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Computational Science:
Computational Methods in Engineering

Implementation of Slab Waveguide Analysis

Outline

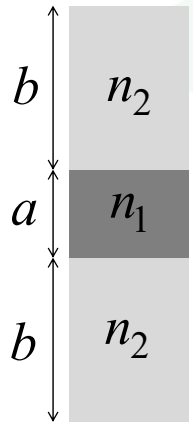
- Implementation in MATLAB
- More About Resolution and Spacer Regions

Implementation in MATLAB

Implementation Outline

1. Initialize MATLAB
2. Dashboard (materials, dimensions, etc.)
3. Calculate Grid
4. Build Device on Grid
5. Perform Finite-Difference Analysis
6. Visualize the Results

Dashboard



How big should b be?

→ Enough to allow the mode to decay to zero before reaching the boundary.

What grid resolution should be used?

→ Convergence

```
% slabdemo.m

% INITIALIZE MATLAB
close all;
clc;
clear all;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% DASHBOARD
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% FREE SPACE WAVELENGTH
lam0 = 1.0;

% SLAB PARAMETERS
n1 = 2.0;
n2 = 1.0;
a  = 3*lam0;

% GRID
b    = 5*lam0;
NRES = 10;
dx   = lam0/NRES;

% NUMBER OF MODES TO CALCULATE
M = 5;
```

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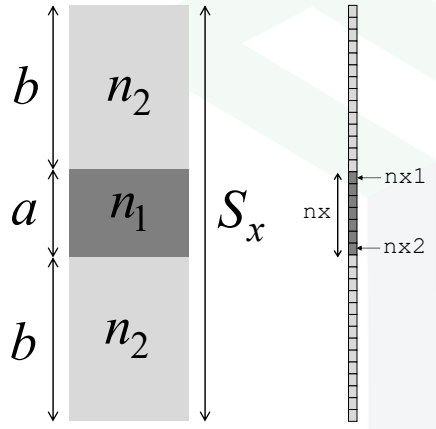
Build Device on Grid




```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% BUILD WAVEGUIDE
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% COMPUTE GRID
Sx = a + 2*b;
Nx = ceil(Sx/dx);
Sx = Nx*dx;

xa = [0.5:Nx-0.5]*dx;
xa = xa - mean(xa);

% COMPUTE START AND STOP INDICES
nx  = round(a/dx);
nx1 = round((Nx - nx)/2);
nx2 = nx1 + nx - 1;

% BUILD N
N          = zeros(Nx,1);
N(1:nx1-1)  = n2;
N(nx1:nx2)  = n1;
N(nx2+1:Nx) = n2;
```

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Perform Finite-Difference Analysis

$$\mathbf{D}_{\tilde{x}}^2 = \frac{1}{\left(k_0 \Delta x\right)^2} \begin{bmatrix} -2 & 1 & & & & & \\ 1 & -2 & 1 & & & & \\ & 1 & -2 & 1 & & & \\ & & & \ddots & & & \\ & & & 1 & -2 & 1 & \\ & & & & 1 & -2 & 1 \\ & & & & & 1 & -2 \end{bmatrix}$$

$$\mathbf{N} = \begin{bmatrix} n(1) & & & & \\ & n(2) & & & \\ & & n(3) & & \\ & & & \ddots & \\ & & & & n(N_x) \end{bmatrix}$$

$$\underbrace{\left(\mathbf{D}_{\tilde{x}}^2 + \mathbf{N}^2\right)}_{\mathbf{A}} \mathbf{e}_y = n_{\text{eff}}^2 \mathbf{e}_y$$

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% PERFORM FD ANALYSIS
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% CALCULATE k0
k0 = 2*pi/lam0;

% BUILD DX2
DX2 = sparse(Nx,Nx);
DX2 = spdiags(+1*ones(Nx,1),-1,DX2);
DX2 = spdiags(-2*ones(Nx,1), 0,DX2);
DX2 = spdiags(+1*ones(Nx,1),+1,DX2);
DX2 = DX2 / (k0*dx)^2;

