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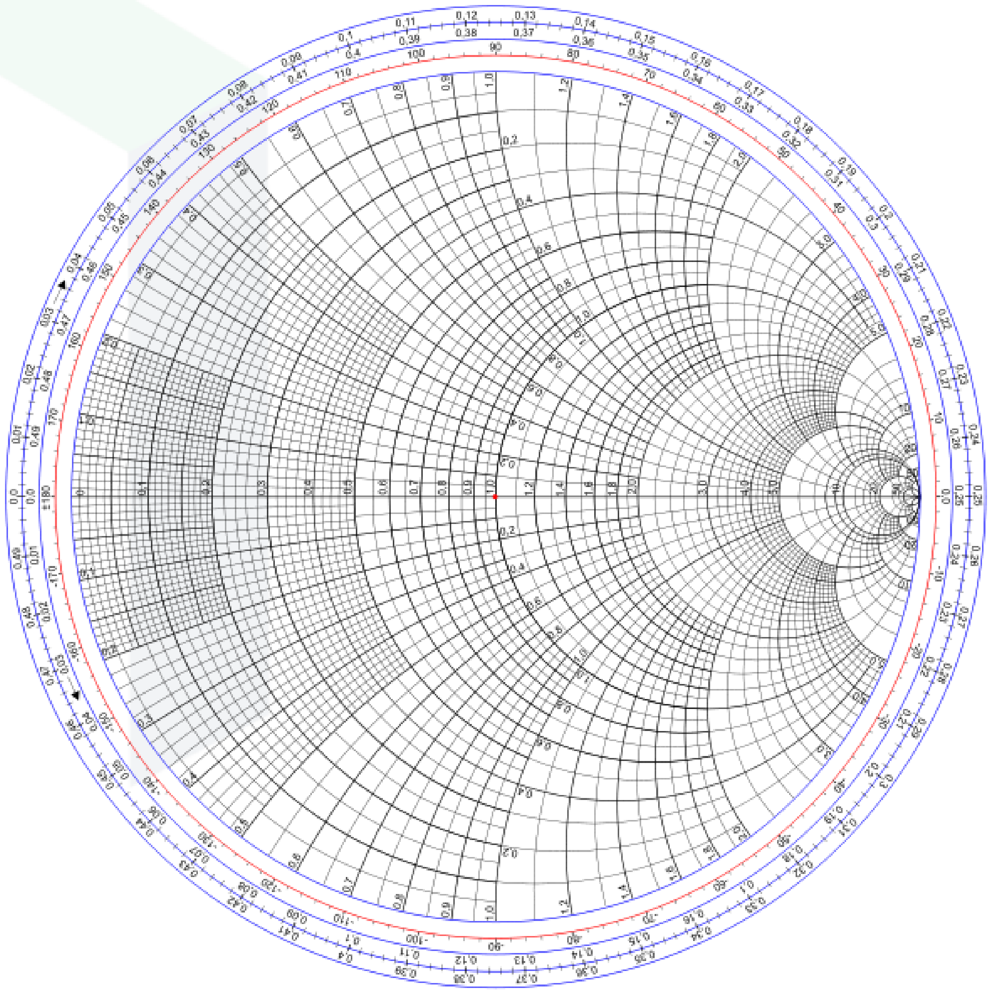
Electromagnetics:

Microwave Engineering

Introduction to Smith Charts

Lecture Outline

- Smith chart is a polar plot of reflection coefficient Γ
- Normalized impedance z
- Derivation of the Smith chart
- Final notes



