



Computational Science:  
Computational Methods in Engineering

## Derivative Matrices on a Staggered Grid



### Outline

- Right-Handed Derivative Matrix  $\left[ D_x^f \right]$
- Right-Handed Derivative Matrix  $\left[ D_y^f \right]$
- Left-Handed Derivative Matrix  $\left[ D_x^g \right]$
- Left-Handed Derivative Matrix  $\left[ D_y^g \right]$
- Final Notes

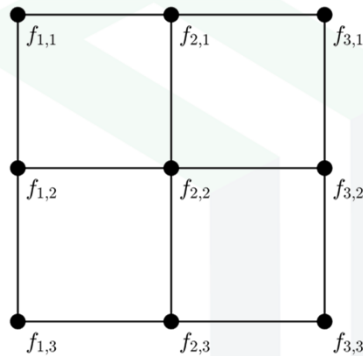


# Right-Handed Derivative Matrix

$$\left[ D_x^f \right]$$



## $\left[ D_x^f \right]$ Derivative Matrix (1 of 11)



$$\frac{\partial f_{i,j}}{\partial x} \approx \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

$$\frac{\partial f_{1,1}}{\partial x} \approx \frac{f_{2,1} - f_{1,1}}{\Delta x}$$

$$\frac{\partial f_{2,1}}{\partial x} \approx \frac{f_{3,1} - f_{2,1}}{\Delta x}$$

$$\frac{\partial f_{3,1}}{\partial x} \approx \frac{0 - f_{3,1}}{\Delta x}$$

$$\frac{\partial f_{1,2}}{\partial x} \approx \frac{f_{2,2} - f_{1,2}}{\Delta x}$$

$$\frac{\partial f_{2,2}}{\partial x} \approx \frac{f_{3,2} - f_{2,2}}{\Delta x}$$

$$\frac{\partial f_{3,2}}{\partial x} \approx \frac{0 - f_{3,2}}{\Delta x}$$

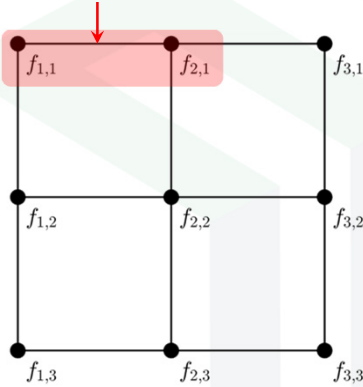
$$\frac{\partial f_{1,3}}{\partial x} \approx \frac{f_{2,3} - f_{1,3}}{\Delta x}$$

$$\frac{\partial f_{2,3}}{\partial x} \approx \frac{f_{3,3} - f_{2,3}}{\Delta x}$$

$$\frac{\partial f_{3,3}}{\partial x} \approx \frac{0 - f_{3,3}}{\Delta x}$$



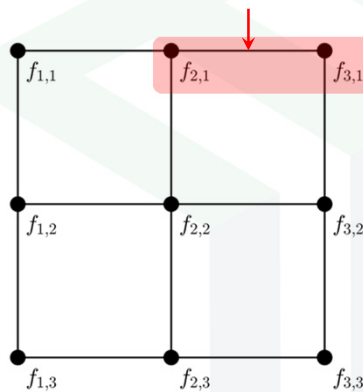
## $[D_x^f]$ Derivative Matrix (2 of 11)



$$\frac{\partial f_{i,j}}{\partial x} \approx \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial f_{1,1}}{\partial x} &\approx \frac{f_{2,1} - f_{1,1}}{\Delta x} \\ \frac{\partial f_{2,1}}{\partial x} &\approx \frac{f_{3,1} - f_{2,1}}{\Delta x} \\ \frac{\partial f_{3,1}}{\partial x} &\approx \frac{0 - f_{3,1}}{\Delta x} \\ \frac{\partial f_{1,2}}{\partial x} &\approx \frac{f_{2,2} - f_{1,2}}{\Delta x} \\ \frac{\partial f_{2,2}}{\partial x} &\approx \frac{f_{3,2} - f_{2,2}}{\Delta x} \\ \frac{\partial f_{3,2}}{\partial x} &\approx \frac{0 - f_{3,2}}{\Delta x} \\ \frac{\partial f_{1,3}}{\partial x} &\approx \frac{f_{2,3} - f_{1,3}}{\Delta x} \\ \frac{\partial f_{2,3}}{\partial x} &\approx \frac{f_{3,3} - f_{2,3}}{\Delta x} \\ \frac{\partial f_{3,3}}{\partial x} &\approx \frac{0 - f_{3,3}}{\Delta x} \end{aligned}$$

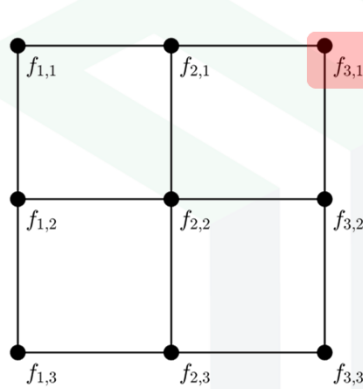
## $[D_x^f]$ Derivative Matrix (3 of 11)



$$\frac{\partial f_{i,j}}{\partial x} \approx \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial f_{1,1}}{\partial x} &\approx \frac{f_{2,1} - f_{1,1}}{\Delta x} \\ \frac{\partial f_{2,1}}{\partial x} &\approx \frac{f_{3,1} - f_{2,1}}{\Delta x} \\ \frac{\partial f_{3,1}}{\partial x} &\approx \frac{0 - f_{3,1}}{\Delta x} \\ \frac{\partial f_{1,2}}{\partial x} &\approx \frac{f_{2,2} - f_{1,2}}{\Delta x} \\ \frac{\partial f_{2,2}}{\partial x} &\approx \frac{f_{3,2} - f_{2,2}}{\Delta x} \\ \frac{\partial f_{3,2}}{\partial x} &\approx \frac{0 - f_{3,2}}{\Delta x} \\ \frac{\partial f_{1,3}}{\partial x} &\approx \frac{f_{2,3} - f_{1,3}}{\Delta x} \\ \frac{\partial f_{2,3}}{\partial x} &\approx \frac{f_{3,3} - f_{2,3}}{\Delta x} \\ \frac{\partial f_{3,3}}{\partial x} &\approx \frac{0 - f_{3,3}}{\Delta x} \end{aligned}$$

## $[D_x^f]$ Derivative Matrix (4 of 11)



$$\frac{\partial f_{i,j}}{\partial x} \approx \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

$$\frac{\partial f_{1,1}}{\partial x} \approx \frac{f_{2,1} - f_{1,1}}{\Delta x}$$

$$\frac{\partial f_{2,1}}{\partial x} \approx \frac{f_{3,1} - f_{2,1}}{\Delta x}$$

$$\frac{\partial f_{3,1}}{\partial x} \approx \frac{0 - f_{3,1}}{\Delta x}$$

$$\frac{\partial f_{1,2}}{\partial x} \approx \frac{f_{2,2} - f_{1,2}}{\Delta x}$$

$$\frac{\partial f_{2,2}}{\partial x} \approx \frac{f_{3,2} - f_{2,2}}{\Delta x}$$

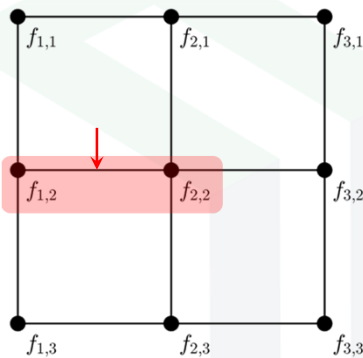
$$\frac{\partial f_{3,2}}{\partial x} \approx \frac{0 - f_{3,2}}{\Delta x}$$

$$\frac{\partial f_{1,3}}{\partial x} \approx \frac{f_{2,3} - f_{1,3}}{\Delta x}$$

$$\frac{\partial f_{2,3}}{\partial x} \approx \frac{f_{3,3} - f_{2,3}}{\Delta x}$$

$$\frac{\partial f_{3,3}}{\partial x} \approx \frac{0 - f_{3,3}}{\Delta x}$$

## $[D_x^f]$ Derivative Matrix (5 of 11)



$$\frac{\partial f_{i,j}}{\partial x} \approx \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

$$\frac{\partial f_{1,1}}{\partial x} \approx \frac{f_{2,1} - f_{1,1}}{\Delta x}$$

$$\frac{\partial f_{2,1}}{\partial x} \approx \frac{f_{3,1} - f_{2,1}}{\Delta x}$$

$$\frac{\partial f_{3,1}}{\partial x} \approx \frac{0 - f_{3,1}}{\Delta x}$$

$$\frac{\partial f_{1,2}}{\partial x} \approx \frac{f_{2,2} - f_{1,2}}{\Delta x}$$

$$\frac{\partial f_{2,2}}{\partial x} \approx \frac{f_{3,2} - f_{2,2}}{\Delta x}$$

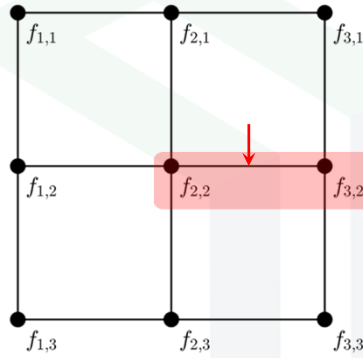
$$\frac{\partial f_{3,2}}{\partial x} \approx \frac{0 - f_{3,2}}{\Delta x}$$

$$\frac{\partial f_{1,3}}{\partial x} \approx \frac{f_{2,3} - f_{1,3}}{\Delta x}$$

$$\frac{\partial f_{2,3}}{\partial x} \approx \frac{f_{3,3} - f_{2,3}}{\Delta x}$$

$$\frac{\partial f_{3,3}}{\partial x} \approx \frac{0 - f_{3,3}}{\Delta x}$$

## $[D_x^f]$ Derivative Matrix (6 of 11)

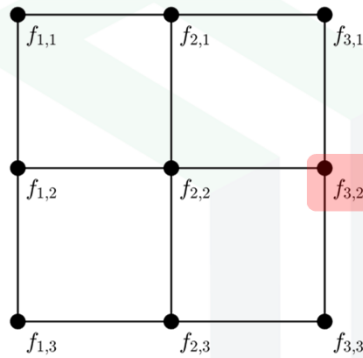


$$\frac{\partial f_{i,j}}{\partial x} \approx \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial f_{1,1}}{\partial x} &\approx \frac{f_{2,1} - f_{1,1}}{\Delta x} \\ \frac{\partial f_{2,1}}{\partial x} &\approx \frac{f_{3,1} - f_{2,1}}{\Delta x} \\ \frac{\partial f_{3,1}}{\partial x} &\approx \frac{0 - f_{3,1}}{\Delta x} \\ \frac{\partial f_{1,2}}{\partial x} &\approx \frac{f_{2,2} - f_{1,2}}{\Delta x} \\ \frac{\partial f_{2,2}}{\partial x} &\approx \frac{f_{3,2} - f_{2,2}}{\Delta x} \\ \frac{\partial f_{3,2}}{\partial x} &\approx \frac{0 - f_{3,2}}{\Delta x} \\ \frac{\partial f_{1,3}}{\partial x} &\approx \frac{f_{2,3} - f_{1,3}}{\Delta x} \\ \frac{\partial f_{2,3}}{\partial x} &\approx \frac{f_{3,3} - f_{2,3}}{\Delta x} \\ \frac{\partial f_{3,3}}{\partial x} &\approx \frac{0 - f_{3,3}}{\Delta x} \end{aligned}$$



