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Electromagnetics:

Microwave Engineering

Impedance Transformation on Smith Charts

Lecture Outline

- Impedance transformation
- Example 1

Impedance Transformation

Normalized Impedance Transformation Formula

The impedance transformation formula was

$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan \beta\ell}{Z_0 + jZ_L \tan \beta\ell}$$

This can now be written in terms of the reflection coefficient Γ.

$$Z_{\text{in}} = Z_0 \frac{Z_L \cos \beta\ell + jZ_0 \sin \beta\ell}{Z_0 \cos \beta\ell + jZ_L \sin \beta\ell} = Z_0 \frac{0.5Z_L \left(e^{j\beta\ell} + e^{-j\beta\ell}\right) + 0.5Z_0 \left(e^{j\beta\ell} - e^{-j\beta\ell}\right)}{0.5Z_0 \left(e^{j\beta\ell} + e^{-j\beta\ell}\right) + 0.5Z_Z \left(e^{j\beta\ell} - e^{-j\beta\ell}\right)}$$

$$= Z_0 \frac{Z_L e^{j\beta\ell} + Z_L e^{-j\beta\ell} + Z_0 e^{j\beta\ell} - Z_0 e^{-j\beta\ell}}{Z_0 e^{j\beta\ell} + Z_0 e^{-j\beta\ell} + Z_L e^{j\beta\ell} - Z_L e^{-j\beta\ell}} = Z_0 \frac{\left(Z_L + Z_0\right) e^{j\beta\ell} + \left(Z_L - Z_0\right) e^{-j\beta\ell}}{\left(Z_L + Z_0\right) e^{j\beta\ell} - \left(Z_L - Z_0\right) e^{-j\beta\ell}}$$

$$= Z_0 \frac{1 + \dfrac{\left(Z_L - Z_0\right) e^{-j\beta\ell}}{\left(Z_L + Z_0\right) e^{j\beta\ell}}}{1 - \dfrac{\left(Z_L - Z_0\right) e^{-j\beta\ell}}{\left(Z_L + Z_0\right) e^{j\beta\ell}}} = Z_0 \frac{1 + \Gamma e^{-j2\beta\ell}}{1 - \Gamma e^{-j2\beta\ell}}$$

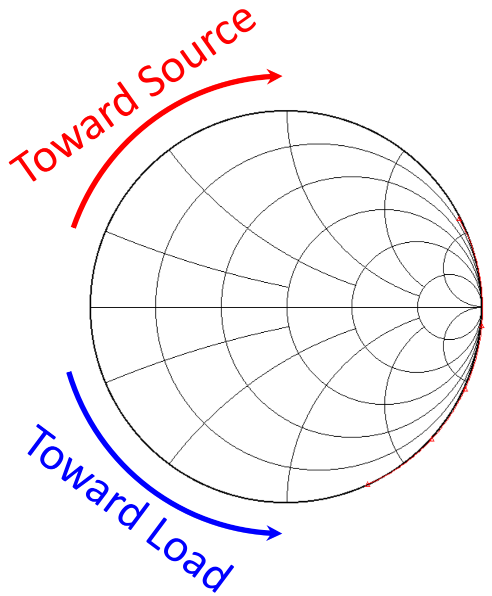
Normalized the input impedance by dividing by Z_0.

$$z_{\text{in}} = \frac{1 + \Gamma e^{-j2\beta\ell}}{1 - \Gamma e^{-j2\beta\ell}}$$

Interpreting the Formula

The normalized impedance transformation formula was

$$z_{\text{in}}\left(\ell\right)=\frac{1+\Gamma e^{-j2\beta\ell}}{1-\Gamma e^{-j2\beta\ell}}$$



