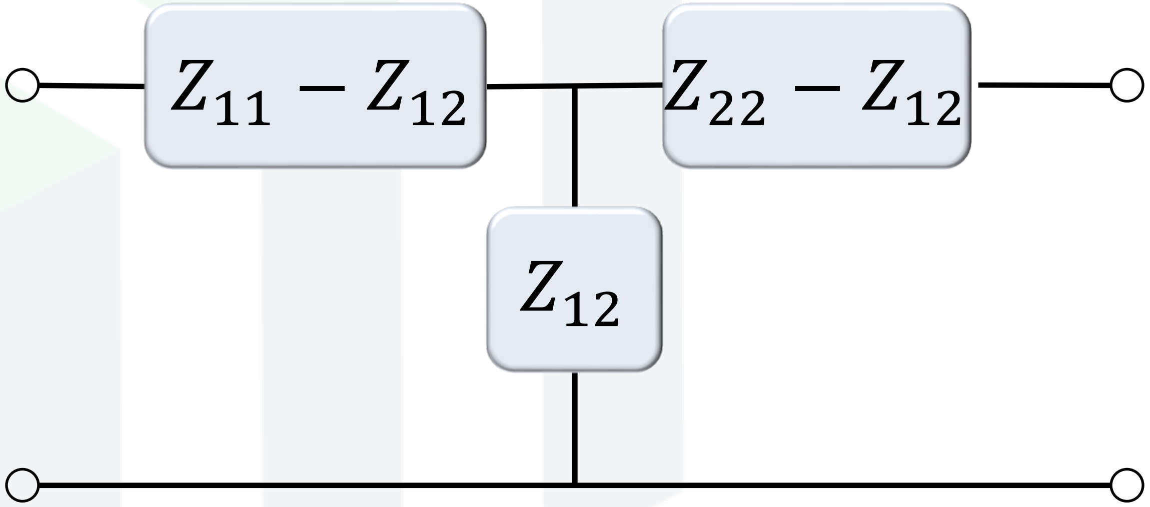
$$[S] = [S_a] \otimes [S_b] = \frac{1}{1 - S_{22}^a S_{11}^b} \begin{bmatrix} S_{11}^a - S_{11}^b \det[S_a] & S_{12}^a S_{12}^b \\ S_{21}^a S_{21}^b & S_{22}^b - S_{22}^a \det[S_b] \end{bmatrix}$$