## $[D_x^f]$ Derivative Matrix (7 of 11)

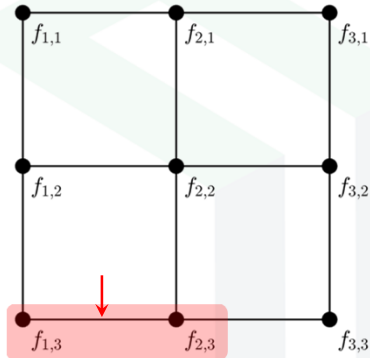


$$\frac{\partial f_{i,j}}{\partial x} \approx \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial f_{1,1}}{\partial x} &\approx \frac{f_{2,1} - f_{1,1}}{\Delta x} \\ \frac{\partial f_{2,1}}{\partial x} &\approx \frac{f_{3,1} - f_{2,1}}{\Delta x} \\ \frac{\partial f_{3,1}}{\partial x} &\approx \frac{0 - f_{3,1}}{\Delta x} \\ \frac{\partial f_{1,2}}{\partial x} &\approx \frac{f_{2,2} - f_{1,2}}{\Delta x} \\ \frac{\partial f_{2,2}}{\partial x} &\approx \frac{f_{3,2} - f_{2,2}}{\Delta x} \\ \frac{\partial f_{3,2}}{\partial x} &\approx \frac{0 - f_{3,2}}{\Delta x} \\ \frac{\partial f_{1,3}}{\partial x} &\approx \frac{f_{2,3} - f_{1,3}}{\Delta x} \\ \frac{\partial f_{2,3}}{\partial x} &\approx \frac{f_{3,3} - f_{2,3}}{\Delta x} \\ \frac{\partial f_{3,3}}{\partial x} &\approx \frac{0 - f_{3,3}}{\Delta x} \end{aligned}$$



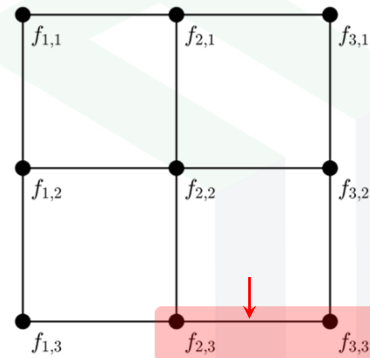
## $[D_x^f]$ Derivative Matrix (8 of 11)



$$\frac{\partial f_{i,j}}{\partial x} \approx \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial f_{1,1}}{\partial x} &\approx \frac{f_{2,1} - f_{1,1}}{\Delta x} \\ \frac{\partial f_{2,1}}{\partial x} &\approx \frac{f_{3,1} - f_{2,1}}{\Delta x} \\ \frac{\partial f_{3,1}}{\partial x} &\approx \frac{0 - f_{3,1}}{\Delta x} \\ \frac{\partial f_{1,2}}{\partial x} &\approx \frac{f_{2,2} - f_{1,2}}{\Delta x} \\ \frac{\partial f_{2,2}}{\partial x} &\approx \frac{f_{3,2} - f_{2,2}}{\Delta x} \\ \frac{\partial f_{3,2}}{\partial x} &\approx \frac{0 - f_{3,2}}{\Delta x} \\ \frac{\partial f_{1,3}}{\partial x} &\approx \frac{f_{2,3} - f_{1,3}}{\Delta x} \\ \frac{\partial f_{2,3}}{\partial x} &\approx \frac{f_{3,3} - f_{2,3}}{\Delta x} \\ \frac{\partial f_{3,3}}{\partial x} &\approx \frac{0 - f_{3,3}}{\Delta x} \end{aligned}$$

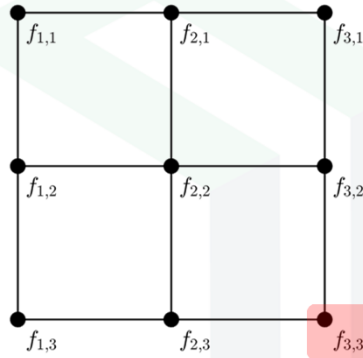
## $[D_x^f]$ Derivative Matrix (9 of 11)



$$\frac{\partial f_{i,j}}{\partial x} \approx \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial f_{1,1}}{\partial x} &\approx \frac{f_{2,1} - f_{1,1}}{\Delta x} \\ \frac{\partial f_{2,1}}{\partial x} &\approx \frac{f_{3,1} - f_{2,1}}{\Delta x} \\ \frac{\partial f_{3,1}}{\partial x} &\approx \frac{0 - f_{3,1}}{\Delta x} \\ \frac{\partial f_{1,2}}{\partial x} &\approx \frac{f_{2,2} - f_{1,2}}{\Delta x} \\ \frac{\partial f_{2,2}}{\partial x} &\approx \frac{f_{3,2} - f_{2,2}}{\Delta x} \\ \frac{\partial f_{3,2}}{\partial x} &\approx \frac{0 - f_{3,2}}{\Delta x} \\ \frac{\partial f_{1,3}}{\partial x} &\approx \frac{f_{2,3} - f_{1,3}}{\Delta x} \\ \frac{\partial f_{2,3}}{\partial x} &\approx \frac{f_{3,3} - f_{2,3}}{\Delta x} \\ \frac{\partial f_{3,3}}{\partial x} &\approx \frac{0 - f_{3,3}}{\Delta x} \end{aligned}$$

# $[D_x^f]$ Derivative Matrix (10 of 11)



$$\frac{\partial f_{i,j}}{\partial x} \approx \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

- $\frac{\partial f_{1,1}}{\partial x} \approx \frac{f_{2,1} - f_{1,1}}{\Delta x}$
- $\frac{\partial f_{2,1}}{\partial x} \approx \frac{f_{3,1} - f_{2,1}}{\Delta x}$
- $\frac{\partial f_{3,1}}{\partial x} \approx \frac{0 - f_{3,1}}{\Delta x}$
- $\frac{\partial f_{1,2}}{\partial x} \approx \frac{f_{2,2} - f_{1,2}}{\Delta x}$
- $\frac{\partial f_{2,2}}{\partial x} \approx \frac{f_{3,2} - f_{2,2}}{\Delta x}$
- $\frac{\partial f_{3,2}}{\partial x} \approx \frac{0 - f_{3,2}}{\Delta x}$
- $\frac{\partial f_{1,3}}{\partial x} \approx \frac{f_{2,3} - f_{1,3}}{\Delta x}$
- $\frac{\partial f_{2,3}}{\partial x} \approx \frac{f_{3,3} - f_{2,3}}{\Delta x}$
- $\frac{\partial f_{3,3}}{\partial x} \approx \frac{0 - f_{3,3}}{\Delta x}$

# $[D_x^f]$ Derivative Matrix (11 of 11)

$$\frac{1}{\Delta x} \begin{bmatrix} f_{1,1} \\ f_{2,1} \\ f_{3,1} \\ f_{1,2} \\ f_{2,2} \\ f_{3,2} \\ f_{1,3} \\ f_{2,3} \\ f_{3,3} \end{bmatrix} = \frac{1}{\Delta x} \begin{bmatrix} f_{2,1} - f_{1,1} \\ f_{3,1} - f_{2,1} \\ 0 - f_{3,1} \\ f_{2,2} - f_{1,2} \\ f_{3,2} - f_{2,2} \\ 0 - f_{3,2} \\ f_{2,3} - f_{1,3} \\ f_{3,3} - f_{2,3} \\ 0 - f_{3,3} \end{bmatrix}$$

- $\frac{\partial f_{1,1}}{\partial x} \approx \frac{f_{2,1} - f_{1,1}}{\Delta x}$
- $\frac{\partial f_{2,1}}{\partial x} \approx \frac{f_{3,1} - f_{2,1}}{\Delta x}$
- $\frac{\partial f_{3,1}}{\partial x} \approx \frac{0 - f_{3,1}}{\Delta x}$
- $\frac{\partial f_{1,2}}{\partial x} \approx \frac{f_{2,2} - f_{1,2}}{\Delta x}$
- $\frac{\partial f_{2,2}}{\partial x} \approx \frac{f_{3,2} - f_{2,2}}{\Delta x}$
- $\frac{\partial f_{3,2}}{\partial x} \approx \frac{0 - f_{3,2}}{\Delta x}$
- $\frac{\partial f_{1,3}}{\partial x} \approx \frac{f_{2,3} - f_{1,3}}{\Delta x}$
- $\frac{\partial f_{2,3}}{\partial x} \approx \frac{f_{3,3} - f_{2,3}}{\Delta x}$
- $\frac{\partial f_{3,3}}{\partial x} \approx \frac{0 - f_{3,3}}{\Delta x}$