Recognizing that $\Gamma = |\Gamma|e^{j\theta}$, this equation can be written as

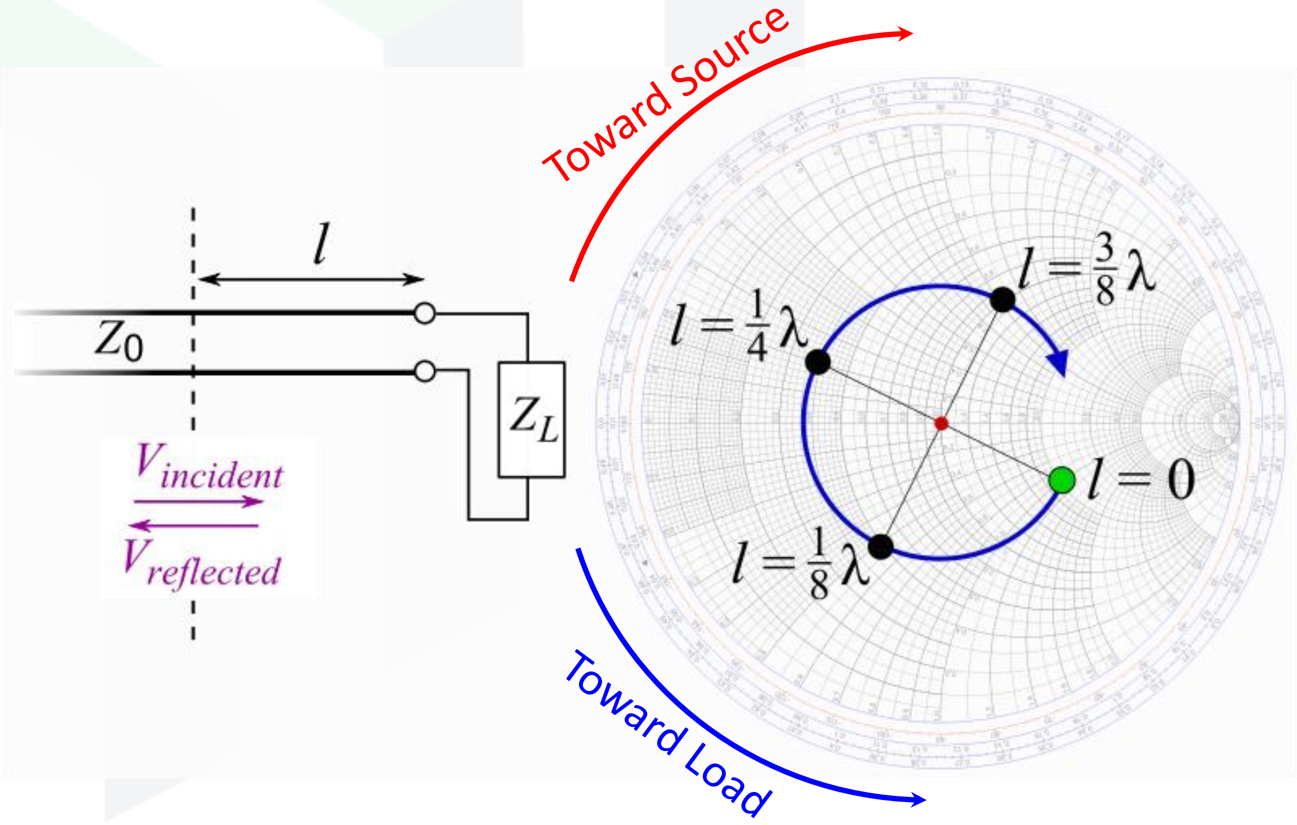
$$z_{\text{in}}\left(\ell\right)=\frac{1+\left|\Gamma\right|e^{j\theta}e^{-j2\beta\ell}}{1-\left|\Gamma\right|e^{j\theta}e^{-j2\beta\ell}}=\frac{1+\left|\Gamma\right|e^{j(\theta-2\beta\ell)}}{1-\left|\Gamma\right|e^{j(\theta-2\beta\ell)}}$$

Thus we see that traversing along the transmission line simply changes the phase of the reflection coefficient.

