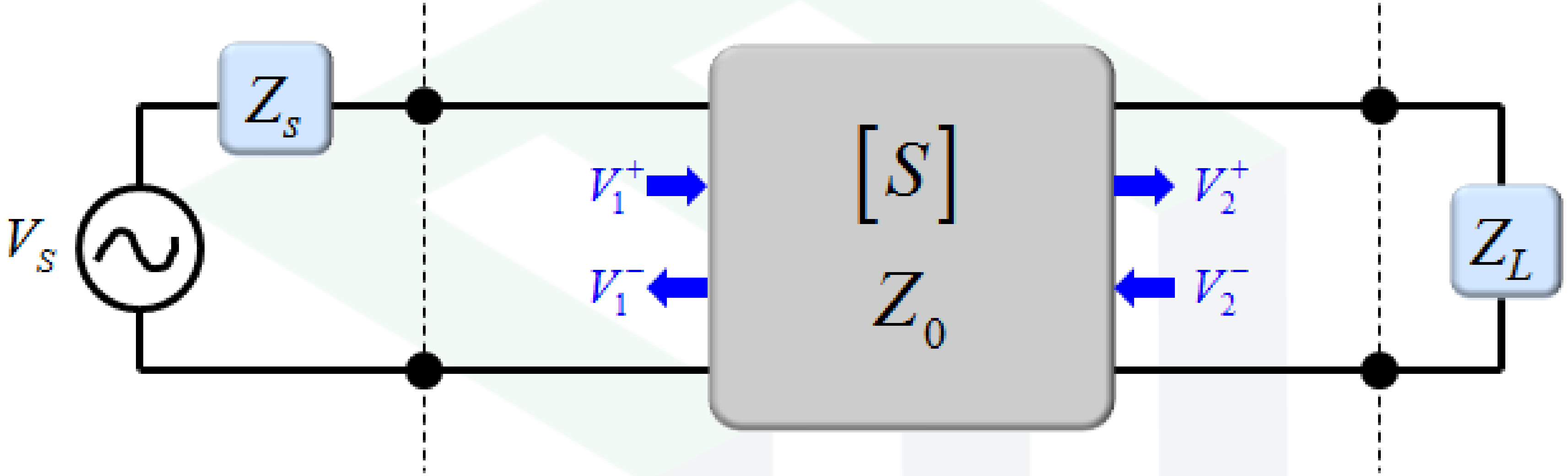
Electromagnetics:

Microwave Engineering

Power in Two-Port Networks

Local Parameters



The impedances of the source and load can be written as

$$Z_s = R_s + jX_s$$
$$Z_L = R_L + jX_L$$

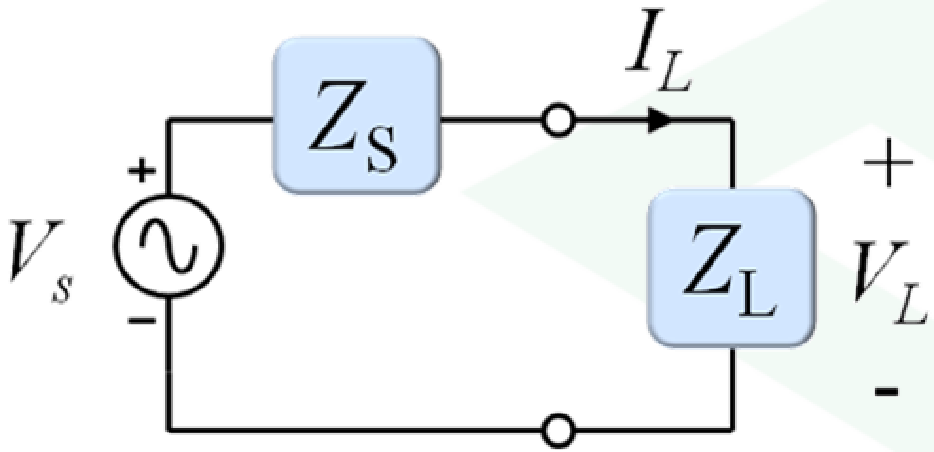
The reflection coefficient seen from the network to the source is

$$\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0} \quad \text{(local reflection)}$$

The reflection coefficient seen from the network to the load is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \text{(local reflection)}$$

Available Power and Impedance Matching



This is the case when the two-port network between the source and load is nothing.