Matrix and Circuit Conversions

Matrix-to-Circuit Conversion

For reciprocal networks, we have

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$



Matrix to Circuit Conversion

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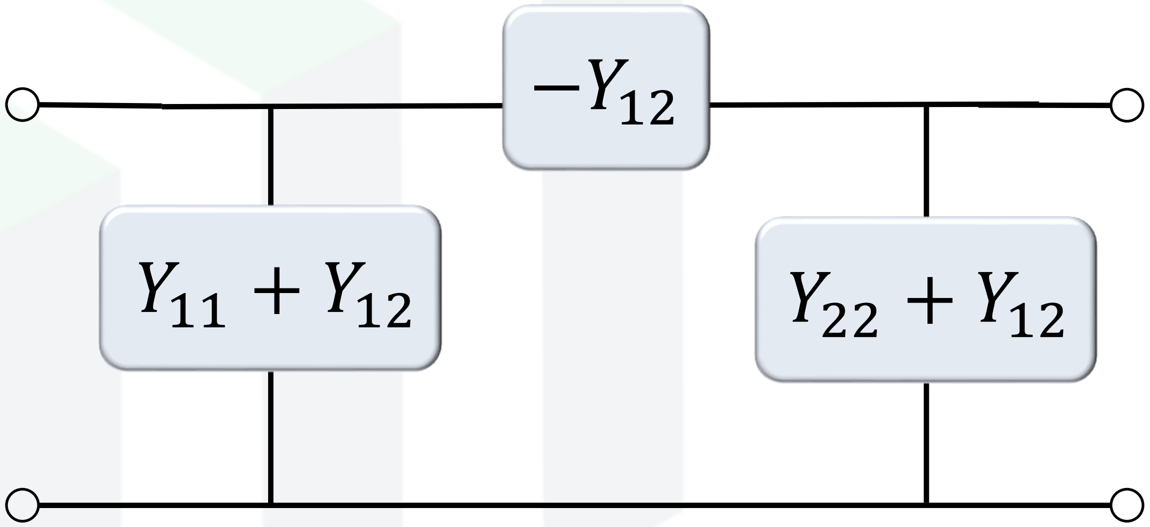
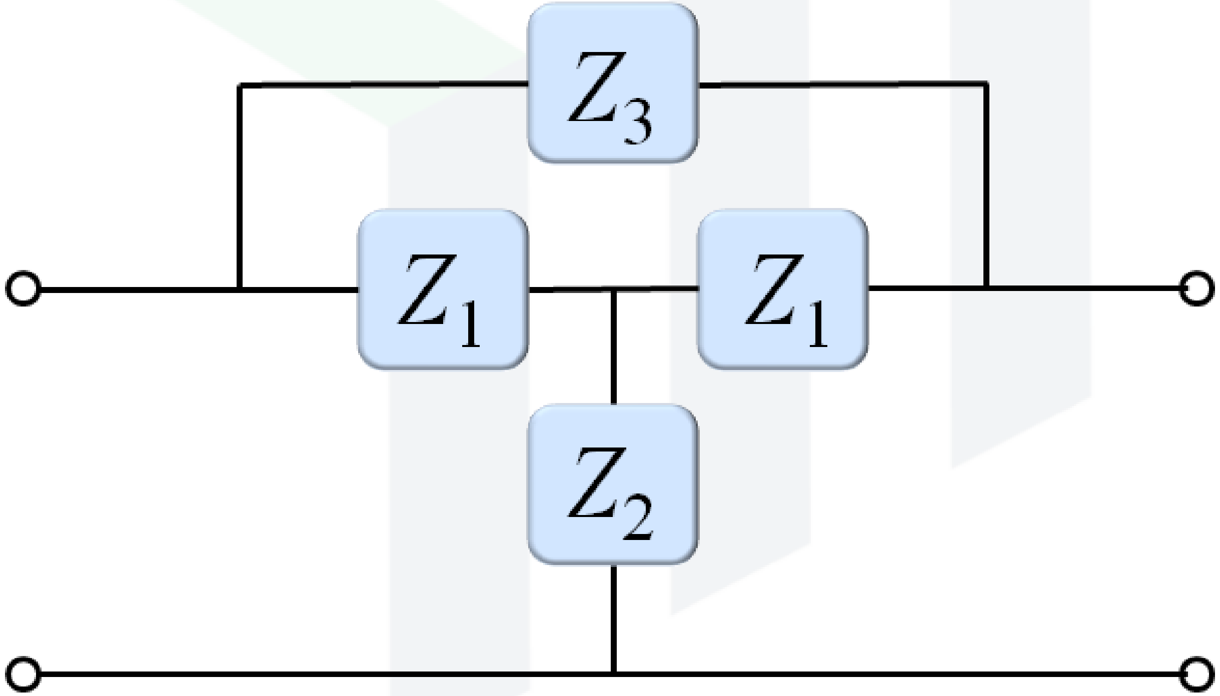
$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$



Circuit to Matrix Conversion

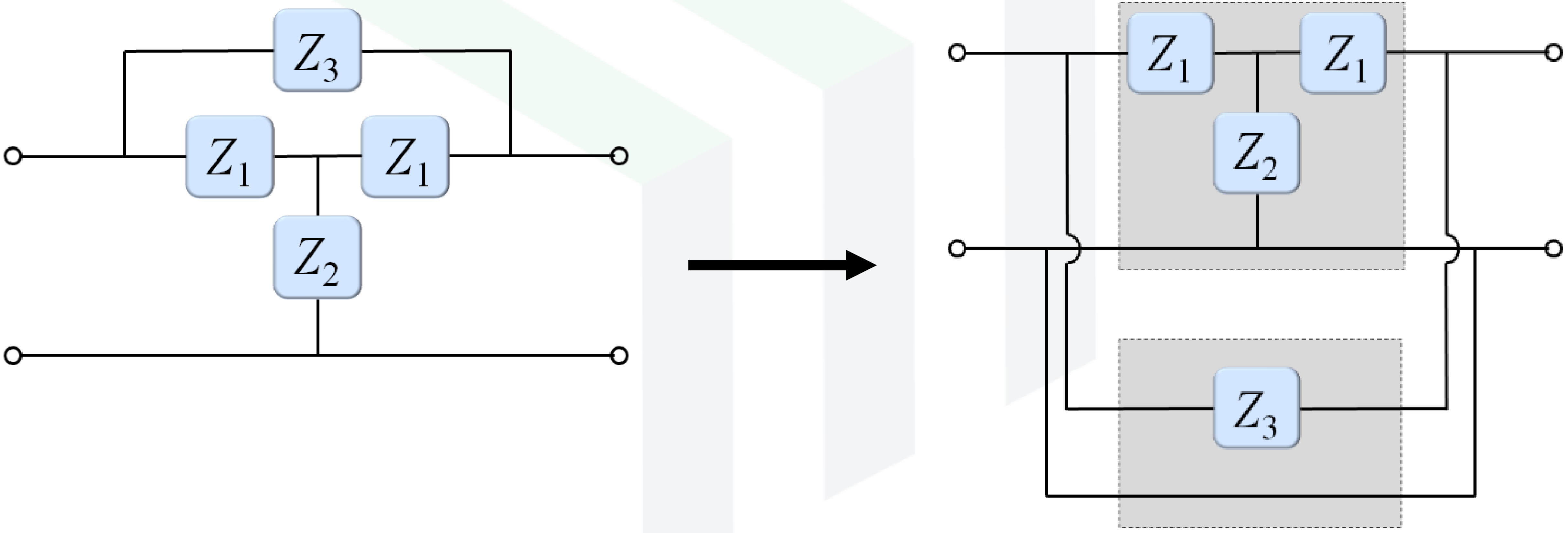
To convert from circuit to matrix, the trick is to rearrange the circuit to get combinations of known configurations.

