Suppose it is desired to calculate the second-order derivative of some function that is known only at seven discrete points.

\[ \frac{d^2 f(x)}{dx^2} \approx ? \]
The Finite-Difference Approximation

The second-order derivative can be estimated with a 3-point finite-difference approximation.

\[
\frac{d^2 f_i}{dx^2} \approx \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2}
\]

The Middle Points

The derivatives are approximated at each intermediate point by applying the finite-difference approximation using the surrounding points.

\[
\frac{d^2 f_2}{dx^2} \approx \frac{f_1 - 2f_2 + f_3}{h^2}
\]

\[
\frac{d^2 f_5}{dx^2} \approx \frac{f_4 - 2f_5 + f_6}{h^2}
\]
Problem at the Boundaries

How are the finite-differences evaluated at $i = 1$ and $i = 7$?

New finite-difference approximations must be derived for each boundary point.

One Possible Boundary Fix
Summary of Finite-Difference Approximations

Below are all of the equations across the entire grid to numerically calculate the second-order derivative. The boundary points get their own special equations.

\[
\frac{d^2 f_i}{dx^2} \approx \frac{f_{i-1} - 2f_i + f_{i+1}}{h^2}
\]

\[
\frac{d^2 f_1}{dx^2} \approx \frac{2f_1 - 5f_2 + 4f_3 - f_4}{h^2}
\]

\[
\frac{d^2 f_7}{dx^2} \approx \frac{-f_4 + 4f_5 - 5f_6 + 2f_7}{h^2}
\]