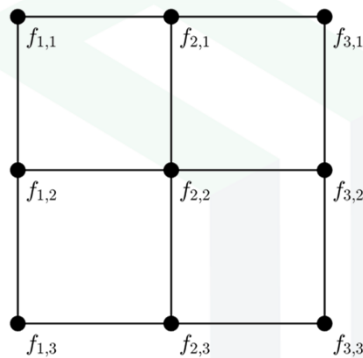


# Right-Handed Derivative Matrix

$$\left[ D_y^f \right]$$

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## $\left[ D_y^f \right]$ Derivative Matrix (1 of 11)



$$\frac{\partial f_{i,j}}{\partial y} \approx \frac{f_{i,j+1} - f_{i,j}}{\Delta y}$$

$$\frac{\partial f_{1,1}}{\partial y} \approx \frac{f_{1,2} - f_{1,1}}{\Delta y}$$

$$\frac{\partial f_{2,1}}{\partial y} \approx \frac{f_{2,2} - f_{2,1}}{\Delta y}$$

$$\frac{\partial f_{3,1}}{\partial y} \approx \frac{f_{3,2} - f_{3,1}}{\Delta y}$$

$$\frac{\partial f_{1,2}}{\partial y} \approx \frac{f_{1,3} - f_{1,2}}{\Delta y}$$

$$\frac{\partial f_{2,2}}{\partial y} \approx \frac{f_{2,3} - f_{2,2}}{\Delta y}$$

$$\frac{\partial f_{3,2}}{\partial y} \approx \frac{f_{3,3} - f_{3,2}}{\Delta y}$$

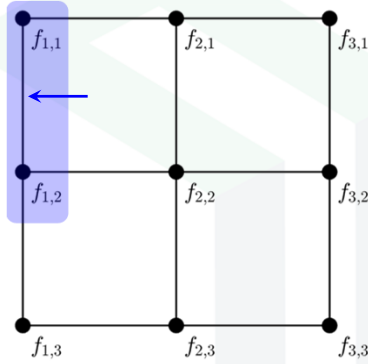
$$\frac{\partial f_{1,3}}{\partial y} \approx \frac{0 - f_{1,3}}{\Delta y}$$

$$\frac{\partial f_{2,3}}{\partial y} \approx \frac{0 - f_{2,3}}{\Delta y}$$

$$\frac{\partial f_{3,3}}{\partial y} \approx \frac{0 - f_{3,3}}{\Delta y}$$

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## $[D_y^f]$ Derivative Matrix (2 of 11)



$$\frac{\partial f_{i,j}}{\partial y} \approx \frac{f_{i,j+1} - f_{i,j}}{\Delta y}$$

$$\frac{\partial f_{1,1}}{\partial y} \approx \frac{f_{1,2} - f_{1,1}}{\Delta y}$$

$$\frac{\partial f_{2,1}}{\partial y} \approx \frac{f_{2,2} - f_{2,1}}{\Delta y}$$

$$\frac{\partial f_{3,1}}{\partial y} \approx \frac{f_{3,2} - f_{3,1}}{\Delta y}$$

$$\frac{\partial f_{1,2}}{\partial y} \approx \frac{f_{1,3} - f_{1,2}}{\Delta y}$$

$$\frac{\partial f_{2,2}}{\partial y} \approx \frac{f_{2,3} - f_{2,2}}{\Delta y}$$

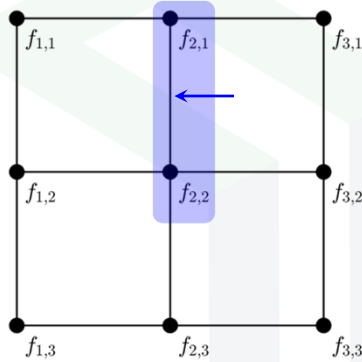
$$\frac{\partial f_{3,2}}{\partial y} \approx \frac{f_{3,3} - f_{3,2}}{\Delta y}$$

$$\frac{\partial f_{1,3}}{\partial y} \approx \frac{0 - f_{1,3}}{\Delta y}$$

$$\frac{\partial f_{2,3}}{\partial y} \approx \frac{0 - f_{2,3}}{\Delta y}$$

$$\frac{\partial f_{3,3}}{\partial y} \approx \frac{0 - f_{3,3}}{\Delta y}$$

## $[D_y^f]$ Derivative Matrix (3 of 11)



$$\frac{\partial f_{i,j}}{\partial y} \approx \frac{f_{i,j+1} - f_{i,j}}{\Delta y}$$

$$\frac{\partial f_{1,1}}{\partial y} \approx \frac{f_{1,2} - f_{1,1}}{\Delta y}$$

$$\frac{\partial f_{2,1}}{\partial y} \approx \frac{f_{2,2} - f_{2,1}}{\Delta y}$$

$$\frac{\partial f_{3,1}}{\partial y} \approx \frac{f_{3,2} - f_{3,1}}{\Delta y}$$

$$\frac{\partial f_{1,2}}{\partial y} \approx \frac{f_{1,3} - f_{1,2}}{\Delta y}$$

$$\frac{\partial f_{2,2}}{\partial y} \approx \frac{f_{2,3} - f_{2,2}}{\Delta y}$$

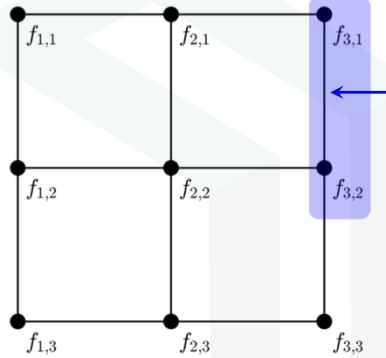
$$\frac{\partial f_{3,2}}{\partial y} \approx \frac{f_{3,3} - f_{3,2}}{\Delta y}$$

$$\frac{\partial f_{1,3}}{\partial y} \approx \frac{0 - f_{1,3}}{\Delta y}$$

$$\frac{\partial f_{2,3}}{\partial y} \approx \frac{0 - f_{2,3}}{\Delta y}$$

$$\frac{\partial f_{3,3}}{\partial y} \approx \frac{0 - f_{3,3}}{\Delta y}$$

## $[D_y^f]$ Derivative Matrix (4 of 11)



$$\frac{\partial f_{i,j}}{\partial y} \approx \frac{f_{i,j+1} - f_{i,j}}{\Delta y}$$

$$\frac{\partial f_{1,1}}{\partial y} \approx \frac{f_{1,2} - f_{1,1}}{\Delta y}$$

$$\frac{\partial f_{2,1}}{\partial y} \approx \frac{f_{2,2} - f_{2,1}}{\Delta y}$$

$$\frac{\partial f_{3,1}}{\partial y} \approx \frac{f_{3,2} - f_{3,1}}{\Delta y}$$

$$\frac{\partial f_{1,2}}{\partial y} \approx \frac{f_{1,3} - f_{1,2}}{\Delta y}$$

$$\frac{\partial f_{2,2}}{\partial y} \approx \frac{f_{2,3} - f_{2,2}}{\Delta y}$$

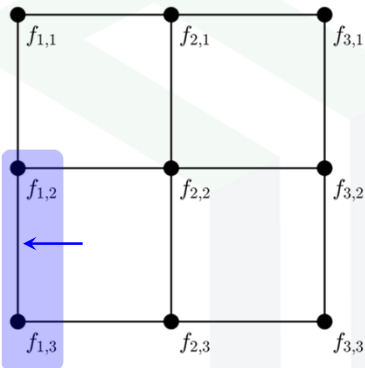
$$\frac{\partial f_{3,2}}{\partial y} \approx \frac{f_{3,3} - f_{3,2}}{\Delta y}$$

$$\frac{\partial f_{1,3}}{\partial y} \approx \frac{0 - f_{1,3}}{\Delta y}$$

$$\frac{\partial f_{2,3}}{\partial y} \approx \frac{0 - f_{2,3}}{\Delta y}$$

$$\frac{\partial f_{3,3}}{\partial y} \approx \frac{0 - f_{3,3}}{\Delta y}$$

## $[D_y^f]$ Derivative Matrix (5 of 11)



$$\frac{\partial f_{i,j}}{\partial y} \approx \frac{f_{i,j+1} - f_{i,j}}{\Delta y}$$

$$\frac{\partial f_{1,1}}{\partial y} \approx \frac{f_{1,2} - f_{1,1}}{\Delta y}$$

$$\frac{\partial f_{2,1}}{\partial y} \approx \frac{f_{2,2} - f_{2,1}}{\Delta y}$$

$$\frac{\partial f_{3,1}}{\partial y} \approx \frac{f_{3,2} - f_{3,1}}{\Delta y}$$

$$\frac{\partial f_{1,2}}{\partial y} \approx \frac{f_{1,3} - f_{1,2}}{\Delta y}$$

$$\frac{\partial f_{2,2}}{\partial y} \approx \frac{f_{2,3} - f_{2,2}}{\Delta y}$$

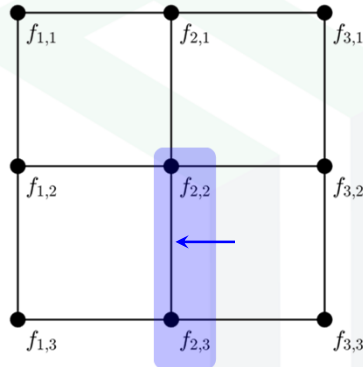
$$\frac{\partial f_{3,2}}{\partial y} \approx \frac{f_{3,3} - f_{3,2}}{\Delta y}$$

$$\frac{\partial f_{1,3}}{\partial y} \approx \frac{0 - f_{1,3}}{\Delta y}$$

$$\frac{\partial f_{2,3}}{\partial y} \approx \frac{0 - f_{2,3}}{\Delta y}$$

$$\frac{\partial f_{3,3}}{\partial y} \approx \frac{0 - f_{3,3}}{\Delta y}$$

## $[D_y^f]$ Derivative Matrix (6 of 11)



$$\frac{\partial f_{i,j}}{\partial y} \approx \frac{f_{i,j+1} - f_{i,j}}{\Delta y}$$

$$\frac{\partial f_{1,1}}{\partial y} \approx \frac{f_{1,2} - f_{1,1}}{\Delta y}$$

$$\frac{\partial f_{2,1}}{\partial y} \approx \frac{f_{2,2} - f_{2,1}}{\Delta y}$$

$$\frac{\partial f_{3,1}}{\partial y} \approx \frac{f_{3,2} - f_{3,1}}{\Delta y}$$

$$\frac{\partial f_{1,2}}{\partial y} \approx \frac{f_{1,3} - f_{1,2}}{\Delta y}$$

$$\frac{\partial f_{2,2}}{\partial y} \approx \frac{f_{2,3} - f_{2,2}}{\Delta y}$$

