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Computational Science:
Computational Methods in Engineering

Convergence of Numerical Integration

What is Convergence?

Convergence is the tendency of a numerical algorithm to approach a specific value as the resolution of the algorithm is increased.

This does NOT imply the answer gets more correct.

There may still be something wrong with your calculation!

Demonstration of Convergence

Suppose it is desired to evaluate the following integral with discrete integration:

$$\int_0^{\pi} \sin x dx$$

How many rectangles are necessary?

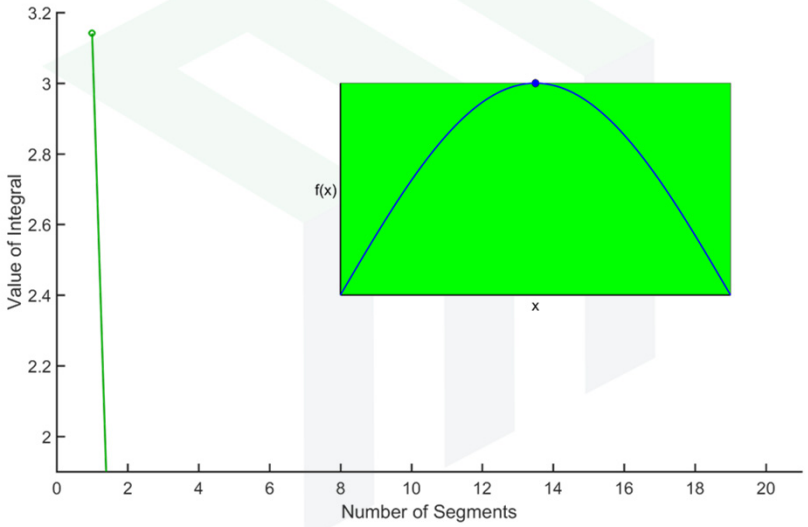
A convergence study must be performed!

Analytical Answer

To check the numerical answer, solve the integral analytically…

$$\begin{aligned}\int_0^{\pi} \sin x dx &= -\cos x\Big|_0^{\pi} \\ &= (-\cos\pi) - (-\cos 0) \\ &= 2\end{aligned}$$

Convergence Study



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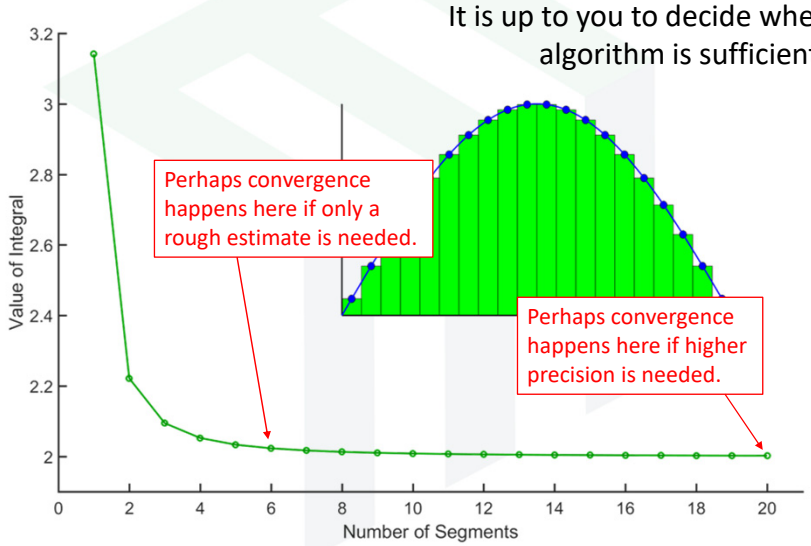
Convergence Study

It is up to you to decide when a numerical algorithm is sufficiently converged.



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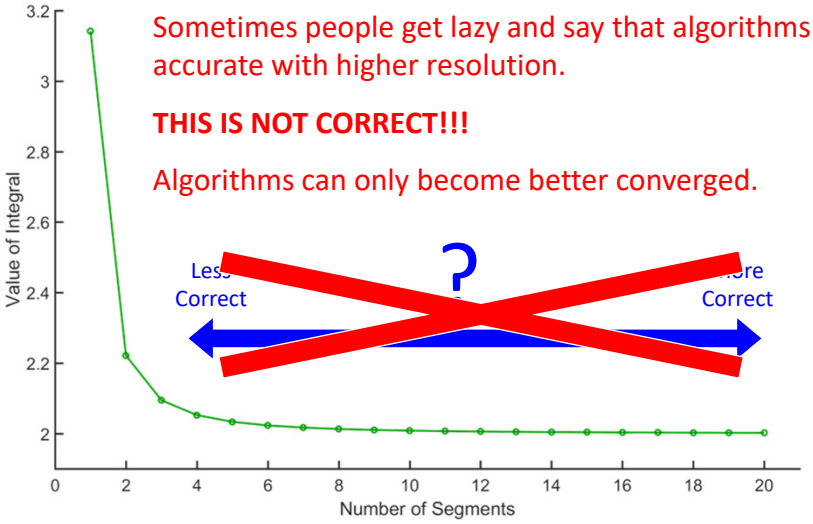
Convergence Does NOT Imply Correctness



Sometimes people get lazy and say that algorithms get more accurate with higher resolution.

THIS IS NOT CORRECT!!!

Algorithms can only become better converged.

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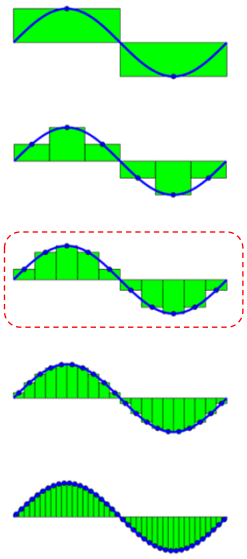
Rule-of-Thumb for Resolution

For calculations involving waves, the resolution begins to converge when one wave cycle is resolved with about 10 divisions.

$$\Delta \approx \frac{\lambda}{10} \qquad \lambda \equiv \text{wavelength}$$



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