As we move away from the load and toward the source, we subtract phase from θ. On the Smith chart, we rotate clockwise (CW) around the constant VSWR circle by an amount $2\beta l$. A complete rotation corresponds to $\lambda/2$.

Impedance Transformation on the Smith chart

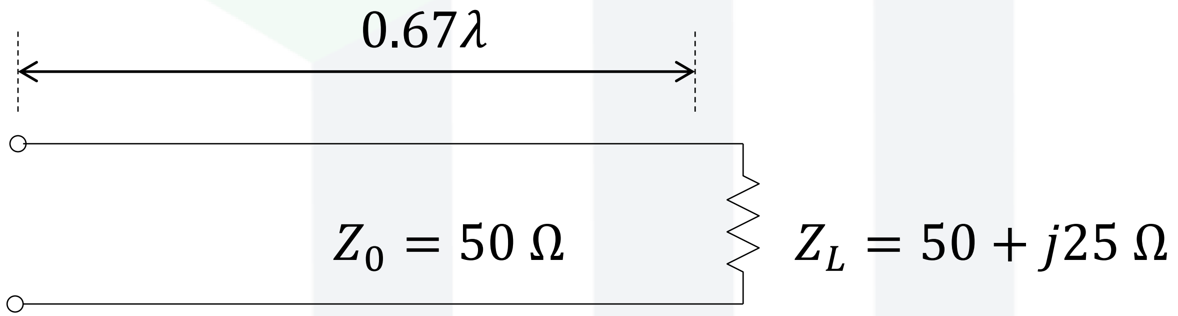
1. Plot the normalized load impedance on the Smith chart.
2. Move clockwise around the middle of the Smith chart as we move away from the load (toward generator). One rotation is $\lambda/2$ in the transmission line.
3. The final point is the input impedance of the line.



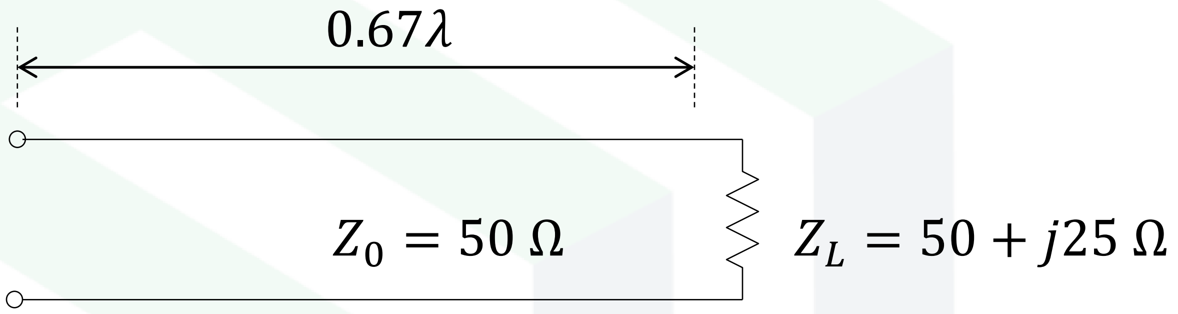
Example 1

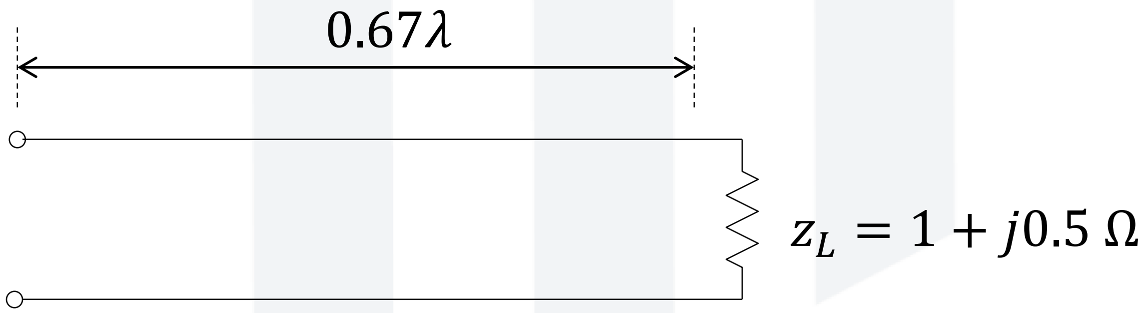
Example #1 – Impedance Transformation: Normalize the Parameters

What is the impedance of a transmission line with intrinsic impedance $Z_0 = 50\ \Omega$ and terminated in a load with impedance $Z_L = 50 + j25\ \Omega$, at a distance 0.67λ away from the load?



