Computational Science:
Computational Methods in Engineering

Finite-Difference Approximations

Outline

- What are finite-difference approximations?
- Generalizing the concept of finite-differences.

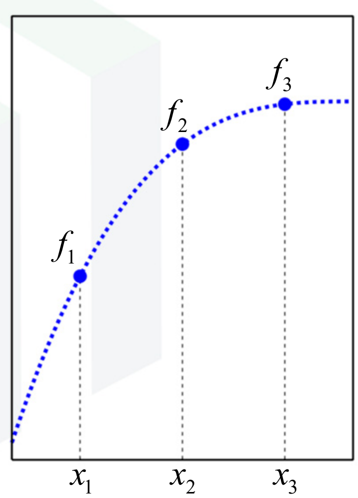
What are Finite-Difference Approximations?

What are Finite-Difference Approximations? (1 of 3)

Very often in science and engineering we must calculate a derivative.

When we are processing data from measurements or simulations, there may not be an analytical equation to work with symbolically.

Typically, we only know the function at discrete points.

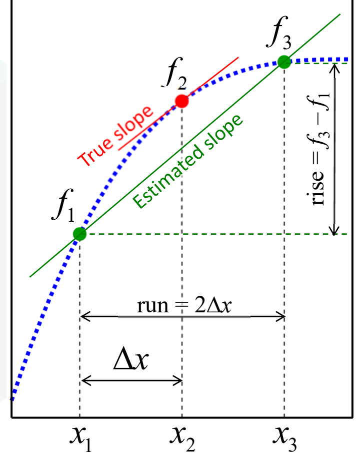


What are Finite-Difference Approximations? (2 of 3)

Suppose we wish to numerically calculate the first-order derivative at x_2.

The first-order derivative is slope. We can estimate the slope as rise÷run using information from surrounding points.

$$f'(x_2) \approx \frac{\text{rise}}{\text{run}} = \frac{f_3 - f_1}{2\Delta x}$$



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What are Finite-Difference Approximations? (3 of 3)

We can estimate the derivative at the midpoint between data points.

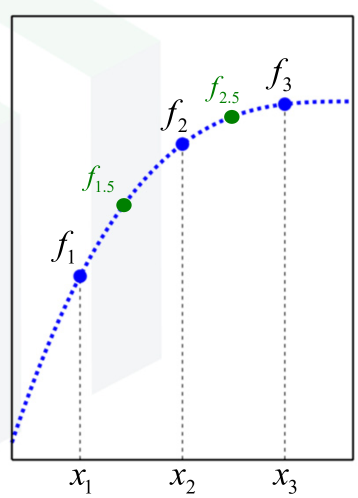
$$f'(x_{1.5}) = \frac{f_2 - f_1}{\Delta x} \qquad f'(x_{2.5}) = \frac{f_3 - f_2}{\Delta x}$$

The second-order derivative is the slope of the slope.

$$f''(x_2) = \frac{f'(x_{2.5}) - f'(x_{1.5})}{\Delta x}$$

$$= \frac{\dfrac{f_3 - f_2}{\Delta x} - \dfrac{f_2 - f_1}{\Delta x}}{\Delta x}$$

$$= \frac{f_3 - 2f_2 + f_1}{\Delta x^2}$$



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First Two Finite-Difference Approximations

Two finite-difference approximations have already been derived!

$$f'(x_2) \approx \frac{f_3 - f_1}{2\Delta x}$$

$$f''(x_2) \approx \frac{f_3 - 2f_2 + f_1}{\Delta x^2}$$

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Generalizing the Concept of Finite-Differences

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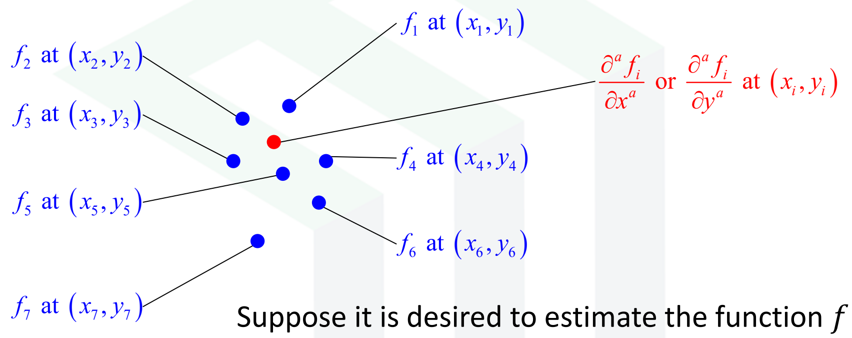
General Concept of Finite-Difference Approximations (1 of 2)



Suppose it is desired to estimate the function $f(x)$ or one of its derivatives at location (x_i, y_i).