% MAKE N DIAGONAL
N = diag(sparse(N(:)));

% SOLVE EIGEN-VALUE PROBLEM
A     = DX2 + N^2;
[V,D] = eig(full(A));
NEFF  = sqrt(diag(D));
```

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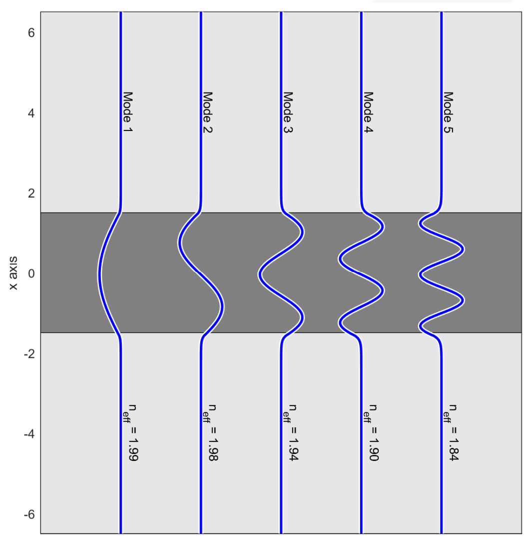
Visualize the Results




```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% VISUALIZE
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% SORT MODES
[~,ind] = sort(real(NEFF),'descend');
V       = V(:,ind);
NEFF    = NEFF(ind);

% OPEN FIGURE WINDOW
figure('Color','w');
hold on;

% DRAW SLAB WAVEGUIDE
x = [0 2*(M+1) 2*(M+1) 0 0];
y = [ -b-a/2 -b-a/2 b+a/2 b+a/2 -b-a/2 ];
fill(x,y,0.9*[1 1 1]);
y = [ -a/2 -a/2 a/2 a/2 -a/2 ];
fill(x,y,0.5*[1 1 1]);

% DRAW AND LABEL MODES
for m = 1 : M
    x0 = 2*m;
    y0 = (a + b)/2;
    x = x0 + 3*V(:,m);
    y = linspace(-b-a/2,b+a/2,Nx);
    line(x,y,'Color','w','LineWidth',4);
    h = line(x,y,'Color','b','LineWidth',2);
    text(x0,y0,['Mode ' num2str(m)],'Rotation',-90,...
         'HorizontalAlignment','center','VerticalAlignment','bottom');
    text(x0,-y0,['n_{eff} = ' num2str(NEFF(m),'%4.2f')],'Rotation',-90,...
         'HorizontalAlignment','center','VerticalAlignment','bottom');
end