<u>Example:</u> What is $[Y]$?



Circuit to Matrix Conversion

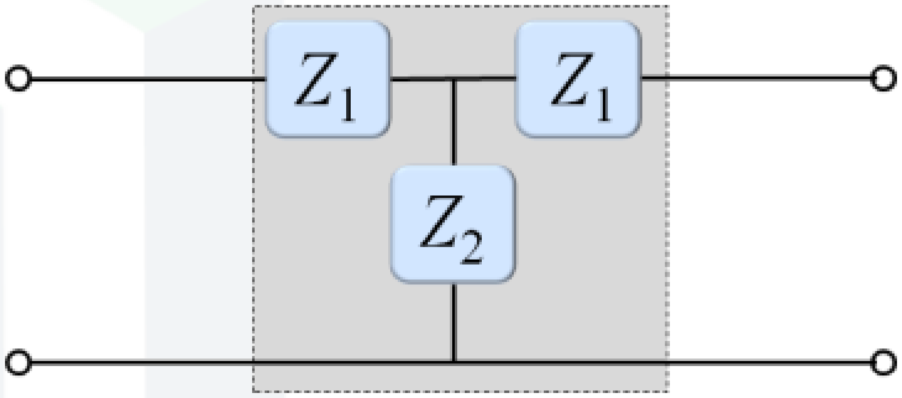
We can view this as a parallel connection of two circuits.



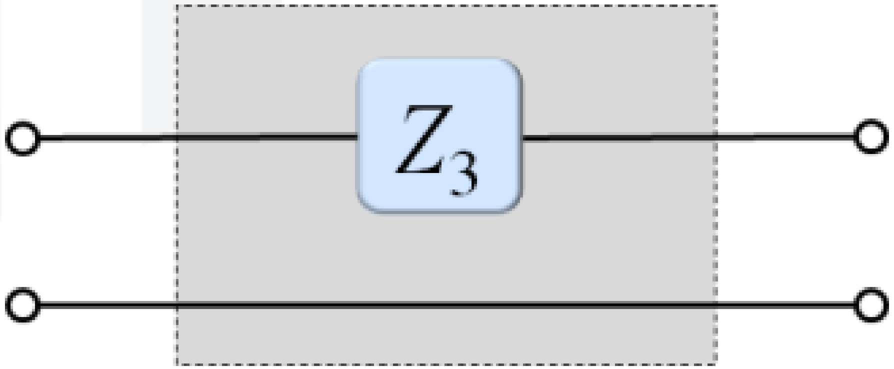
Circuit to Matrix Conversion

We can analyze each circuit separately.

$$[Z_a] = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_1 + Z_2 \end{bmatrix}$$



$$[Y_b] = \begin{bmatrix} 1/Z_3 & -1/Z_3 \\ -1/Z_3 & 1/Z_3 \end{bmatrix}$$



Circuit to Matrix Conversion

A parallel connection is done in terms of the admittance $[Y]$ so the first network is inverted.

$$[Y] = [Z_a]^{-1} + [Y_b] = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_1 + Z_2 \end{bmatrix}^{-1} + \begin{bmatrix} 1/Z_3 & -1/Z_3 \\ -1/Z_3 & 1/Z_3 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{Z_1 + Z_2}{Z_1(Z_1 + 2Z_2)} & \dfrac{-Z_2}{Z_1(Z_1 + 2Z_2)} \\ \dfrac{-Z_2}{Z_1(Z_1 + 2Z_2)} & \dfrac{Z_1 + Z_2}{Z_1(Z_1 + 2Z_2)} \end{bmatrix}^{-1} + \begin{bmatrix} 1/Z_3 & -1/Z_3 \\ -1/Z_3 & 1/Z_3 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{1}{Z_3} + \dfrac{Z_1 + Z_2}{Z_1(Z_1 + 2Z_2)} & -\dfrac{1}{Z_3} - \dfrac{-Z_2}{Z_1(Z_1 + 2Z_2)} \\ -\dfrac{1}{Z_3} - \dfrac{-Z_2}{Z_1(Z_1 + 2Z_2)} & \dfrac{1}{Z_3} + \dfrac{Z_1 + Z_2}{Z_1(Z_1 + 2Z_2)} \end{bmatrix}$$