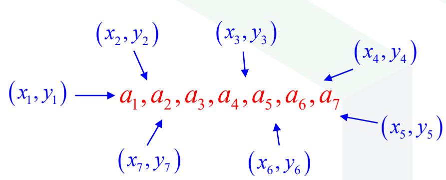
$$\frac{\partial^a f_i}{\partial x^a} \text{ or } \frac{\partial^a f_i}{\partial y^a} = a_1 f_1 + a_2 f_2 + a_3 f_3 + a_4 f_4 + a_5 f_5 + a_6 f_6 + a_7 f_7$$

It is always possible to estimate this from a linear sum of the function values at surrounding points.

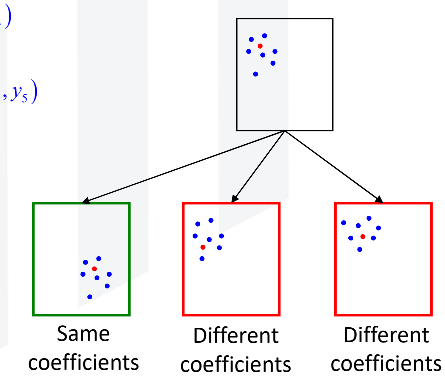
General Concept of Finite-Difference Approximations (2 of 2)

The trick is, how arethe coefficients a_n calculated?

These are a function of the positions of the points.

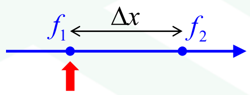


Finite-difference coefficients depend only on the relative position of the points. They do not depend on the absolute positions.



Types of Finite-Differences

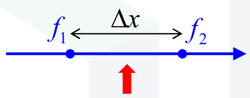
$$\frac{df_1}{dx} \approx \frac{f_2 - f_1}{\Delta x}$$



Forward Finite-Difference

Reaches ahead to use data in the forward direction.

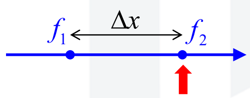
$$\frac{df_{1.5}}{dx} \approx \frac{f_2 - f_1}{\Delta x}$$



Central Finite-Difference

Reaches symmetrically to use data in both directions for highest accuracy.

$$\frac{df_2}{dx} \approx \frac{f_2 - f_1}{\Delta x}$$



Backward Finite-Difference

Reaches behind to use data in the backward direction.

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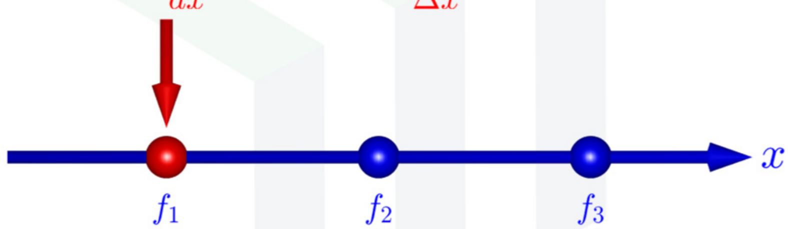
Continuum of Finite-Difference Approximations (1 of 2)

$$\frac{df_{1.0}}{dx} \simeq \frac{-1.5 f_1 + 2.0 f_2 - 0.5 f_3}{\Delta x}$$



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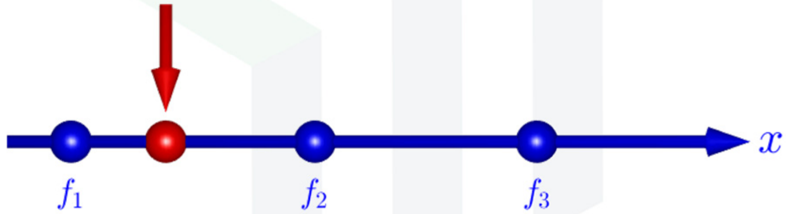
Continuum of Finite-Difference Approximations (2 of 2)

$$\frac{df_{1.0}}{dx} \simeq \frac{-1.0f_1 + 1.1f_2 - 0.1f_3}{\Delta x}$$



Two Key Considerations

1. The position of the points from which the finite-difference approximation is calculated. More closely spaced points improves accuracy, but typically leads to more computations.

2. The location of the point where the finite-difference is being evaluated. It is usually desired to have this point as centered as possible for best accuracy.