% SET GRAPHICS VIEW
hold off;
h2 = get(h,'Parent');
set(h2,'XTick',[]);
axis equal tight;
ylabel('x axis');
```

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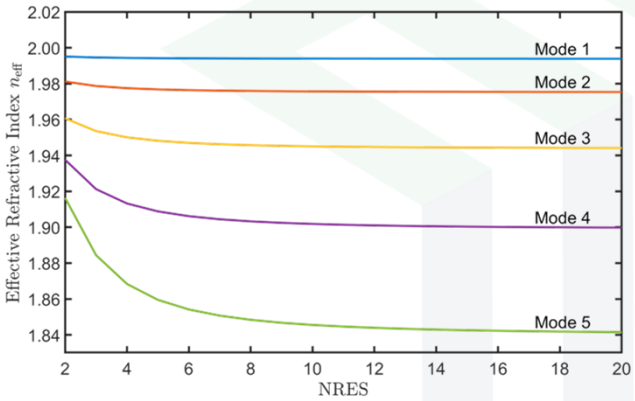
More About Resolution and Spacer Regions

Convergence Study for NRES



$$\Delta x = \frac{\lambda_0}{\text{NRES}}$$

Notes
- Higher-order modes converge slower.
- Higher-order modes have a smaller n_{eff}.

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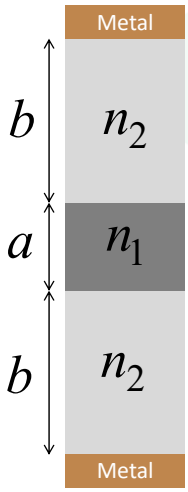
Spacer Region b



Remember the Dirichlet boundary conditions? Values outside of the grid are forced to zero.

This means what is being simulated is a slab waveguide inside of a large metal waveguide.

It is only possible to get an accurate simulation of the slab waveguide when the metal waveguide is large enough to not matter.

Choose b to be large enough to ensure the metal waveguide is insignificant.

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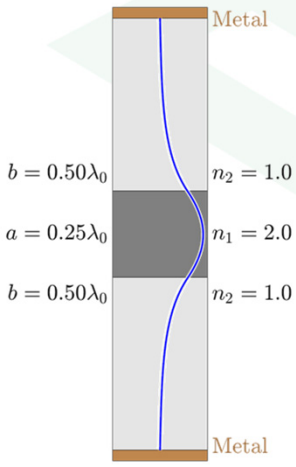
Effect of Spacer Region Size



$n_{\text{eff}} = 1.6622$

If the spacer region b is too small, the outer metal waveguide becomes significant and the results for the slab are not accurate.

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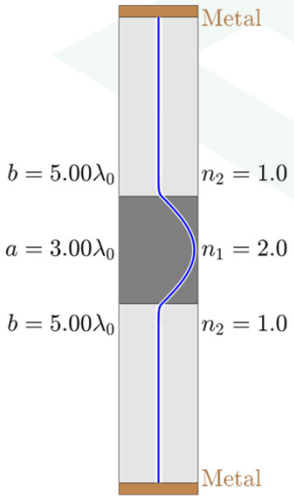
Conditions for Large Evanescent Fields: *Thin Waveguides*



Thin dielectric waveguides have large evanescent fields.

The spacer region b must be big enough to sufficiently encompass the evanescent field in order to give an accurate simulation.

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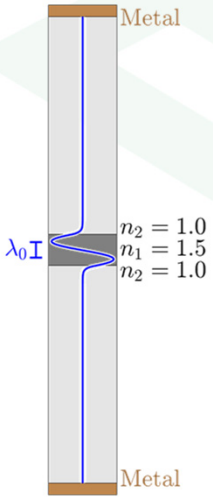
Conditions for Large Evanescent Fields: *Modes Near Cutoff*



Guided modes operating near cutoff have very large evanescent fields.

The spacer region b must be big enough to sufficiently encompass the evanescent field in order to give an accurate simulation.

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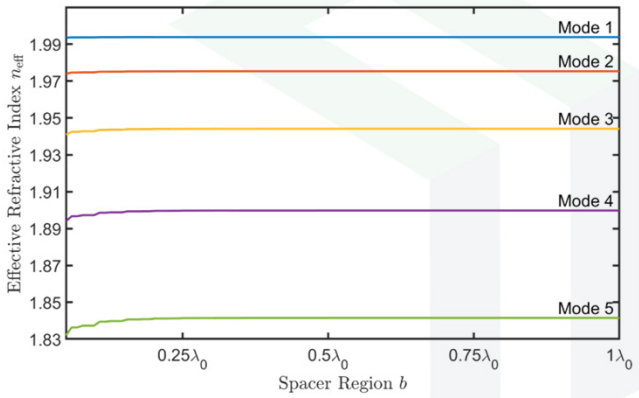
Convergence Study for Spacer Region b



Notes

- Under normal circumstances, the spacer region size can be ~$0.25\lambda_0$.
- Modes near cutoff require larger spacer regions to resolve.
- Thin waveguides may require larger spacer regions.
- Always check for convergence of spacer region size.

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