



Computational Science:
Computational Methods in Engineering

Boundary Conditions



High-Order Boundary Conditions

Here, the derivative at the boundaries is calculated using special finite-difference equations derived specifically for just these points.

$$\frac{d^2 f_i}{dx^2} \cong \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2}$$

The diagram shows a horizontal axis labeled x with seven discrete points marked as x_1 through x_7 . Above each point is a corresponding function value f_1 through f_7 . A blue bracket spans from x_2 to x_6 , indicating the range of the standard second-order finite-difference stencil. Below the axis, two special high-order finite-difference formulas are provided for the boundary points x_1 and x_7 , with blue arrows pointing from the f_1 and f_7 labels to their respective formulas.

$$\frac{d^2 f_1}{dx^2} \cong \frac{2f_1 - 5f_2 + 4f_3 - f_4}{h^2}$$

$$\frac{d^2 f_7}{dx^2} \cong \frac{-f_4 + 4f_5 - 5f_6 + 2f_7}{h^2}$$



Dirichlet Boundary Conditions

The simplest boundary condition is to assume all function values outside of the grid are zero.

$$\frac{d^2 f_i}{dx^2} \cong \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2}$$

Diagram illustrating Dirichlet Boundary Conditions. A horizontal axis x shows nodes x_1 through x_7 with corresponding function values f_1 through f_7 . The second derivative formula is shown for interior nodes: $\frac{d^2 f_i}{dx^2} \cong \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2}$. At the boundaries, the values outside the grid are assumed to be zero:

$$\frac{d^2 f_1}{dx^2} \cong \frac{0 - 2f_1 + f_2}{h^2}$$

$$\frac{d^2 f_7}{dx^2} \cong \frac{f_6 - 2f_7 + 0}{h^2}$$

Periodic Boundary Conditions

If the problem is periodic (i.e. keeps repeating), then the value outside of the grid is the same as the value at the opposite side of the grid.

$$\frac{d^2 f_i}{dx^2} \cong \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2}$$

Diagram illustrating Periodic Boundary Conditions. A horizontal axis x shows nodes x_1 through x_7 with corresponding function values f_1 through f_7 . The second derivative formula is shown for interior nodes: $\frac{d^2 f_i}{dx^2} \cong \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2}$. At the boundaries, the values outside the grid are assumed to be the same as the value at the opposite side of the grid:

$$\frac{d^2 f_1}{dx^2} \cong \frac{f_7 - 2f_1 + f_2}{h^2}$$

$$\frac{d^2 f_7}{dx^2} \cong \frac{f_6 - 2f_7 + f_1}{h^2}$$

Neuman Boundary Conditions

The Neuman boundary condition allows functions to continue linearly off of the grid as if to infinity.

$$\frac{df_i}{dx} \cong \frac{f_{i+1} - f_{i-1}}{2h} \quad \frac{d^2 f_i}{dx^2} \cong \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2}$$

The diagram shows a horizontal axis labeled x with seven nodes marked by dots and labeled x_1 through x_7 . Above each node is a function value f_1 through f_7 . A blue bracket above the grid spans from x_2 to x_6 , indicating the stencil for the central difference formulas. Below the grid, two sets of derivative formulas are shown, connected to the first and last nodes by blue arrows:

$$\frac{df_1}{dx} \cong \frac{f_2 - f_1}{h} \quad \frac{d^2 f_1}{dx^2} \cong 0$$

$$\frac{df_7}{dx} \cong \frac{f_7 - f_6}{h} \quad \frac{d^2 f_7}{dx^2} \cong 0$$