Smith Chart is a Polar Plot of Reflection

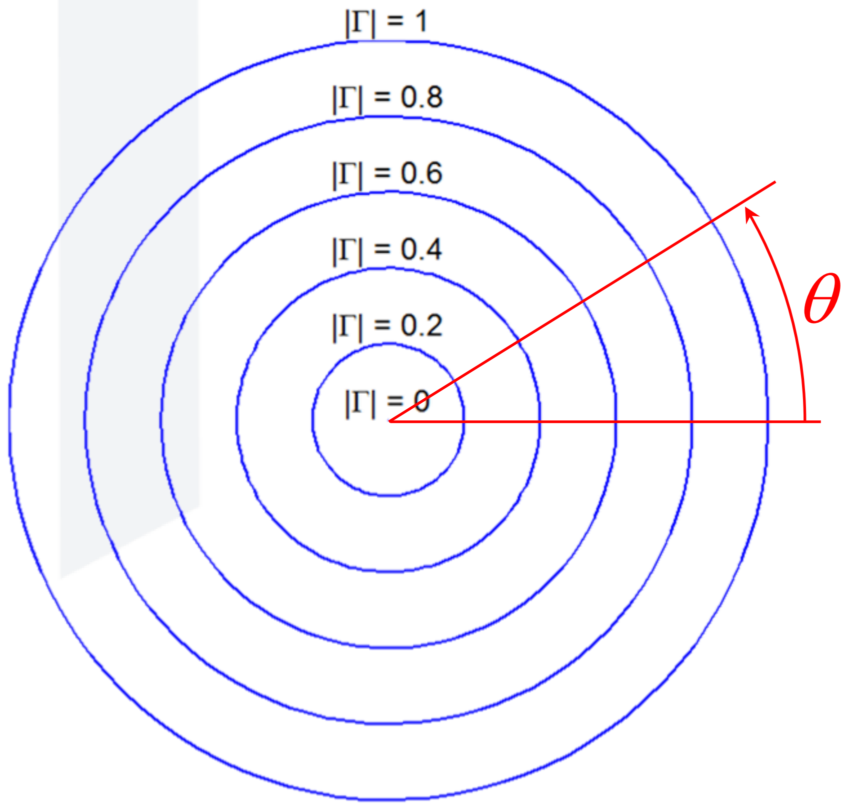
Polar Plot of Reflection Coefficient Γ

The Smith chart is based on a polar plot of the voltage reflection coefficient Γ. The outer boundary corresponds to $|\Gamma| = 1$. The reflection coefficient in any passive system must be $|\Gamma| \leq 1$.

$$\Gamma = |\Gamma| e^{j\theta}$$

$|\Gamma| \equiv$ radius on Smith chart

$\theta \equiv$ angle measured CCW from right side of chart



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Normalized Impedance z

Normalized Impedance z

All impedances on the Smith Chart are normalized. This is usually done with respect to the characteristic impedance of the transmission line Z_0.

$$z = \frac{Z}{Z_0}$$

Reflection Coefficient Γ from Normalized Impedance

The reflection coefficient Γ from a load can be written in terms of normalized impedances.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L/Z_0 - Z_0/Z_0}{Z_L/Z_0 + Z_0/Z_0} = \frac{z_L - 1}{z_L + 1}$$

$$Z_L \equiv \text{Load impedance}$$

$$z_L \equiv \text{Normalized load impedance}$$

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Derivation of the Smith Chart

Solve for Normalized Load Impedance z_{L}

Solve the previous equation for normalized load impedance z_{L} to get

$$\Gamma = \frac{z_{\mathrm{L}} - 1}{z_{\mathrm{L}} + 1}$$

$$z_{\mathrm{L}} = \frac{1+\Gamma}{1-\Gamma}$$

Real & Imaginary Parts of Normalized Load Impedance

The normalized load impedance z_L and reflection coefficient Γ can be written in terms of real and imaginary parts.

$$z_L = r_L + jx_L \qquad\qquad \Gamma = \Gamma_r + j\Gamma_i$$

Substituting these into the normalized load impedance equation yields

$$z_L = \frac{1+\Gamma}{1-\Gamma}$$

$$r_L + jx_L = \frac{1+\left(\Gamma_r + j\Gamma_i\right)}{1-\left(\Gamma_r + j\Gamma_i\right)}$$

$$r_L + jx_L = \frac{\left(1+\Gamma_r\right)+ j\Gamma_i}{\left(1-\Gamma_r\right)- j\Gamma_i}$$

Solve for r_L and x_L

Solve the previous equation for r_L and x_L by setting the real and imaginary parts equal.

$$r_L + jx_L = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i}$$

$$= \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i} \cdot \frac{(1-\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) + j\Gamma_i}$$

$$= \frac{(1+\Gamma_r)(1-\Gamma_r) + j(1+\Gamma_r)\Gamma_i + j\Gamma_i(1-\Gamma_r) - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$= \frac{1-\Gamma_r^2 + j\Gamma_i + j\Gamma_r\Gamma_i + j\Gamma_i - j\Gamma_r\Gamma_i - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$= \frac{1-\Gamma_r^2 - \Gamma_i^2 + j2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$= \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} + j\frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$r_L = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

Rearrange Equation Containing r_L

Rearrange the equation for r_L so that it has the form of a circle.