Example #1 – Impedance Transformation: Normalize the Parameters





$$z_{\text{in}}(\ell) = \frac{z_L + j\tan\beta\ell}{1 + jz_L\tan\beta\ell} = \frac{(1 + j0.5) + j\tan(2\pi\cdot 0.67)}{1 + j(1 + j0.5)\tan(2\pi\cdot 0.67)} = 1.299 - j0.485$$

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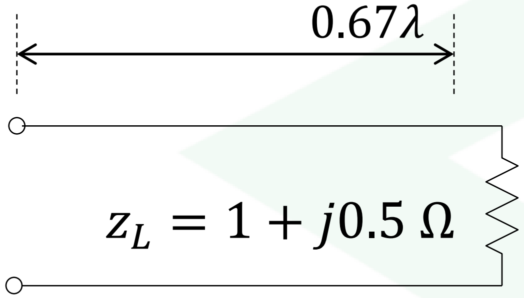
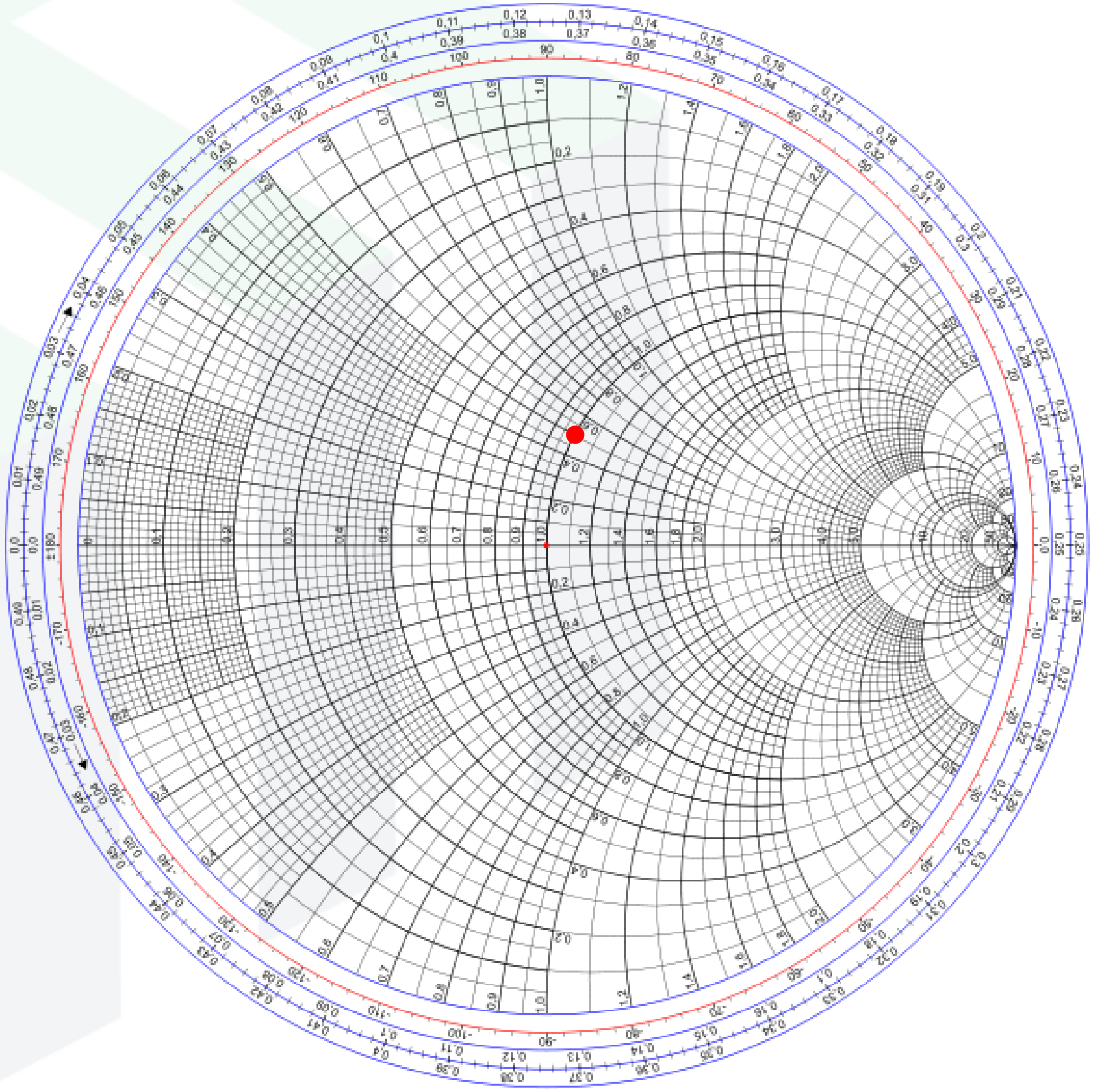
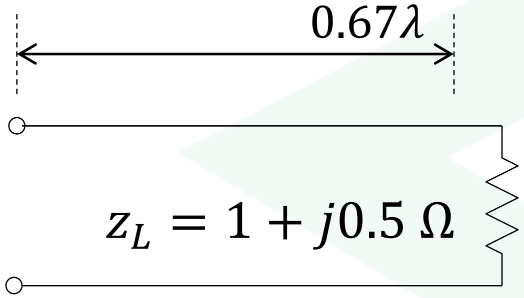
Example #1 – Impedance Transformation: Plot load impedance





