Electromagnetics: 
Microwave Engineering

Network Performance Metrics
Network Performance Metrics

Performance without network

Performance with network
Insertion Loss

Quantifies how much less power is delivered to the load due to inserting the network between the source and load.

\[ L_I = 10 \log_{10} \left( \frac{P_L \text{ w/o network}}{P_L \text{ w/ network}} \right) = 10 \log_{10} \left| \frac{AZ_L + B + CZ_S Z_L + DZ_S}{Z_S + Z_L} \right|^2 \]

We can invert this to calculate insertion gain

\[ L_I = 10 \log_{10} \left( \frac{P_L \text{ w/ network}}{P_L \text{ w/o network}} \right) = 10 \log_{10} \left| \frac{Z_S + Z_L}{AZ_L + B + CZ_S Z_L + DZ_S} \right|^2 \] (\text{~relative to reality})
Insertion Phase

Quantifies how much the phase changes at the load when the network is inserted between the source and the load

\[ \theta_l = \theta_{after} - \theta_{before} = \tan^{-1}\left(\frac{\text{Im}[AZ_L + B + CZ_s Z_L + DZ_s]}{\text{Re}[AZ_L + B + CZ_s Z_L + DZ_s]}\right) \]

(~relative to reality)
Transducer Loss

The network (or transducer) can perform impedance matching and actually increase the power delivered to the load. Despite this, the transducer probably also dissipates energy and therefore is responsible for some loss.

\[ L_T = 10 \log_{10} \left( \frac{P_A}{P_L} \right) = 10 \log_{10} \left( \frac{|AZ_L + B + CZ_S Z_L + DZ_s|^2}{4R_s R_L} \right) \]  \( (~\text{relative to ideal}) \)

We can invert this to calculate transducer gain.

\[ G_T = 10 \log_{10} \left( \frac{P_L}{P_A} \right) = 10 \log_{10} \left( \frac{4R_s R_L}{|AZ_L + B + CZ_S Z_L + DZ_s|^2} \right) \]

\[ G_T = \frac{P_L}{P_{s,max}} = \frac{|S_{21}|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - S_{22} \Gamma_L^2||1 - \Gamma_S \Gamma_{in}|^2} \]
Transducer Loss

Note that $L_I = L_T$ when $Z_s = Z_L$.

The actual power gain through the network is

$$G = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2 (1 - |\Gamma_s|^2)(1 - |\Gamma_L|^2)}{|1 - S_{22}\Gamma_L|^2 |1 - \Gamma_s\Gamma_{in}|^2}$$

$G \leq 1$ for passive networks

The maximum gain available from the network is then

$$G_A = \frac{P_{L,max}}{P_{s,max}} = \frac{|S_{21}|^2 (1 - |\Gamma_s|^2)}{|1 - S_{11}\Gamma_s|^2 (1 - |\Gamma_{out}|)^2}$$
Mismatch Loss

Mismatch loss is how much less power is delivered to the load due to reflection.

\[ L_M = 10 \log_{10} \left( \frac{1}{1 - |\Gamma_{in}|^2} \right) \]
Dissipative Loss

The overall loss from source to load is the transducer loss. The power loss either reflects back to the source \( (L_M) \) or is dissipated as heat inside the network or transducer \( (L_\sigma) \)

\[
L_T = L_M + L_\sigma
\]
Special Case: Conjugate Matched System

When both the input and output ports are matched, we have

\[ \Gamma_s = \Gamma_L = 0 \]  
(local reflection)

\[ \Gamma_{in} = S_{11} \]  
(global reflection)

\[ \Gamma_{out} = S_{22} \]  
(global reflection)

\[ P_A = \frac{|V_s|^2}{8Z_0} \]

\[ P_{in} = \frac{|V_s|^2}{8Z_0} (1 - |S_{11}|^2) \]

\[ P_L = \frac{|V_s|^2}{8Z_0} |S_{21}|^2 \]

\[ G = \frac{|S_{21}|^2}{1 - |S_{11}|^2} \]

\[ P_{L,\text{max}} = \frac{|V_s|^2}{8Z_0} \frac{|S_{21}|^2}{1 - |S_{11}|^2} \]

\[ G_T = |S_{21}|^2 \]