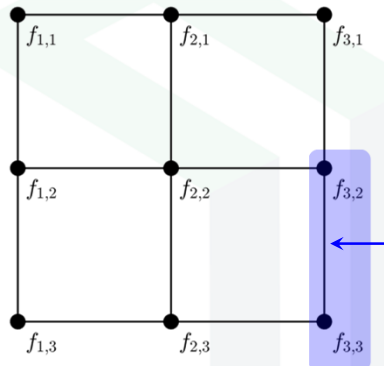
$$\frac{\partial f_{3,2}}{\partial y} \approx \frac{f_{3,3} - f_{3,2}}{\Delta y}$$

$$\frac{\partial f_{1,3}}{\partial y} \approx \frac{0 - f_{1,3}}{\Delta y}$$

$$\frac{\partial f_{2,3}}{\partial y} \approx \frac{0 - f_{2,3}}{\Delta y}$$

$$\frac{\partial f_{3,3}}{\partial y} \approx \frac{0 - f_{3,3}}{\Delta y}$$

## $[D_y^f]$ Derivative Matrix (7 of 11)



$$\frac{\partial f_{i,j}}{\partial y} \approx \frac{f_{i,j+1} - f_{i,j}}{\Delta y}$$

$$\frac{\partial f_{1,1}}{\partial y} \approx \frac{f_{1,2} - f_{1,1}}{\Delta y}$$

$$\frac{\partial f_{2,1}}{\partial y} \approx \frac{f_{2,2} - f_{2,1}}{\Delta y}$$

$$\frac{\partial f_{3,1}}{\partial y} \approx \frac{f_{3,2} - f_{3,1}}{\Delta y}$$

$$\frac{\partial f_{1,2}}{\partial y} \approx \frac{f_{1,3} - f_{1,2}}{\Delta y}$$

$$\frac{\partial f_{2,2}}{\partial y} \approx \frac{f_{2,3} - f_{2,2}}{\Delta y}$$

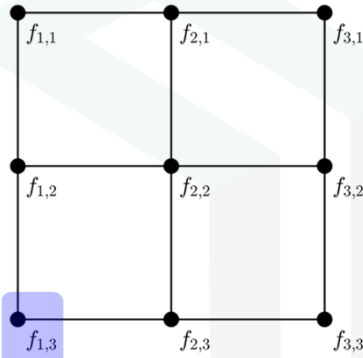
$$\frac{\partial f_{3,2}}{\partial y} \approx \frac{f_{3,3} - f_{3,2}}{\Delta y}$$

$$\frac{\partial f_{1,3}}{\partial y} \approx \frac{0 - f_{1,3}}{\Delta y}$$

$$\frac{\partial f_{2,3}}{\partial y} \approx \frac{0 - f_{2,3}}{\Delta y}$$

$$\frac{\partial f_{3,3}}{\partial y} \approx \frac{0 - f_{3,3}}{\Delta y}$$

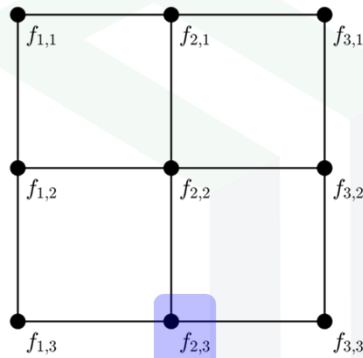
# $[D_y^f]$ Derivative Matrix (8 of 11)



$$\frac{\partial f_{i,j}}{\partial y} \approx \frac{f_{i,j+1} - f_{i,j}}{\Delta y}$$

- $\frac{\partial f_{1,1}}{\partial y} \approx \frac{f_{1,2} - f_{1,1}}{\Delta y}$
- $\frac{\partial f_{2,1}}{\partial y} \approx \frac{f_{2,2} - f_{2,1}}{\Delta y}$
- $\frac{\partial f_{3,1}}{\partial y} \approx \frac{f_{3,2} - f_{3,1}}{\Delta y}$
- $\frac{\partial f_{1,2}}{\partial y} \approx \frac{f_{1,3} - f_{1,2}}{\Delta y}$
- $\frac{\partial f_{2,2}}{\partial y} \approx \frac{f_{2,3} - f_{2,2}}{\Delta y}$
- $\frac{\partial f_{3,2}}{\partial y} \approx \frac{f_{3,3} - f_{3,2}}{\Delta y}$
- $\frac{\partial f_{1,3}}{\partial y} \approx \frac{0 - f_{1,3}}{\Delta y}$
- $\frac{\partial f_{2,3}}{\partial y} \approx \frac{0 - f_{2,3}}{\Delta y}$
- $\frac{\partial f_{3,3}}{\partial y} \approx \frac{0 - f_{3,3}}{\Delta y}$

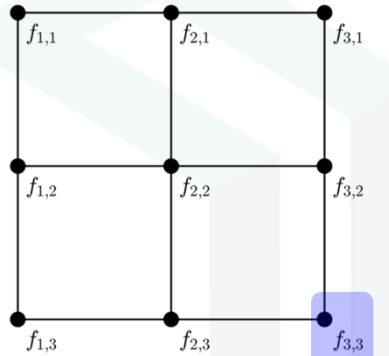
# $[D_y^f]$ Derivative Matrix (9 of 11)



$$\frac{\partial f_{i,j}}{\partial y} \approx \frac{f_{i,j+1} - f_{i,j}}{\Delta y}$$

- $\frac{\partial f_{1,1}}{\partial y} \approx \frac{f_{1,2} - f_{1,1}}{\Delta y}$
- $\frac{\partial f_{2,1}}{\partial y} \approx \frac{f_{2,2} - f_{2,1}}{\Delta y}$
- $\frac{\partial f_{3,1}}{\partial y} \approx \frac{f_{3,2} - f_{3,1}}{\Delta y}$
- $\frac{\partial f_{1,2}}{\partial y} \approx \frac{f_{1,3} - f_{1,2}}{\Delta y}$
- $\frac{\partial f_{2,2}}{\partial y} \approx \frac{f_{2,3} - f_{2,2}}{\Delta y}$
- $\frac{\partial f_{3,2}}{\partial y} \approx \frac{f_{3,3} - f_{3,2}}{\Delta y}$
- $\frac{\partial f_{1,3}}{\partial y} \approx \frac{0 - f_{1,3}}{\Delta y}$
- $\frac{\partial f_{2,3}}{\partial y} \approx \frac{0 - f_{2,3}}{\Delta y}$
- $\frac{\partial f_{3,3}}{\partial y} \approx \frac{0 - f_{3,3}}{\Delta y}$

# $[D_y^f]$ Derivative Matrix (10 of 11)



$$\frac{\partial f_{i,j}}{\partial y} \approx \frac{f_{i,j+1} - f_{i,j}}{\Delta y}$$

- $\frac{\partial f_{1,1}}{\partial y} \approx \frac{f_{1,2} - f_{1,1}}{\Delta y}$
- $\frac{\partial f_{2,1}}{\partial y} \approx \frac{f_{2,2} - f_{2,1}}{\Delta y}$
- $\frac{\partial f_{3,1}}{\partial y} \approx \frac{f_{3,2} - f_{3,1}}{\Delta y}$
- $\frac{\partial f_{1,2}}{\partial y} \approx \frac{f_{1,3} - f_{1,2}}{\Delta y}$
- $\frac{\partial f_{2,2}}{\partial y} \approx \frac{f_{2,3} - f_{2,2}}{\Delta y}$
- $\frac{\partial f_{3,2}}{\partial y} \approx \frac{f_{3,3} - f_{3,2}}{\Delta y}$
- $\frac{\partial f_{1,3}}{\partial y} \approx \frac{0 - f_{1,3}}{\Delta y}$
- $\frac{\partial f_{2,3}}{\partial y} \approx \frac{0 - f_{2,3}}{\Delta y}$
- $\frac{\partial f_{3,3}}{\partial y} \approx \frac{0 - f_{3,3}}{\Delta y}$

# $[D_y^f]$ Derivative Matrix (11 of 11)

$$\frac{1}{\Delta y}$$

$$\begin{bmatrix} f_{1,1} \\ f_{2,1} \\ f_{3,1} \\ f_{1,2} \\ f_{2,2} \\ f_{3,2} \\ f_{1,3} \\ f_{2,3} \\ f_{3,3} \end{bmatrix} = \frac{1}{\Delta y} \begin{bmatrix} f_{1,2} - f_{1,1} \\ f_{2,2} - f_{2,1} \\ f_{3,2} - f_{3,1} \\ f_{1,3} - f_{1,2} \\ f_{2,3} - f_{2,2} \\ f_{3,3} - f_{3,2} \\ 0 - f_{1,3} \\ 0 - f_{2,3} \\ 0 - f_{3,3} \end{bmatrix}$$



- $\frac{\partial f_{1,1}}{\partial y} \approx \frac{f_{1,2} - f_{1,1}}{\Delta y}$
- $\frac{\partial f_{2,1}}{\partial y} \approx \frac{f_{2,2} - f_{2,1}}{\Delta y}$
- $\frac{\partial f_{3,1}}{\partial y} \approx \frac{f_{3,2} - f_{3,1}}{\Delta y}$
- $\frac{\partial f_{1,2}}{\partial y} \approx \frac{f_{1,3} - f_{1,2}}{\Delta y}$
- $\frac{\partial f_{2,2}}{\partial y} \approx \frac{f_{2,3} - f_{2,2}}{\Delta y}$
- $\frac{\partial f_{3,2}}{\partial y} \approx \frac{f_{3,3} - f_{3,2}}{\Delta y}$
- $\frac{\partial f_{1,3}}{\partial y} \approx \frac{0 - f_{1,3}}{\Delta y}$
- $\frac{\partial f_{2,3}}{\partial y} \approx \frac{0 - f_{2,3}}{\Delta y}$
- $\frac{\partial f_{3,3}}{\partial y} \approx \frac{0 - f_{3,3}}{\Delta y}$