The current and voltage across the load are

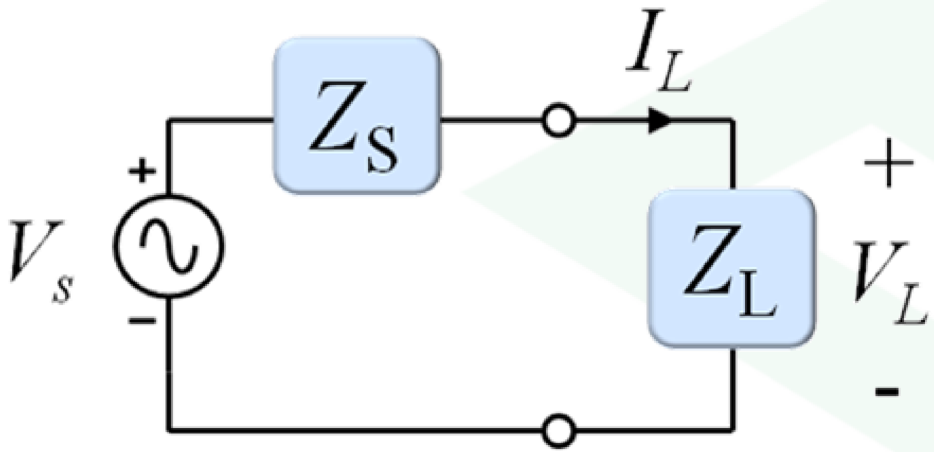
$$I_L = \frac{V_s}{Z_s + Z_L} \qquad V_L = \frac{V_s Z_L}{Z_s + Z_L}$$

The RMS power delivered to the load is then

$$P_L = \frac{1}{2}\text{Re}\left[V_L I_L^*\right] = \frac{1}{2}\text{Re}\left[\frac{V_s Z_L}{Z_s + Z_L}\frac{V_s^*}{(Z_s + Z_L)^*}\right] = \frac{|V_s|^2}{2}\frac{R_L}{|Z_s + Z_L|^2}$$

$$= \frac{|V_s|^2}{2}\frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

Available Power and Impedance Matching



P_L is maximized when $X_S = -X_L$

Then the total available power to the load is

$$P_A = \frac{|V_s|^2}{2} \frac{R_L}{(R_s + R_L)^2}$$

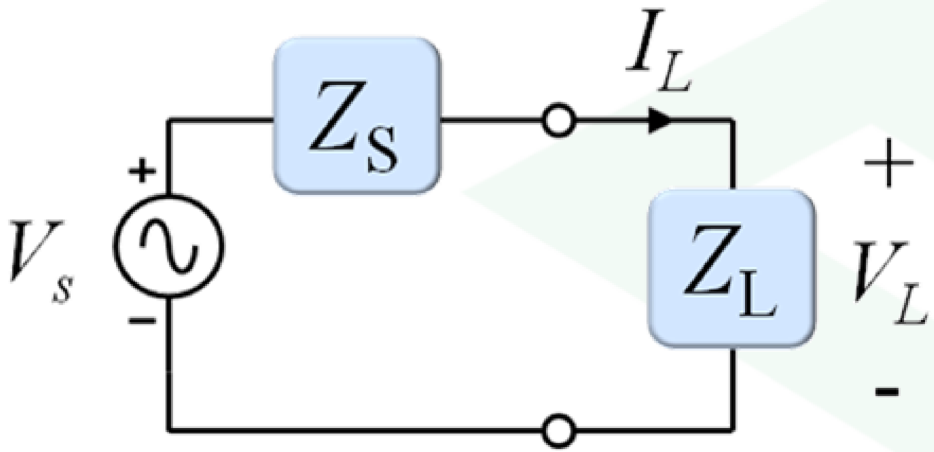
This function has a maximum when

$$\frac{\partial P_A}{\partial R_L} = 0 = \frac{|V_s|^2}{2} \frac{(R_s + R_L)^2 - 2R_L(R_s + R_L)}{(R_s + R_L)^4}$$

$$0 = (R_s + R_L)^2 - 2R_L(R_s + R_L)$$

$$R_L = R_s$$

Available Power and Impedance Matching



This is called conjugate matching.