$$r_L = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}$$

$$(1-\Gamma_r)^2+\Gamma_i^2 = \frac{1-\Gamma_r^2-\Gamma_i^2}{r_L}$$

$$(1-\Gamma_r)^2+\Gamma_i^2 - \frac{1}{r_L} + \frac{\Gamma_r^2}{r_L} + \frac{\Gamma_i^2}{r_L} = 0$$

$$-2\Gamma_r + \Gamma_r^2 + \frac{\Gamma_r^2}{r_L} + \Gamma_i^2 + \frac{\Gamma_i^2}{r_L} + 1 - \frac{1}{r_L} = 0$$

$$-2r_L\Gamma_r + r_L\Gamma_r^2 + \Gamma_r^2 + r_L\Gamma_i^2 + \Gamma_i^2 + r_L - 1 = 0$$

$$-2r_L\Gamma_r + (r_L+1)\Gamma_r^2 + (r_L+1)\Gamma_i^2 + r_L - 1 = 0$$

$$\underbrace{\Gamma_r^2 - 2\frac{r_L\Gamma_r}{r_L+1}}_{\text{can be factored}} + \Gamma_i^2 + \frac{r_L-1}{r_L+1} = 0$$

Rearrange Equation Containing r_L

Rearrange the equation for r_L so that it has the form of a circle.

$$\left(\Gamma_r - \frac{r_L}{r_L+1}\right)^2 - \left(\frac{r_L}{r_L+1}\right)^2 + \Gamma_i^2 + \frac{r_L-1}{r_L+1} = 0$$

$$\left(\Gamma_r - \frac{r_L}{r_L+1}\right)^2 + \Gamma_i^2 = \left(\frac{r_L}{r_L+1}\right)^2 - \frac{r_L-1}{r_L+1}$$

$$\left(\Gamma_r - \frac{r_L}{r_L+1}\right)^2 + \Gamma_i^2 = \frac{r_L^2}{(r_L+1)^2} - \frac{(r_L-1)(r_L+1)}{(r_L+1)^2}$$

$$\left(\Gamma_r - \frac{r_L}{r_L+1}\right)^2 + \Gamma_i^2 = \frac{r_L^2}{(r_L+1)^2} - \frac{r_L^2-1}{(r_L+1)^2}$$

$$\left(\Gamma_r - \frac{r_L}{r_L+1}\right)^2 + \Gamma_i^2 = \frac{1}{(r_L+1)^2}$$

Rearrange Equation Containing x_L

Rearrange the equation for x_L so that it has the form of a circle.

$$x_\mathrm{L} = \frac{2\Gamma_\mathrm{i}}{(1-\Gamma_\mathrm{r})^2 + \Gamma_\mathrm{i}^2}$$

$$(1-\Gamma_\mathrm{r})^2 + \Gamma_\mathrm{i}^2 = \frac{2\Gamma_\mathrm{i}}{x_\mathrm{L}}$$

$$\underbrace{(1-\Gamma_\mathrm{r})^2}_{\text{swap terms}} + \underbrace{\Gamma_\mathrm{i}^2 - \frac{2}{x_\mathrm{L}}\Gamma_\mathrm{i}}_{\text{can be factored}} = 0$$

$$(\Gamma_\mathrm{r}-1)^2 + \left(\Gamma_\mathrm{i} - \frac{1}{x_\mathrm{L}}\right)^2 - \frac{1}{x_\mathrm{L}^2} = 0$$

Two Families of Circles

Constant Resistance Circles

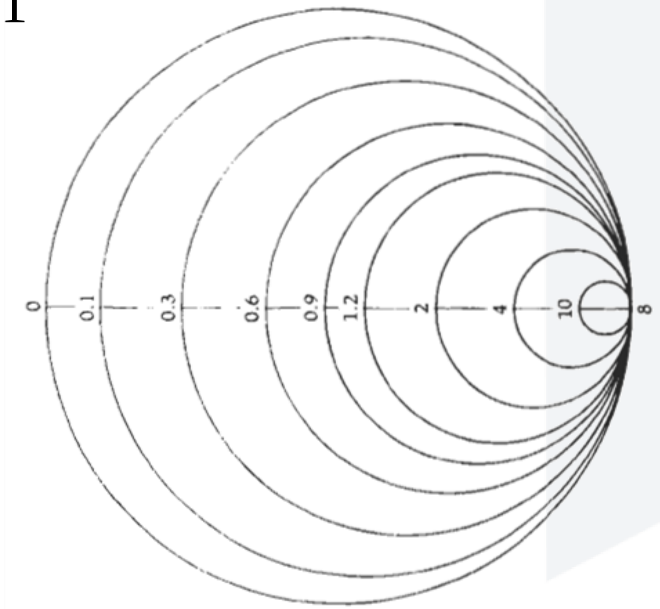
$$\left(\Gamma_r - \frac{r_L}{r_L+1}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r_L}\right)^2$$