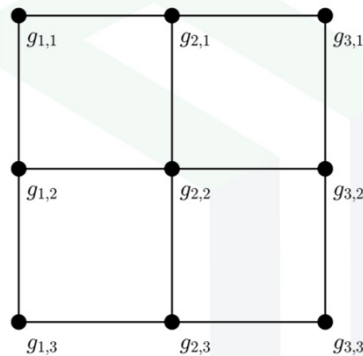


# Left-Handed Derivative Matrix

$$\left[ D_x^g \right]$$



## $\left[ D_x^g \right]$ Derivative Matrix (1 of 11)



$$\frac{\partial g_{i,j}}{\partial x} \approx \frac{g_{i,j} - g_{i-1,j}}{\Delta x}$$

$$\frac{\partial g_{1,1}}{\partial x} \approx \frac{g_{1,1} - 0}{\Delta x}$$

$$\frac{\partial g_{2,1}}{\partial x} \approx \frac{g_{2,1} - g_{1,1}}{\Delta x}$$

$$\frac{\partial g_{3,1}}{\partial x} \approx \frac{g_{3,1} - g_{2,1}}{\Delta x}$$

$$\frac{\partial g_{1,2}}{\partial x} \approx \frac{g_{1,2} - 0}{\Delta x}$$

$$\frac{\partial g_{2,2}}{\partial x} \approx \frac{g_{2,2} - g_{1,2}}{\Delta x}$$

$$\frac{\partial g_{3,2}}{\partial x} \approx \frac{g_{3,2} - g_{2,2}}{\Delta x}$$

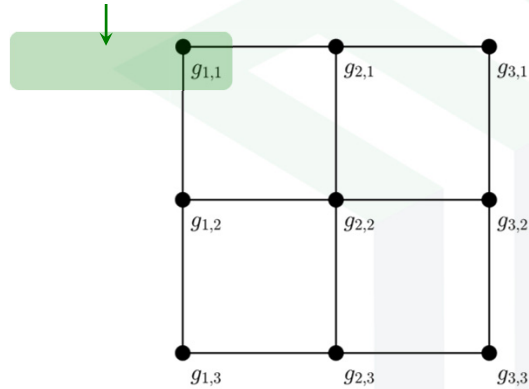
$$\frac{\partial g_{1,3}}{\partial x} \approx \frac{g_{1,3} - 0}{\Delta x}$$

$$\frac{\partial g_{2,3}}{\partial x} \approx \frac{g_{2,3} - g_{1,3}}{\Delta x}$$

$$\frac{\partial g_{3,3}}{\partial x} \approx \frac{g_{3,3} - g_{2,3}}{\Delta x}$$



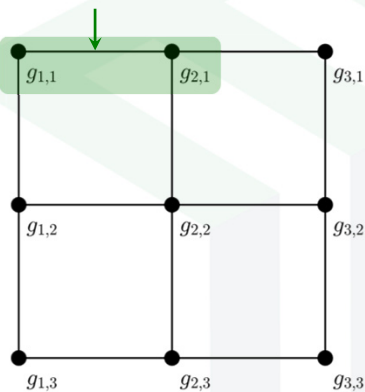
## $[D_x^g]$ Derivative Matrix (2 of 11)



$$\frac{\partial g_{i,j}}{\partial x} \approx \frac{g_{i,j} - g_{i-1,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial g_{1,1}}{\partial x} &\approx \frac{g_{1,1} - 0}{\Delta x} \\ \frac{\partial g_{2,1}}{\partial x} &\approx \frac{g_{2,1} - g_{1,1}}{\Delta x} \\ \frac{\partial g_{3,1}}{\partial x} &\approx \frac{g_{3,1} - g_{2,1}}{\Delta x} \\ \frac{\partial g_{1,2}}{\partial x} &\approx \frac{g_{1,2} - 0}{\Delta x} \\ \frac{\partial g_{2,2}}{\partial x} &\approx \frac{g_{2,2} - g_{1,2}}{\Delta x} \\ \frac{\partial g_{3,2}}{\partial x} &\approx \frac{g_{3,2} - g_{2,2}}{\Delta x} \\ \frac{\partial g_{1,3}}{\partial x} &\approx \frac{g_{1,3} - 0}{\Delta x} \\ \frac{\partial g_{2,3}}{\partial x} &\approx \frac{g_{2,3} - g_{1,3}}{\Delta x} \\ \frac{\partial g_{3,3}}{\partial x} &\approx \frac{g_{3,3} - g_{2,3}}{\Delta x} \end{aligned}$$

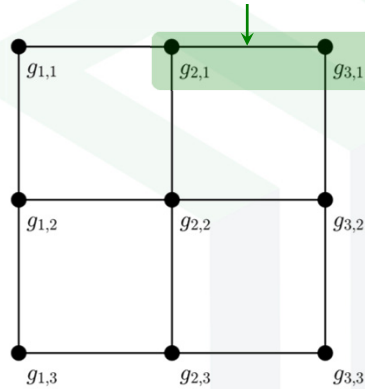
## $[D_x^g]$ Derivative Matrix (3 of 11)



$$\frac{\partial g_{i,j}}{\partial x} \approx \frac{g_{i,j} - g_{i-1,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial g_{1,1}}{\partial x} &\approx \frac{g_{1,1} - 0}{\Delta x} \\ \frac{\partial g_{2,1}}{\partial x} &\approx \frac{g_{2,1} - g_{1,1}}{\Delta x} \\ \frac{\partial g_{3,1}}{\partial x} &\approx \frac{g_{3,1} - g_{2,1}}{\Delta x} \\ \frac{\partial g_{1,2}}{\partial x} &\approx \frac{g_{1,2} - 0}{\Delta x} \\ \frac{\partial g_{2,2}}{\partial x} &\approx \frac{g_{2,2} - g_{1,2}}{\Delta x} \\ \frac{\partial g_{3,2}}{\partial x} &\approx \frac{g_{3,2} - g_{2,2}}{\Delta x} \\ \frac{\partial g_{1,3}}{\partial x} &\approx \frac{g_{1,3} - 0}{\Delta x} \\ \frac{\partial g_{2,3}}{\partial x} &\approx \frac{g_{2,3} - g_{1,3}}{\Delta x} \\ \frac{\partial g_{3,3}}{\partial x} &\approx \frac{g_{3,3} - g_{2,3}}{\Delta x} \end{aligned}$$

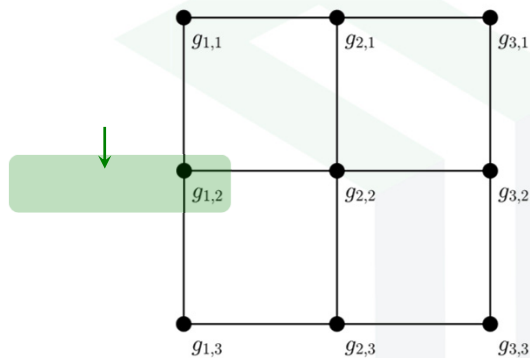
## $[D_x^g]$ Derivative Matrix (4 of 11)



$$\frac{\partial g_{i,j}}{\partial x} \approx \frac{g_{i,j} - g_{i-1,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial g_{1,1}}{\partial x} &\approx \frac{g_{1,1} - 0}{\Delta x} \\ \frac{\partial g_{2,1}}{\partial x} &\approx \frac{g_{2,1} - g_{1,1}}{\Delta x} \\ \frac{\partial g_{3,1}}{\partial x} &\approx \frac{g_{3,1} - g_{2,1}}{\Delta x} \\ \frac{\partial g_{1,2}}{\partial x} &\approx \frac{g_{1,2} - 0}{\Delta x} \\ \frac{\partial g_{2,2}}{\partial x} &\approx \frac{g_{2,2} - g_{1,2}}{\Delta x} \\ \frac{\partial g_{3,2}}{\partial x} &\approx \frac{g_{3,2} - g_{2,2}}{\Delta x} \\ \frac{\partial g_{1,3}}{\partial x} &\approx \frac{g_{1,3} - 0}{\Delta x} \\ \frac{\partial g_{2,3}}{\partial x} &\approx \frac{g_{2,3} - g_{1,3}}{\Delta x} \\ \frac{\partial g_{3,3}}{\partial x} &\approx \frac{g_{3,3} - g_{2,3}}{\Delta x} \end{aligned}$$

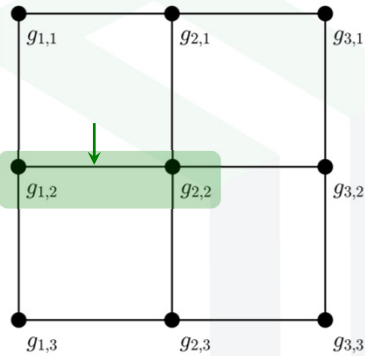
## $[D_x^g]$ Derivative Matrix (5 of 11)



$$\frac{\partial g_{i,j}}{\partial x} \approx \frac{g_{i,j} - g_{i-1,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial g_{1,1}}{\partial x} &\approx \frac{g_{1,1} - 0}{\Delta x} \\ \frac{\partial g_{2,1}}{\partial x} &\approx \frac{g_{2,1} - g_{1,1}}{\Delta x} \\ \frac{\partial g_{3,1}}{\partial x} &\approx \frac{g_{3,1} - g_{2,1}}{\Delta x} \\ \frac{\partial g_{1,2}}{\partial x} &\approx \frac{g_{1,2} - 0}{\Delta x} \\ \frac{\partial g_{2,2}}{\partial x} &\approx \frac{g_{2,2} - g_{1,2}}{\Delta x} \\ \frac{\partial g_{3,2}}{\partial x} &\approx \frac{g_{3,2} - g_{2,2}}{\Delta x} \\ \frac{\partial g_{1,3}}{\partial x} &\approx \frac{g_{1,3} - 0}{\Delta x} \\ \frac{\partial g_{2,3}}{\partial x} &\approx \frac{g_{2,3} - g_{1,3}}{\Delta x} \\ \frac{\partial g_{3,3}}{\partial x} &\approx \frac{g_{3,3} - g_{2,3}}{\Delta x} \end{aligned}$$