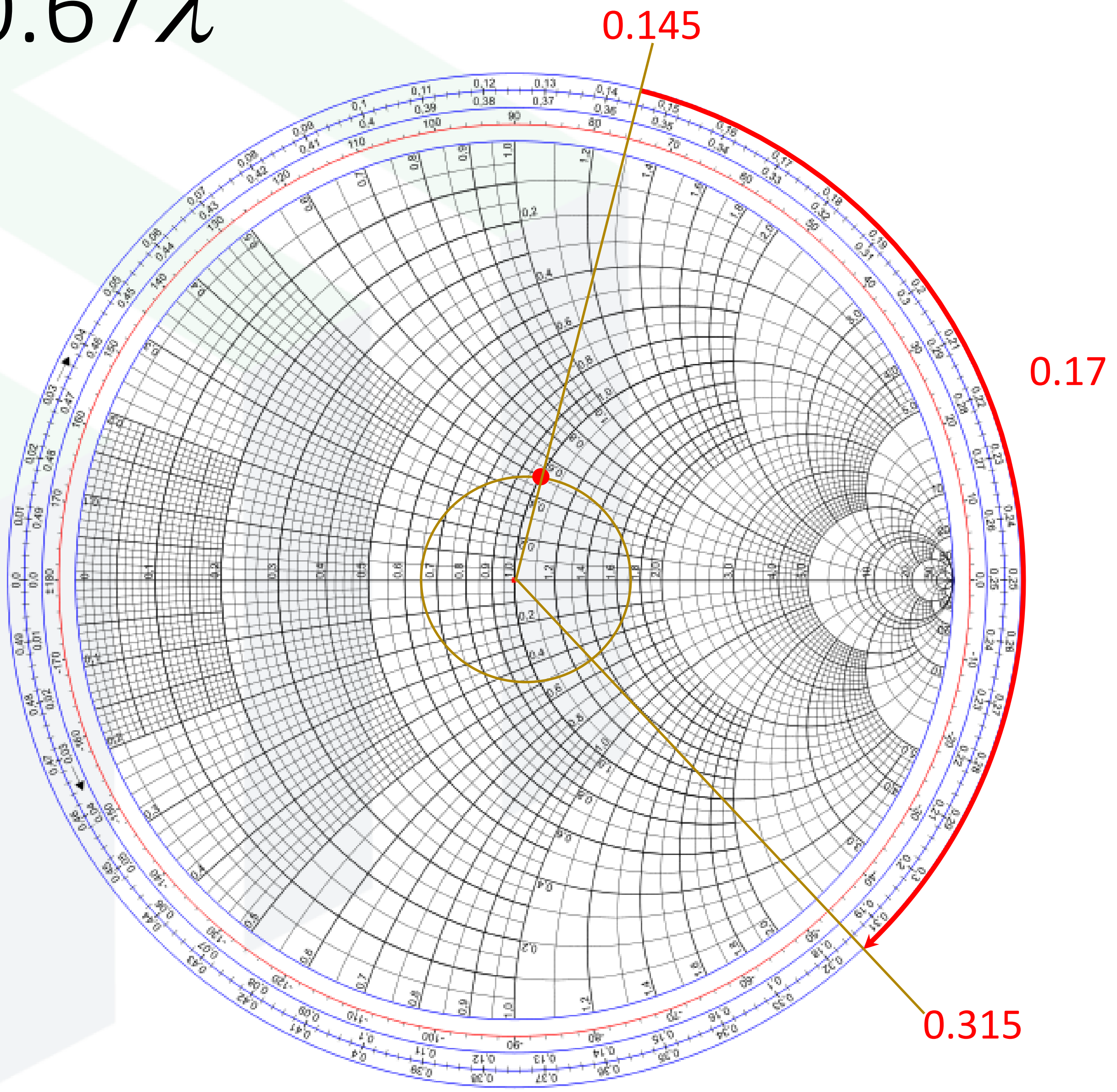
Example #1 – Impedance Transformation: Walk away from load 0.67λ