These have centers at

$$\Gamma_r = \frac{r_L}{r_L+1} \qquad \Gamma_i = 0$$

Radii

$$\frac{1}{1+r_L}$$



Constant Reactance Circles

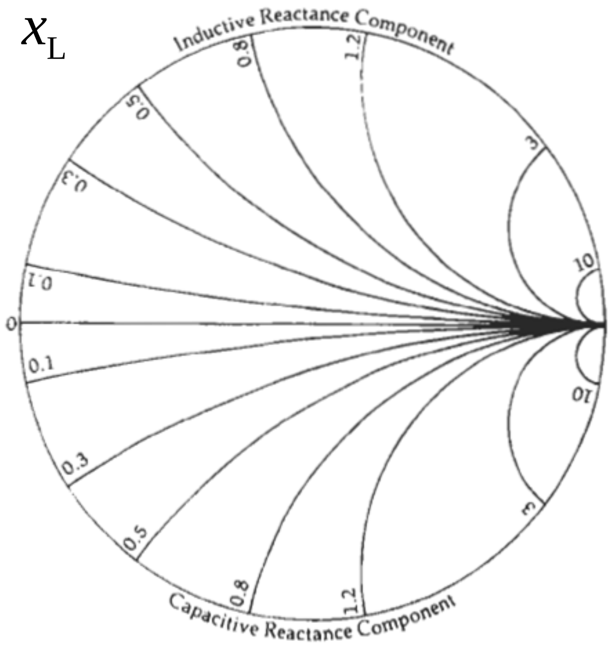
$$\left(\Gamma_r - 1\right)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2$$

These have centers at

$$\Gamma_r = 1 \qquad \Gamma_i = \frac{1}{x_L}$$

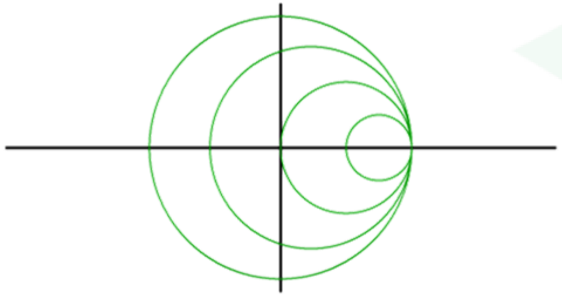
Radii

$$\frac{1}{x_L}$$



Putting It All Together

Lines of constant resistance



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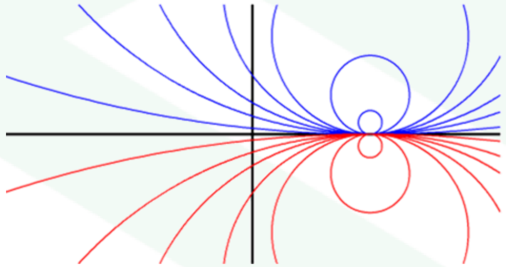
Lines of constant inductive reactance



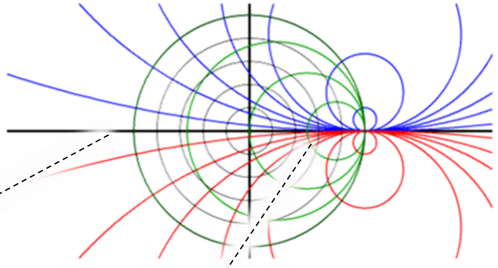
Lines of constant capacitive reactance

+

Lines of constant reflection coefficient

=

Superposition



Ignore what is outside the $|\Gamma| = 1$ circle.

Do not draw the constant $|\Gamma|$ circles.

This is the Smith chart!

