



Computational Science:
Computational Methods in Engineering

Introduction & Discrete Integration



Why Use Numerical Integration?

How can the following integral be evaluated?

$$\int_a^b e^{x^2} dx = ?$$

No analytical solution exists to perform this integration by hand.

This must be integrated by other means.



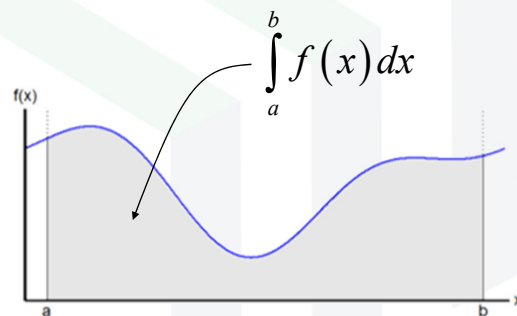
Discrete Integration

...also called a *Riemann Sum*



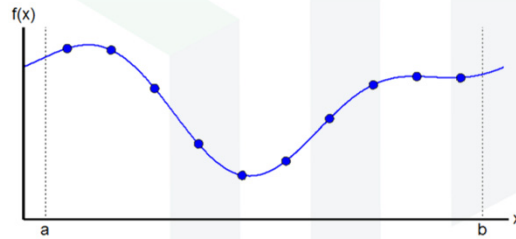
Problem Setup

Suppose a function $f(x)$ and is to be integrated from a to b .



Solution (1 of 2)

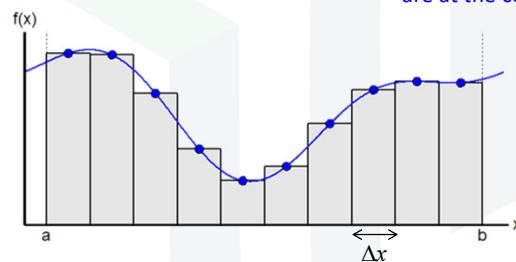
When this problem is solved on a computer, it is most likely that the function is only known at discrete points.



Solution (2 of 2)

A simple approach to approximate this integral is to represent the area under this function as a series of adjacent rectangles.

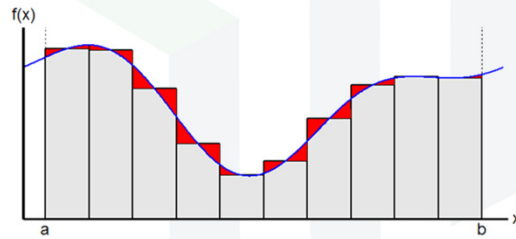
Observe that the position of the points are at the center of the rectangles.



$$\int_a^b f(x) dx \approx \sum_{n=1}^N f(x_n) \Delta x = \frac{b-a}{N} \sum_{n=1}^N f(x_n) \quad \Delta x = \frac{b-a}{N}$$

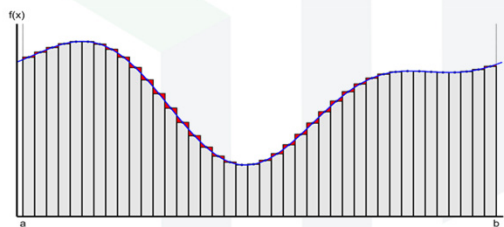
Error

Approximating the integral this way produces some **error**.
Gaps between the true curve and the rectangles leads to **error**.



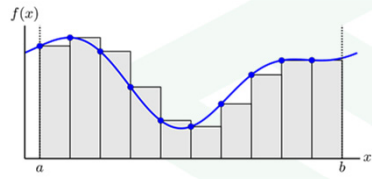
Reducing the Errors

The only way to reduce the error is to use thinner rectangles. However, this increases the number of computations that have to be performed which increases calculation time and memory required and could lead to larger round-off error.

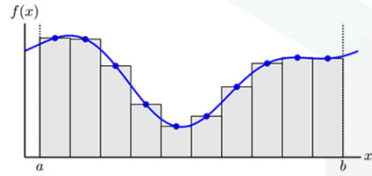


Or, use a different numerical integration technique altogether!

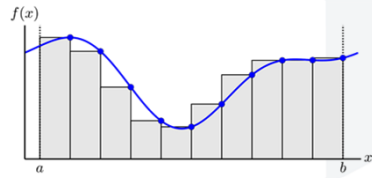
Terminology



Left-Hand Riemann Sum



Center Riemann Sum



Right-Hand Riemann Sum