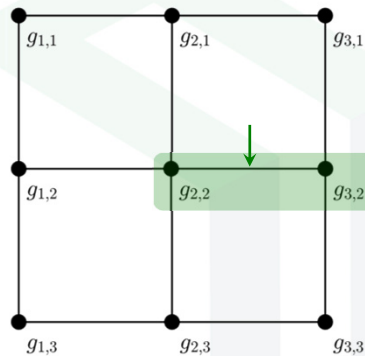
## $[D_x^g]$ Derivative Matrix (6 of 11)



$$\frac{\partial g_{i,j}}{\partial x} \approx \frac{g_{i,j} - g_{i-1,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial g_{1,1}}{\partial x} &\approx \frac{g_{1,1} - 0}{\Delta x} \\ \frac{\partial g_{2,1}}{\partial x} &\approx \frac{g_{2,1} - g_{1,1}}{\Delta x} \\ \frac{\partial g_{3,1}}{\partial x} &\approx \frac{g_{3,1} - g_{2,1}}{\Delta x} \\ \frac{\partial g_{1,2}}{\partial x} &\approx \frac{g_{1,2} - 0}{\Delta x} \\ \frac{\partial g_{2,2}}{\partial x} &\approx \frac{g_{2,2} - g_{1,2}}{\Delta x} \\ \frac{\partial g_{3,2}}{\partial x} &\approx \frac{g_{3,2} - g_{2,2}}{\Delta x} \\ \frac{\partial g_{1,3}}{\partial x} &\approx \frac{g_{1,3} - 0}{\Delta x} \\ \frac{\partial g_{2,3}}{\partial x} &\approx \frac{g_{2,3} - g_{1,3}}{\Delta x} \\ \frac{\partial g_{3,3}}{\partial x} &\approx \frac{g_{3,3} - g_{2,3}}{\Delta x} \end{aligned}$$

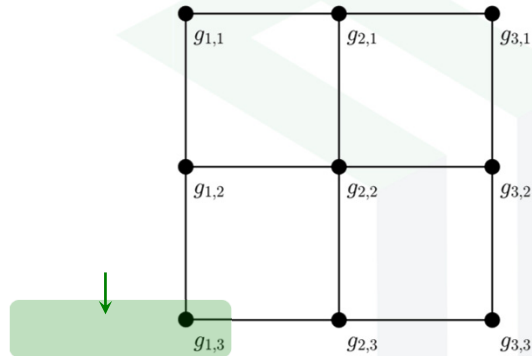
## $[D_x^g]$ Derivative Matrix (7 of 11)



$$\frac{\partial g_{i,j}}{\partial x} \approx \frac{g_{i,j} - g_{i-1,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial g_{1,1}}{\partial x} &\approx \frac{g_{1,1} - 0}{\Delta x} \\ \frac{\partial g_{2,1}}{\partial x} &\approx \frac{g_{2,1} - g_{1,1}}{\Delta x} \\ \frac{\partial g_{3,1}}{\partial x} &\approx \frac{g_{3,1} - g_{2,1}}{\Delta x} \\ \frac{\partial g_{1,2}}{\partial x} &\approx \frac{g_{1,2} - 0}{\Delta x} \\ \frac{\partial g_{2,2}}{\partial x} &\approx \frac{g_{2,2} - g_{1,2}}{\Delta x} \\ \frac{\partial g_{3,2}}{\partial x} &\approx \frac{g_{3,2} - g_{2,2}}{\Delta x} \\ \frac{\partial g_{1,3}}{\partial x} &\approx \frac{g_{1,3} - 0}{\Delta x} \\ \frac{\partial g_{2,3}}{\partial x} &\approx \frac{g_{2,3} - g_{1,3}}{\Delta x} \\ \frac{\partial g_{3,3}}{\partial x} &\approx \frac{g_{3,3} - g_{2,3}}{\Delta x} \end{aligned}$$

## $[D_x^g]$ Derivative Matrix (8 of 11)

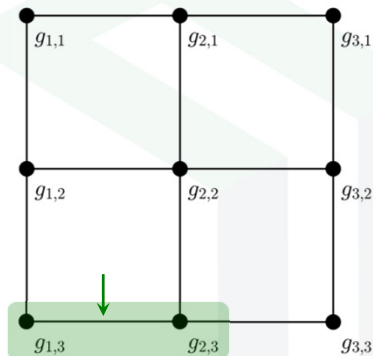


$$\frac{\partial g_{i,j}}{\partial x} \approx \frac{g_{i,j} - g_{i-1,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial g_{1,1}}{\partial x} &\approx \frac{g_{1,1} - 0}{\Delta x} \\ \frac{\partial g_{2,1}}{\partial x} &\approx \frac{g_{2,1} - g_{1,1}}{\Delta x} \\ \frac{\partial g_{3,1}}{\partial x} &\approx \frac{g_{3,1} - g_{2,1}}{\Delta x} \\ \frac{\partial g_{1,2}}{\partial x} &\approx \frac{g_{1,2} - 0}{\Delta x} \\ \frac{\partial g_{2,2}}{\partial x} &\approx \frac{g_{2,2} - g_{1,2}}{\Delta x} \\ \frac{\partial g_{3,2}}{\partial x} &\approx \frac{g_{3,2} - g_{2,2}}{\Delta x} \\ \frac{\partial g_{1,3}}{\partial x} &\approx \frac{g_{1,3} - 0}{\Delta x} \\ \frac{\partial g_{2,3}}{\partial x} &\approx \frac{g_{2,3} - g_{1,3}}{\Delta x} \\ \frac{\partial g_{3,3}}{\partial x} &\approx \frac{g_{3,3} - g_{2,3}}{\Delta x} \end{aligned}$$



## $[D_x^g]$ Derivative Matrix (9 of 11)

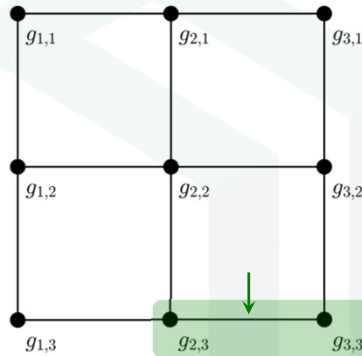


$$\frac{\partial g_{i,j}}{\partial x} \approx \frac{g_{i,j} - g_{i-1,j}}{\Delta x}$$

$$\begin{aligned} \frac{\partial g_{1,1}}{\partial x} &\approx \frac{g_{1,1} - 0}{\Delta x} \\ \frac{\partial g_{2,1}}{\partial x} &\approx \frac{g_{2,1} - g_{1,1}}{\Delta x} \\ \frac{\partial g_{3,1}}{\partial x} &\approx \frac{g_{3,1} - g_{2,1}}{\Delta x} \\ \frac{\partial g_{1,2}}{\partial x} &\approx \frac{g_{1,2} - 0}{\Delta x} \\ \frac{\partial g_{2,2}}{\partial x} &\approx \frac{g_{2,2} - g_{1,2}}{\Delta x} \\ \frac{\partial g_{3,2}}{\partial x} &\approx \frac{g_{3,2} - g_{2,2}}{\Delta x} \\ \frac{\partial g_{1,3}}{\partial x} &\approx \frac{g_{1,3} - 0}{\Delta x} \\ \frac{\partial g_{2,3}}{\partial x} &\approx \frac{g_{2,3} - g_{1,3}}{\Delta x} \\ \frac{\partial g_{3,3}}{\partial x} &\approx \frac{g_{3,3} - g_{2,3}}{\Delta x} \end{aligned}$$



# $[D_x^g]$ Derivative Matrix (10 of 11)



$$\frac{\partial g_{i,j}}{\partial x} \approx \frac{g_{i,j} - g_{i-1,j}}{\Delta x}$$

- $\frac{\partial g_{1,1}}{\partial x} \approx \frac{g_{1,1} - 0}{\Delta x}$
- $\frac{\partial g_{2,1}}{\partial x} \approx \frac{g_{2,1} - g_{1,1}}{\Delta x}$
- $\frac{\partial g_{3,1}}{\partial x} \approx \frac{g_{3,1} - g_{2,1}}{\Delta x}$
- $\frac{\partial g_{1,2}}{\partial x} \approx \frac{g_{1,2} - 0}{\Delta x}$
- $\frac{\partial g_{2,2}}{\partial x} \approx \frac{g_{2,2} - g_{1,2}}{\Delta x}$
- $\frac{\partial g_{3,2}}{\partial x} \approx \frac{g_{3,2} - g_{2,2}}{\Delta x}$
- $\frac{\partial g_{1,3}}{\partial x} \approx \frac{g_{1,3} - 0}{\Delta x}$
- $\frac{\partial g_{2,3}}{\partial x} \approx \frac{g_{2,3} - g_{1,3}}{\Delta x}$
- $\frac{\partial g_{3,3}}{\partial x} \approx \frac{g_{3,3} - g_{2,3}}{\Delta x}$

# $[D_x^g]$ Derivative Matrix (11 of 11)

$$\frac{1}{\Delta x} \begin{bmatrix} g_{1,1} \\ g_{2,1} \\ g_{3,1} \\ g_{1,2} \\ g_{2,2} \\ g_{3,2} \\ g_{1,3} \\ g_{2,3} \\ g_{3,3} \end{bmatrix} = \frac{1}{\Delta x} \begin{bmatrix} g_{1,1} - 0 \\ g_{2,1} - g_{1,1} \\ g_{3,1} - g_{2,1} \\ g_{1,2} - 0 \\ g_{2,2} - g_{1,2} \\ g_{3,2} - g_{2,2} \\ g_{1,3} - 0 \\ g_{2,3} - g_{1,3} \\ g_{3,3} - g_{2,3} \end{bmatrix}$$



- $\frac{\partial g_{1,1}}{\partial x} \approx \frac{g_{1,1} - 0}{\Delta x}$
- $\frac{\partial g_{2,1}}{\partial x} \approx \frac{g_{2,1} - g_{1,1}}{\Delta x}$
- $\frac{\partial g_{3,1}}{\partial x} \approx \frac{g_{3,1} - g_{2,1}}{\Delta x}$
- $\frac{\partial g_{1,2}}{\partial x} \approx \frac{g_{1,2} - 0}{\Delta x}$
- $\frac{\partial g_{2,2}}{\partial x} \approx \frac{g_{2,2} - g_{1,2}}{\Delta x}$
- $\frac{\partial g_{3,2}}{\partial x} \approx \frac{g_{3,2} - g_{2,2}}{\Delta x}$
- $\frac{\partial g_{1,3}}{\partial x} \approx \frac{g_{1,3} - 0}{\Delta x}$
- $\frac{\partial g_{2,3}}{\partial x} \approx \frac{g_{2,3} - g_{1,3}}{\Delta x}$
- $\frac{\partial g_{3,3}}{\partial x} \approx \frac{g_{3,3} - g_{2,3}}{\Delta x}$