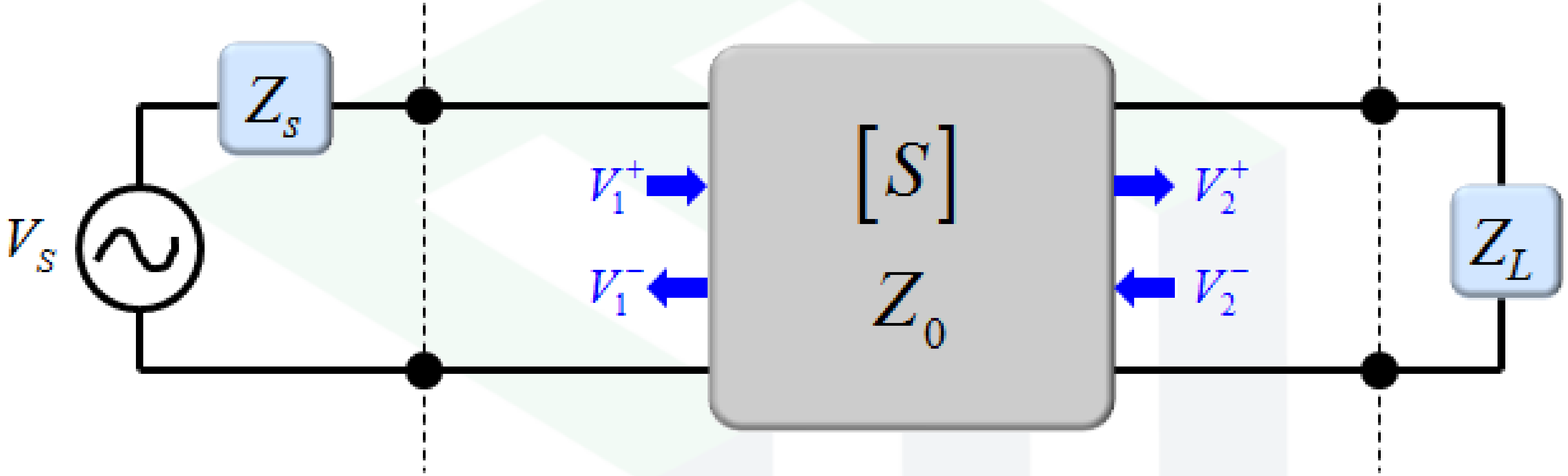
$$Z_L = Z_S^*$$

For maximum power to the load

When $R_L = R_S$, the equation for maximum power reduces to

$$P_A = \frac{|V_S|^2}{8R_S} \qquad \text{when } Z_L = Z_S^*$$

Global Parameters



The input impedance observed by the source into the network/load is

$$Z_{in} = \frac{AZ_L + B}{CZ_L + D}$$

The reflection experienced from the source into the network/load is

$$\Gamma_{in} = \frac{A + BZ_0^{-1} - CZ_0 - D}{A + BZ_0^{-1} + CZ_0 + D} \quad \text{or} \quad \Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

Global Parameters

The reflection coefficient from the network/source to the load is

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$$

The average power delivered by the source is

$$P_{in} = \frac{|V_S|^2}{8Z_0} \frac{|1 - \Gamma_s|^2}{|1 - \Gamma_s\Gamma_{in}|^2}\left(1 - |\Gamma_{in}|^2\right)$$

The average power delivered to the load is

$$P_L = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2\left(1 - |\Gamma_L|^2\right)|1 - \Gamma_s|^2}{|1 - S_{22}\Gamma_L|^2|1 - \Gamma_s\Gamma_{in}|^2}$$

Global Parameters

The maximum possible power P_A that the source can deliver to the load is when the source impedance is conjugate matched to the input of the network.

$$P_A = \frac{|V_s|^2}{8Z_0} \frac{|1 - \Gamma_s|^2}{1 - |\Gamma_s|^2} \qquad P_s \leq P_A$$

If conjugate matched,

$$P_A = \frac{|V_s|^2}{8R_s}$$

The maximum possible power that can be delivered to the load is

$$P_{L,max} = \frac{|V_S|^2}{8Z_0} \frac{|S_{21}|^2 |1 - \Gamma_s|^2}{|1 - S_{22}\Gamma_L|^2 (1 - |\Gamma_{out}|^2)} \qquad P_L \leq P_{L,max}$$