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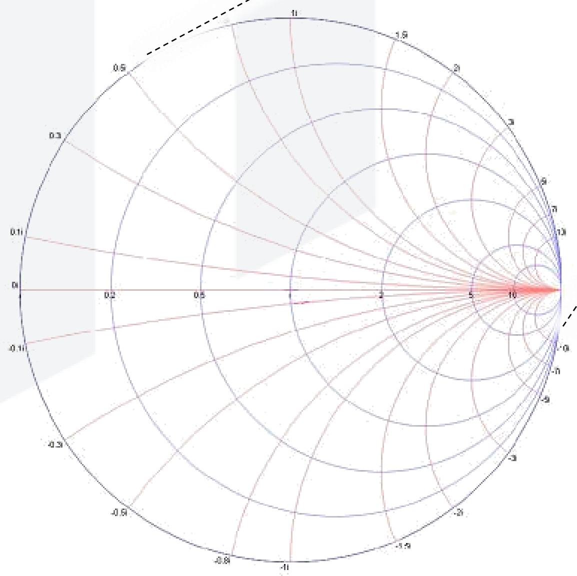
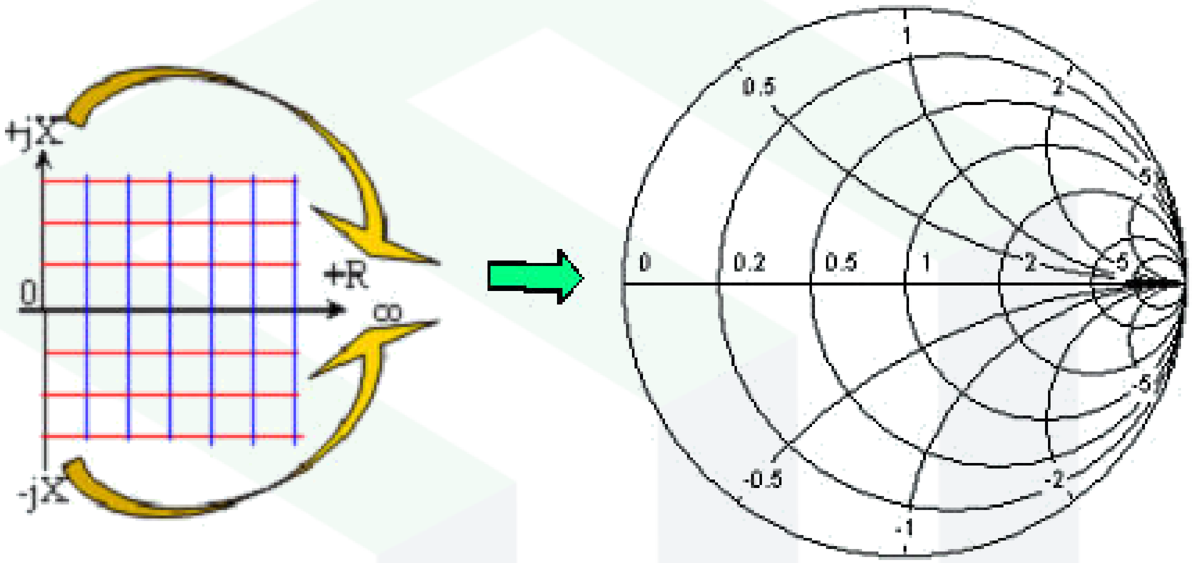
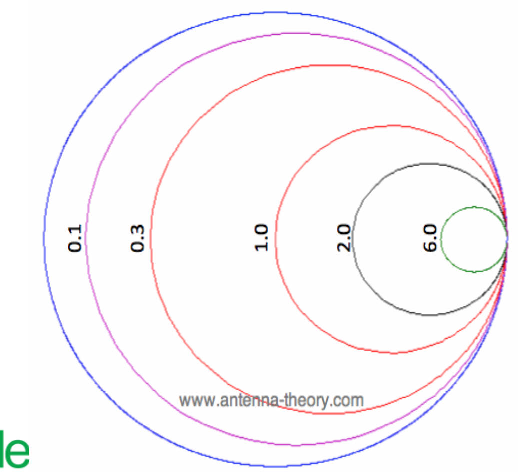