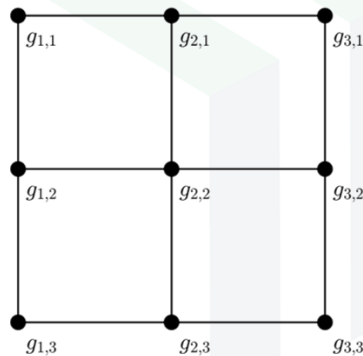
# Left-Handed Derivative Matrix

$$\left[ D_y^g \right]$$



## $\left[ D_y^g \right]$ Derivative Matrix (1 of 11)

$$\frac{\partial g_{i,j}}{\partial y} \cong \frac{g_{i,j} - g_{i,j-1}}{\Delta y}$$



$$\frac{\partial g_{1,1}}{\partial y} \cong \frac{g_{1,1} - 0}{\Delta y}$$

$$\frac{\partial g_{2,1}}{\partial y} \cong \frac{g_{2,1} - 0}{\Delta y}$$

$$\frac{\partial g_{3,1}}{\partial y} \cong \frac{g_{3,1} - 0}{\Delta y}$$

$$\frac{\partial g_{1,2}}{\partial y} \cong \frac{g_{1,2} - g_{1,1}}{\Delta y}$$

$$\frac{\partial g_{2,2}}{\partial y} \cong \frac{g_{2,2} - g_{2,1}}{\Delta y}$$

$$\frac{\partial g_{3,2}}{\partial y} \cong \frac{g_{3,2} - g_{3,1}}{\Delta y}$$

$$\frac{\partial g_{1,3}}{\partial y} \cong \frac{g_{1,3} - g_{1,2}}{\Delta y}$$

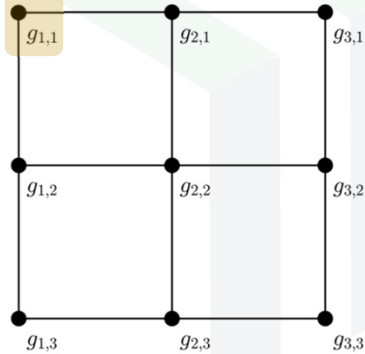
$$\frac{\partial g_{2,3}}{\partial y} \cong \frac{g_{2,3} - g_{2,2}}{\Delta y}$$

$$\frac{\partial g_{3,3}}{\partial y} \cong \frac{g_{3,3} - g_{3,2}}{\Delta y}$$



## $[D_y^g]$ Derivative Matrix (2 of 11)

$$\frac{\partial g_{i,j}}{\partial y} \approx \frac{g_{i,j} - g_{i,j-1}}{\Delta y}$$



$$\frac{\partial g_{1,1}}{\partial y} \approx \frac{g_{1,1} - 0}{\Delta y}$$

$$\frac{\partial g_{2,1}}{\partial y} \approx \frac{g_{2,1} - 0}{\Delta y}$$

$$\frac{\partial g_{3,1}}{\partial y} \approx \frac{g_{3,1} - 0}{\Delta y}$$

$$\frac{\partial g_{1,2}}{\partial y} \approx \frac{g_{1,2} - g_{1,1}}{\Delta y}$$

$$\frac{\partial g_{2,2}}{\partial y} \approx \frac{g_{2,2} - g_{2,1}}{\Delta y}$$

$$\frac{\partial g_{3,2}}{\partial y} \approx \frac{g_{3,2} - g_{3,1}}{\Delta y}$$

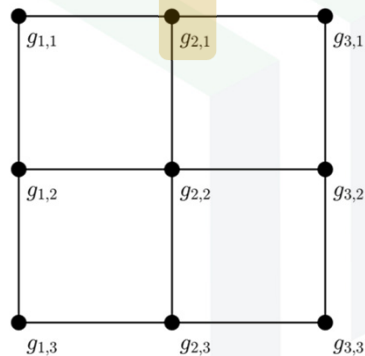
$$\frac{\partial g_{1,3}}{\partial y} \approx \frac{g_{1,3} - g_{1,2}}{\Delta y}$$

$$\frac{\partial g_{2,3}}{\partial y} \approx \frac{g_{2,3} - g_{2,2}}{\Delta y}$$

$$\frac{\partial g_{3,3}}{\partial y} \approx \frac{g_{3,3} - g_{3,2}}{\Delta y}$$

## $[D_y^g]$ Derivative Matrix (3 of 11)

$$\frac{\partial g_{i,j}}{\partial y} \approx \frac{g_{i,j} - g_{i,j-1}}{\Delta y}$$



$$\frac{\partial g_{1,1}}{\partial y} \approx \frac{g_{1,1} - 0}{\Delta y}$$

$$\frac{\partial g_{2,1}}{\partial y} \approx \frac{g_{2,1} - 0}{\Delta y}$$

$$\frac{\partial g_{3,1}}{\partial y} \approx \frac{g_{3,1} - 0}{\Delta y}$$

$$\frac{\partial g_{1,2}}{\partial y} \approx \frac{g_{1,2} - g_{1,1}}{\Delta y}$$

$$\frac{\partial g_{2,2}}{\partial y} \approx \frac{g_{2,2} - g_{2,1}}{\Delta y}$$

$$\frac{\partial g_{3,2}}{\partial y} \approx \frac{g_{3,2} - g_{3,1}}{\Delta y}$$

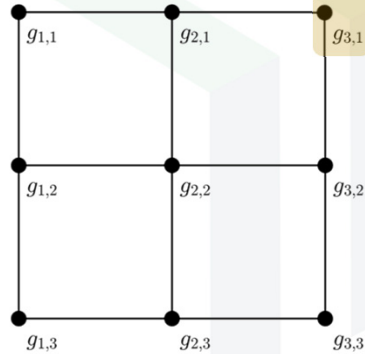
$$\frac{\partial g_{1,3}}{\partial y} \approx \frac{g_{1,3} - g_{1,2}}{\Delta y}$$

$$\frac{\partial g_{2,3}}{\partial y} \approx \frac{g_{2,3} - g_{2,2}}{\Delta y}$$

$$\frac{\partial g_{3,3}}{\partial y} \approx \frac{g_{3,3} - g_{3,2}}{\Delta y}$$

## $[D_y^g]$ Derivative Matrix (4 of 11)

$$\frac{\partial g_{i,j}}{\partial y} \cong \frac{g_{i,j} - g_{i,j-1}}{\Delta y}$$



$$\frac{\partial g_{1,1}}{\partial y} \cong \frac{g_{1,1} - 0}{\Delta y}$$

$$\frac{\partial g_{2,1}}{\partial y} \cong \frac{g_{2,1} - 0}{\Delta y}$$

$$\frac{\partial g_{3,1}}{\partial y} \cong \frac{g_{3,1} - 0}{\Delta y}$$

$$\frac{\partial g_{1,2}}{\partial y} \cong \frac{g_{1,2} - g_{1,1}}{\Delta y}$$

$$\frac{\partial g_{2,2}}{\partial y} \cong \frac{g_{2,2} - g_{2,1}}{\Delta y}$$

$$\frac{\partial g_{3,2}}{\partial y} \cong \frac{g_{3,2} - g_{3,1}}{\Delta y}$$

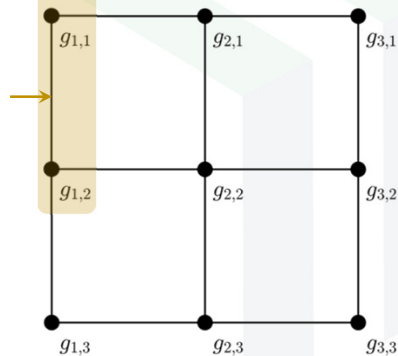
$$\frac{\partial g_{1,3}}{\partial y} \cong \frac{g_{1,3} - g_{1,2}}{\Delta y}$$

$$\frac{\partial g_{2,3}}{\partial y} \cong \frac{g_{2,3} - g_{2,2}}{\Delta y}$$

$$\frac{\partial g_{3,3}}{\partial y} \cong \frac{g_{3,3} - g_{3,2}}{\Delta y}$$

## $[D_y^g]$ Derivative Matrix (5 of 11)

$$\frac{\partial g_{i,j}}{\partial y} \cong \frac{g_{i,j} - g_{i,j-1}}{\Delta y}$$



$$\frac{\partial g_{1,1}}{\partial y} \cong \frac{g_{1,1} - 0}{\Delta y}$$

$$\frac{\partial g_{2,1}}{\partial y} \cong \frac{g_{2,1} - 0}{\Delta y}$$

$$\frac{\partial g_{3,1}}{\partial y} \cong \frac{g_{3,1} - 0}{\Delta y}$$

$$\frac{\partial g_{1,2}}{\partial y} \cong \frac{g_{1,2} - g_{1,1}}{\Delta y}$$

$$\frac{\partial g_{2,2}}{\partial y} \cong \frac{g_{2,2} - g_{2,1}}{\Delta y}$$

$$\frac{\partial g_{3,2}}{\partial y} \cong \frac{g_{3,2} - g_{3,1}}{\Delta y}$$

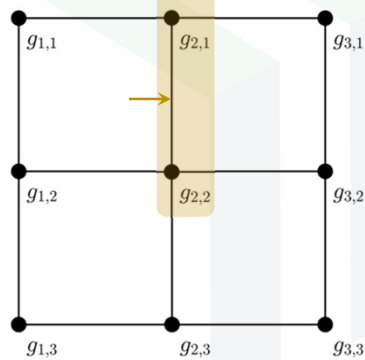
$$\frac{\partial g_{1,3}}{\partial y} \cong \frac{g_{1,3} - g_{1,2}}{\Delta y}$$

$$\frac{\partial g_{2,3}}{\partial y} \cong \frac{g_{2,3} - g_{2,2}}{\Delta y}$$

$$\frac{\partial g_{3,3}}{\partial y} \cong \frac{g_{3,3} - g_{3,2}}{\Delta y}$$

## $[D_y^g]$ Derivative Matrix (6 of 11)

$$\frac{\partial g_{i,j}}{\partial y} \cong \frac{g_{i,j} - g_{i,j-1}}{\Delta y}$$