Since the Smith chart repeats every 0.5λ, traversing 0.67λ is the same as traversing 0.17λ.

Here we start at 0.145 on the Smith chart.

We traverse around the chart to
0.145 + 0.17 = 0.315.



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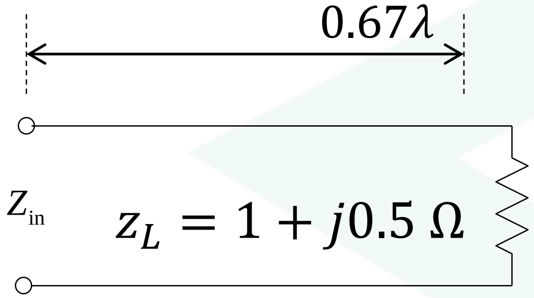
Example #1 – Impedance Transformation: Determine input impedance

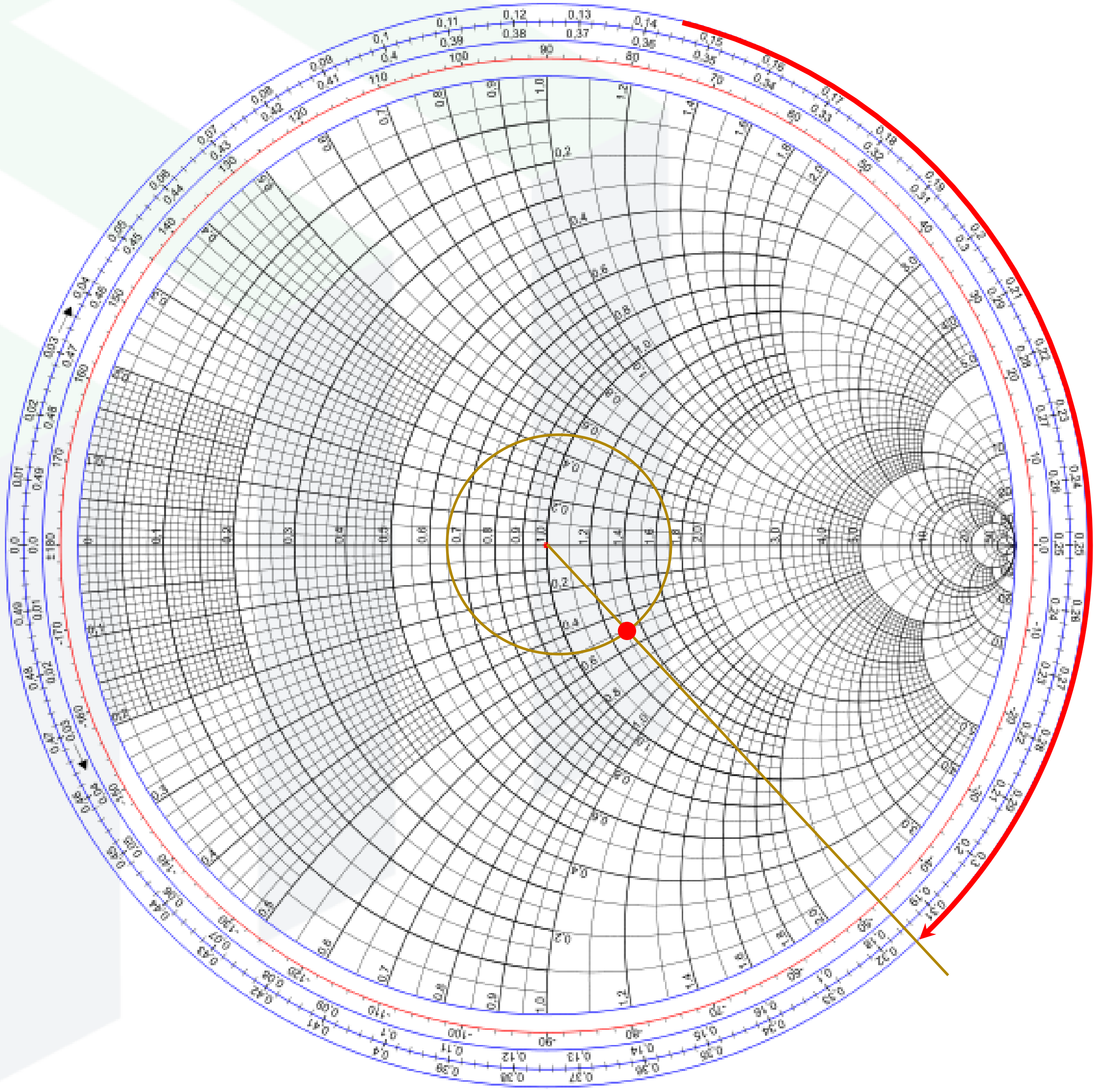


Reflection at the load will be the same regardless of the length of line.

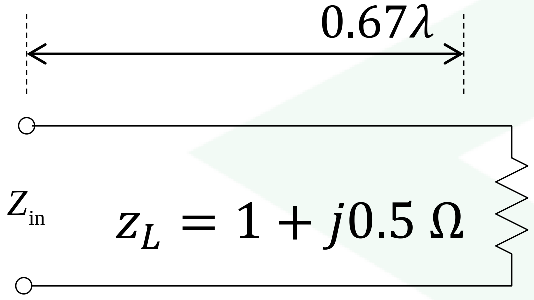
Therefore the VSWR will the same.

The input impedance must lie on the same VSWR plane.

$$z_{\text{in}} \approx 1.3 - j0.5$$



Example #1 – Impedance Transformation: Denormalize



To determine the actual input impedance, we denormalize.

$$Z_{\text{in}} = Z_0 z_{\text{in}} \approx (50\ \Omega)(1.3 - j0.5)$$
$$= \boxed{65 - j25\ \Omega}$$