Final Notes

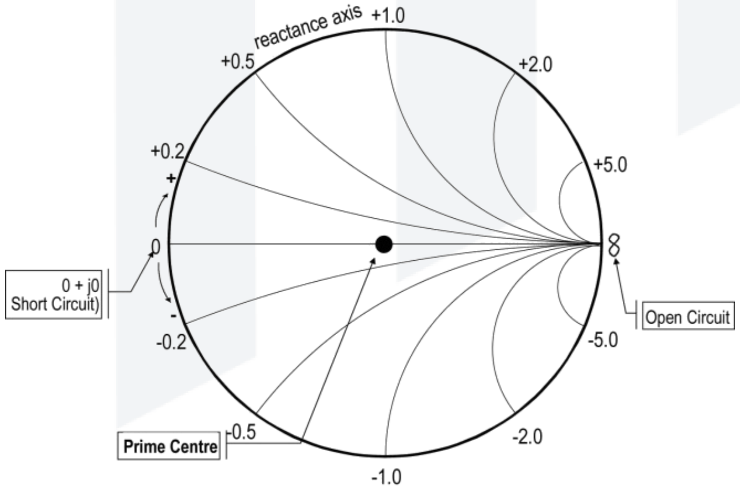
Alternate Way of Visualizing the Smith Chart



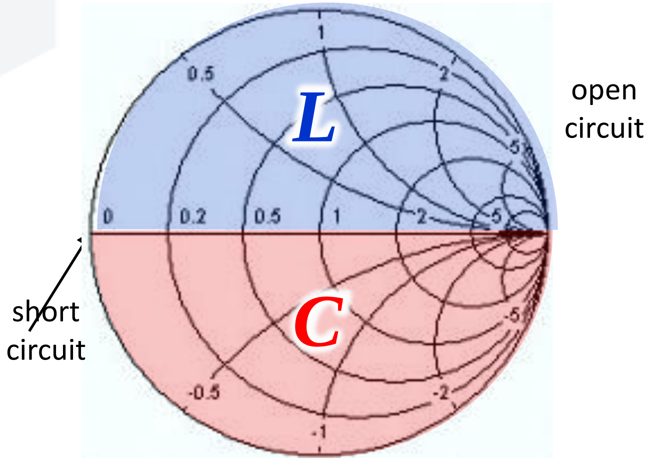
Circles of constant resistance



Circles of constant reactance



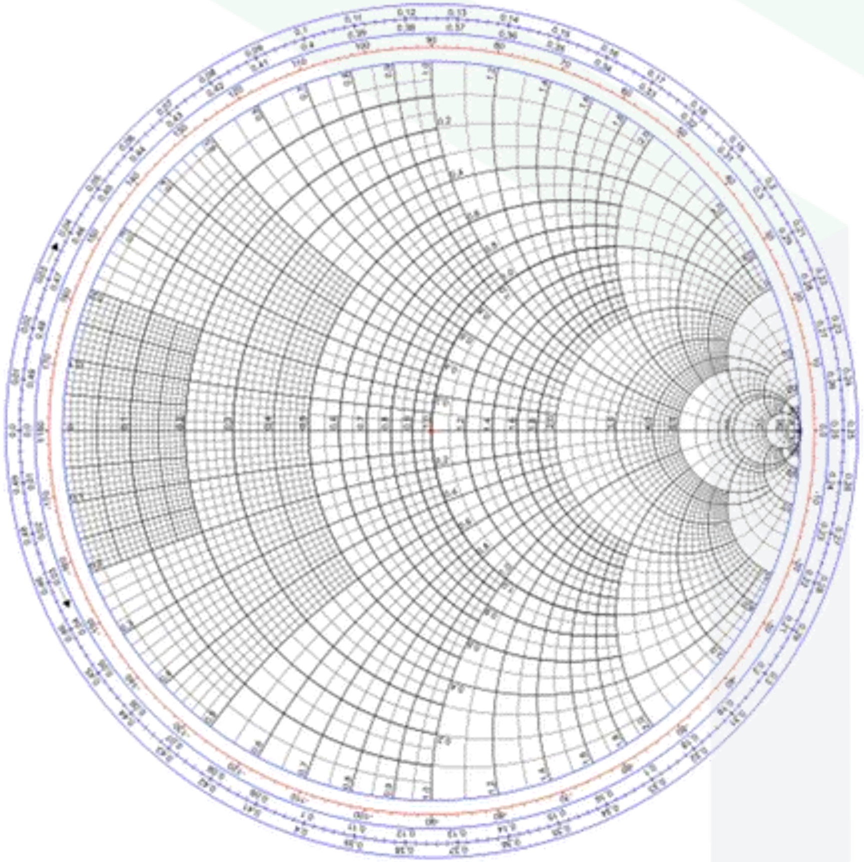
Reactance Regions



3D Smith Chart

The 3D Smith Chart unifies passive and active circuit design.

2D



3D