$$\frac{\partial g_{1,1}}{\partial y} \cong \frac{g_{1,1} - 0}{\Delta y}$$

$$\frac{\partial g_{2,1}}{\partial y} \cong \frac{g_{2,1} - 0}{\Delta y}$$

$$\frac{\partial g_{3,1}}{\partial y} \cong \frac{g_{3,1} - 0}{\Delta y}$$

$$\frac{\partial g_{1,2}}{\partial y} \cong \frac{g_{1,2} - g_{1,1}}{\Delta y}$$

$$\frac{\partial g_{2,2}}{\partial y} \cong \frac{g_{2,2} - g_{2,1}}{\Delta y}$$

$$\frac{\partial g_{3,2}}{\partial y} \cong \frac{g_{3,2} - g_{3,1}}{\Delta y}$$

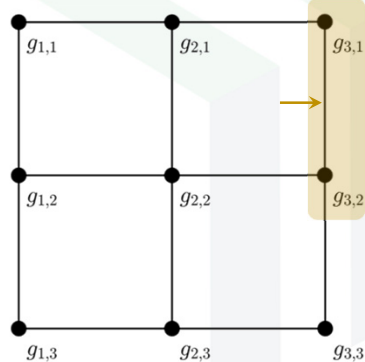
$$\frac{\partial g_{1,3}}{\partial y} \cong \frac{g_{1,3} - g_{1,2}}{\Delta y}$$

$$\frac{\partial g_{2,3}}{\partial y} \cong \frac{g_{2,3} - g_{2,2}}{\Delta y}$$

$$\frac{\partial g_{3,3}}{\partial y} \cong \frac{g_{3,3} - g_{3,2}}{\Delta y}$$

## $[D_y^g]$ Derivative Matrix (7 of 11)

$$\frac{\partial g_{i,j}}{\partial y} \cong \frac{g_{i,j} - g_{i,j-1}}{\Delta y}$$



$$\frac{\partial g_{1,1}}{\partial y} \cong \frac{g_{1,1} - 0}{\Delta y}$$

$$\frac{\partial g_{2,1}}{\partial y} \cong \frac{g_{2,1} - 0}{\Delta y}$$

$$\frac{\partial g_{3,1}}{\partial y} \cong \frac{g_{3,1} - 0}{\Delta y}$$

$$\frac{\partial g_{1,2}}{\partial y} \cong \frac{g_{1,2} - g_{1,1}}{\Delta y}$$

$$\frac{\partial g_{2,2}}{\partial y} \cong \frac{g_{2,2} - g_{2,1}}{\Delta y}$$

$$\frac{\partial g_{3,2}}{\partial y} \cong \frac{g_{3,2} - g_{3,1}}{\Delta y}$$

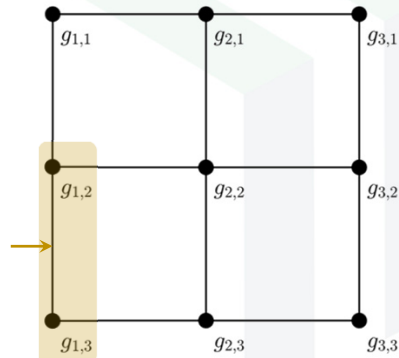
$$\frac{\partial g_{1,3}}{\partial y} \cong \frac{g_{1,3} - g_{1,2}}{\Delta y}$$

$$\frac{\partial g_{2,3}}{\partial y} \cong \frac{g_{2,3} - g_{2,2}}{\Delta y}$$

$$\frac{\partial g_{3,3}}{\partial y} \cong \frac{g_{3,3} - g_{3,2}}{\Delta y}$$

## $[D_y^g]$ Derivative Matrix (8 of 11)

$$\frac{\partial g_{i,j}}{\partial y} \cong \frac{g_{i,j} - g_{i,j-1}}{\Delta y}$$



$$\frac{\partial g_{1,1}}{\partial y} \cong \frac{g_{1,1} - 0}{\Delta y}$$

$$\frac{\partial g_{2,1}}{\partial y} \cong \frac{g_{2,1} - 0}{\Delta y}$$

$$\frac{\partial g_{3,1}}{\partial y} \cong \frac{g_{3,1} - 0}{\Delta y}$$

$$\frac{\partial g_{1,2}}{\partial y} \cong \frac{g_{1,2} - g_{1,1}}{\Delta y}$$

$$\frac{\partial g_{2,2}}{\partial y} \cong \frac{g_{2,2} - g_{2,1}}{\Delta y}$$

$$\frac{\partial g_{3,2}}{\partial y} \cong \frac{g_{3,2} - g_{3,1}}{\Delta y}$$

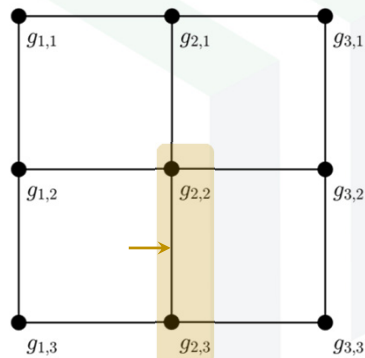
$$\frac{\partial g_{1,3}}{\partial y} \cong \frac{g_{1,3} - g_{1,2}}{\Delta y}$$

$$\frac{\partial g_{2,3}}{\partial y} \cong \frac{g_{2,3} - g_{2,2}}{\Delta y}$$

$$\frac{\partial g_{3,3}}{\partial y} \cong \frac{g_{3,3} - g_{3,2}}{\Delta y}$$

## $[D_y^g]$ Derivative Matrix (9 of 11)

$$\frac{\partial g_{i,j}}{\partial y} \cong \frac{g_{i,j} - g_{i,j-1}}{\Delta y}$$



$$\frac{\partial g_{1,1}}{\partial y} \cong \frac{g_{1,1} - 0}{\Delta y}$$

$$\frac{\partial g_{2,1}}{\partial y} \cong \frac{g_{2,1} - 0}{\Delta y}$$

$$\frac{\partial g_{3,1}}{\partial y} \cong \frac{g_{3,1} - 0}{\Delta y}$$

$$\frac{\partial g_{1,2}}{\partial y} \cong \frac{g_{1,2} - g_{1,1}}{\Delta y}$$

$$\frac{\partial g_{2,2}}{\partial y} \cong \frac{g_{2,2} - g_{2,1}}{\Delta y}$$

$$\frac{\partial g_{3,2}}{\partial y} \cong \frac{g_{3,2} - g_{3,1}}{\Delta y}$$

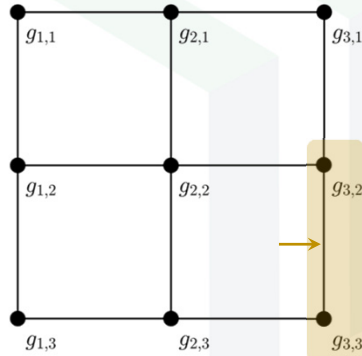
$$\frac{\partial g_{1,3}}{\partial y} \cong \frac{g_{1,3} - g_{1,2}}{\Delta y}$$

$$\frac{\partial g_{2,3}}{\partial y} \cong \frac{g_{2,3} - g_{2,2}}{\Delta y}$$

$$\frac{\partial g_{3,3}}{\partial y} \cong \frac{g_{3,3} - g_{3,2}}{\Delta y}$$

# $[D_y^g]$ Derivative Matrix (10 of 11)

$$\frac{\partial g_{i,j}}{\partial y} \cong \frac{g_{i,j} - g_{i,j-1}}{\Delta y}$$



- $\frac{\partial g_{1,1}}{\partial y} \cong \frac{g_{1,1} - 0}{\Delta y}$
- $\frac{\partial g_{2,1}}{\partial y} \cong \frac{g_{2,1} - 0}{\Delta y}$
- $\frac{\partial g_{3,1}}{\partial y} \cong \frac{g_{3,1} - 0}{\Delta y}$
- $\frac{\partial g_{1,2}}{\partial y} \cong \frac{g_{1,2} - g_{1,1}}{\Delta y}$
- $\frac{\partial g_{2,2}}{\partial y} \cong \frac{g_{2,2} - g_{2,1}}{\Delta y}$
- $\frac{\partial g_{3,2}}{\partial y} \cong \frac{g_{3,2} - g_{3,1}}{\Delta y}$
- $\frac{\partial g_{1,3}}{\partial y} \cong \frac{g_{1,3} - g_{1,2}}{\Delta y}$
- $\frac{\partial g_{2,3}}{\partial y} \cong \frac{g_{2,3} - g_{2,2}}{\Delta y}$
- $\frac{\partial g_{3,3}}{\partial y} \cong \frac{g_{3,3} - g_{3,2}}{\Delta y}$

# $[D_y^g]$ Derivative Matrix (11 of 11)

$$\left[ \frac{1}{\Delta y} \right]$$

$$\begin{bmatrix} g_{1,1} \\ g_{2,1} \\ g_{3,1} \\ g_{1,2} \\ g_{2,2} \\ g_{3,2} \\ g_{1,3} \\ g_{2,3} \\ g_{3,3} \end{bmatrix} = \frac{1}{\Delta y} \begin{bmatrix} g_{1,1} - 0 \\ g_{2,1} - 0 \\ g_{3,1} - 0 \\ g_{1,2} - g_{1,1} \\ g_{2,2} - g_{2,1} \\ g_{3,2} - g_{3,1} \\ g_{1,3} - g_{1,2} \\ g_{2,3} - g_{2,2} \\ g_{3,3} - g_{3,2} \end{bmatrix}$$



- $\frac{\partial g_{1,1}}{\partial y} \cong \frac{g_{1,1} - 0}{\Delta y}$
- $\frac{\partial g_{2,1}}{\partial y} \cong \frac{g_{2,1} - 0}{\Delta y}$
- $\frac{\partial g_{3,1}}{\partial y} \cong \frac{g_{3,1} - 0}{\Delta y}$
- $\frac{\partial g_{1,2}}{\partial y} \cong \frac{g_{1,2} - g_{1,1}}{\Delta y}$
- $\frac{\partial g_{2,2}}{\partial y} \cong \frac{g_{2,2} - g_{2,1}}{\Delta y}$
- $\frac{\partial g_{3,2}}{\partial y} \cong \frac{g_{3,2} - g_{3,1}}{\Delta y}$
- $\frac{\partial g_{1,3}}{\partial y} \cong \frac{g_{1,3} - g_{1,2}}{\Delta y}$
- $\frac{\partial g_{2,3}}{\partial y} \cong \frac{g_{2,3} - g_{2,2}}{\Delta y}$
- $\frac{\partial g_{3,3}}{\partial y} \cong \frac{g_{3,3} - g_{3,2}}{\Delta y}$







## The Rule Using Dirichlet Boundary Conditions

$$\text{If } N_x = 1, \text{ then } [D_x^f] = [D_x^g] = \dots = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

$$\text{If } N_y = 1, \text{ then } [D_y^f] = [D_y^g] = \dots = \begin{bmatrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

These are all sparse matrices of all zeros. 