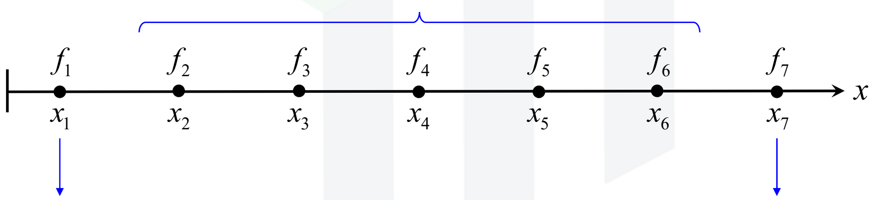
Computational Science:
Computational Methods in Engineering

# Boundary Conditions

## High-Order Boundary Conditions

Here, the derivative at the boundaries is calculated using special finite-difference equations derived specifically for just these points.

$$\frac{d^2 f_i}{dx^2} \cong \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2}$$



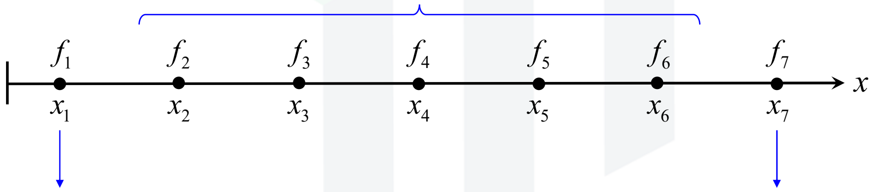

$$\frac{d^2 f_1}{dx^2} \cong \frac{2f_1 - 5f_2 + 4f_3 - f_4}{h^2}$$

$$\frac{d^2 f_7}{dx^2} \cong \frac{-f_4 + 4f_5 - 5f_6 + 2f_7}{h^2}$$

# Dirichlet Boundary Conditions

The simplest boundary condition is to assume all function values outside of the grid are zero.

$$\frac{d^2 f_i}{dx^2} \cong \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2}$$




$$\frac{d^2 f_1}{dx^2} \cong \frac{0 - 2f_1 + f_2}{h^2} \qquad \frac{d^2 f_7}{dx^2} \cong \frac{f_6 - 2f_7 + 0}{h^2}$$

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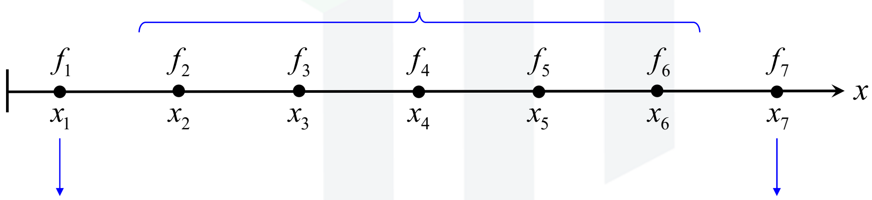
# Periodic Boundary Conditions

If the problem is periodic (i.e. keeps repeating), then the value outside of the grid is the same as the value at the opposite side of the grid.

$$\frac{d^2 f_i}{dx^2} \cong \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2}$$




$$\frac{d^2 f_1}{dx^2} \cong \frac{f_7 - 2f_1 + f_2}{h^2} \qquad \frac{d^2 f_7}{dx^2} \cong \frac{f_6 - 2f_7 + f_1}{h^2}$$

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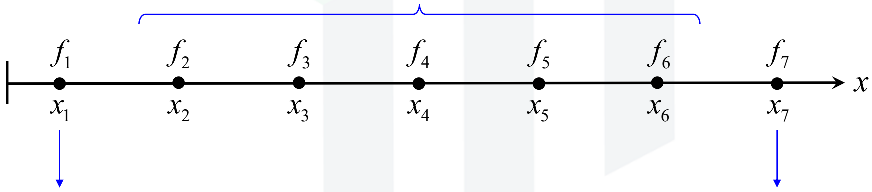
# Neuman Boundary Conditions

The Neuman boundary condition allows functions to continue linearly off of the grid as if to infinity.

$$\frac{df_i}{dx} \cong \frac{f_{i+1} - f_{i-1}}{2h} \qquad \frac{d^2 f_i}{dx^2} \cong \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2}$$




$$\frac{df_1}{dx} \cong \frac{f_2 - f_1}{h} \qquad \frac{d^2 f_1}{dx^2} \cong 0$$

$$\frac{df_7}{dx} \cong \frac{f_7 - f_6}{h} \qquad \frac{d^2 f_7}{dx^2} \cong 0$$

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