Computational Science:
Computational Methods in Engineering

Introduction to Two-Dimensional Finite-Difference Method

Interpretation of Matrices

$$a_{11}x + a_{12}y + a_{13}z = b_1$$
$$a_{21}x + a_{22}y + a_{23}z = b_2$$
$$a_{31}x + a_{32}y + a_{33}z = b_3$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

EQUATION FOR…

Equation for $x \rightarrow$ $[a_{11} \quad a_{12} \quad a_{13}]$

Equation for $y \rightarrow$ $[a_{21} \quad a_{22} \quad a_{23}]$

Equation for $z \rightarrow$ $[a_{31} \quad a_{32} \quad a_{33}]$

RELATION TO…

Relation to x ↓ $\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}$, Relation to y ↓ $\begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}$, Relation to z ↓ $\begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$

Slide 2

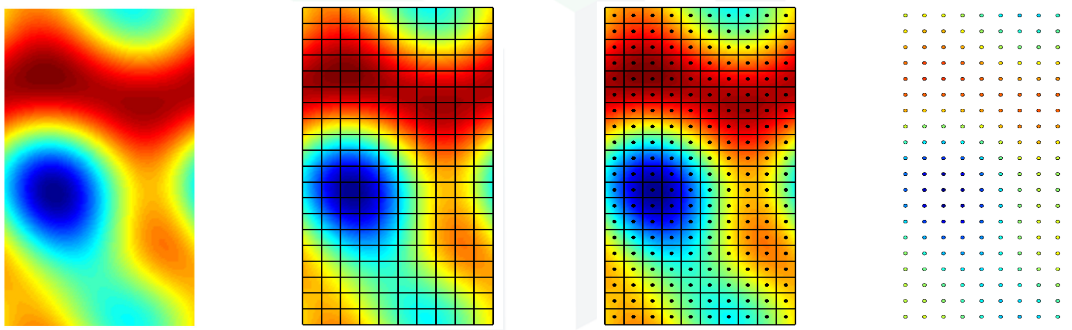
Representing Functions on a Grid

Example physical (continuous) 2D function

A grid is constructed by dividing space into discrete cells

Function is known only at discrete points

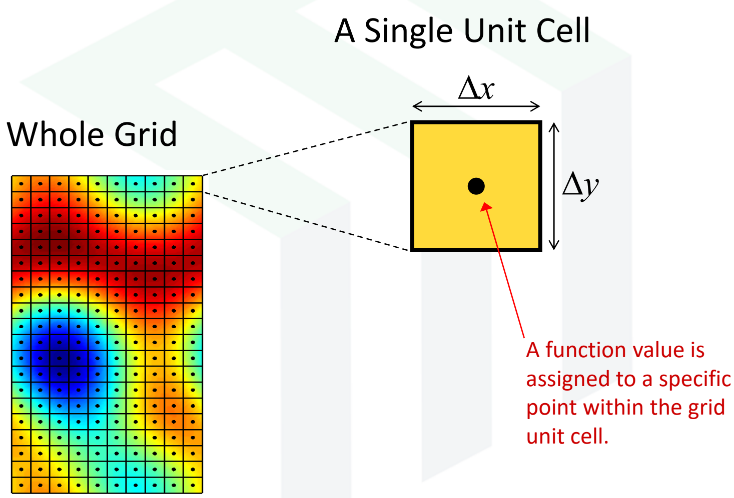
Representation of what is actually stored in memory



EMPossible

Slide 3

Grid Unit Cell



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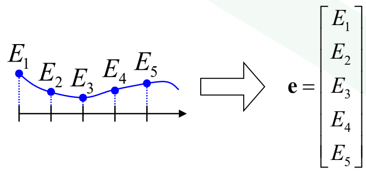
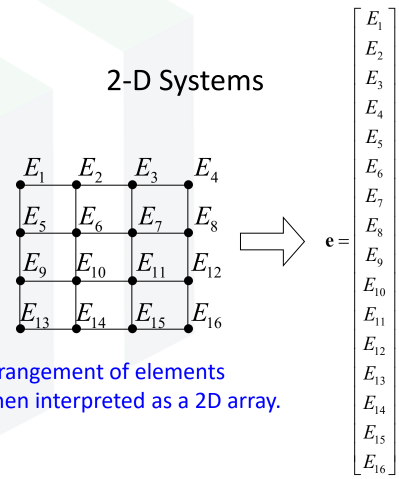
Slide 4

Functions are Stored in Column Vectors

1-D Systems



2-D Systems



Arrangement of elements when interpreted as a 2D array.

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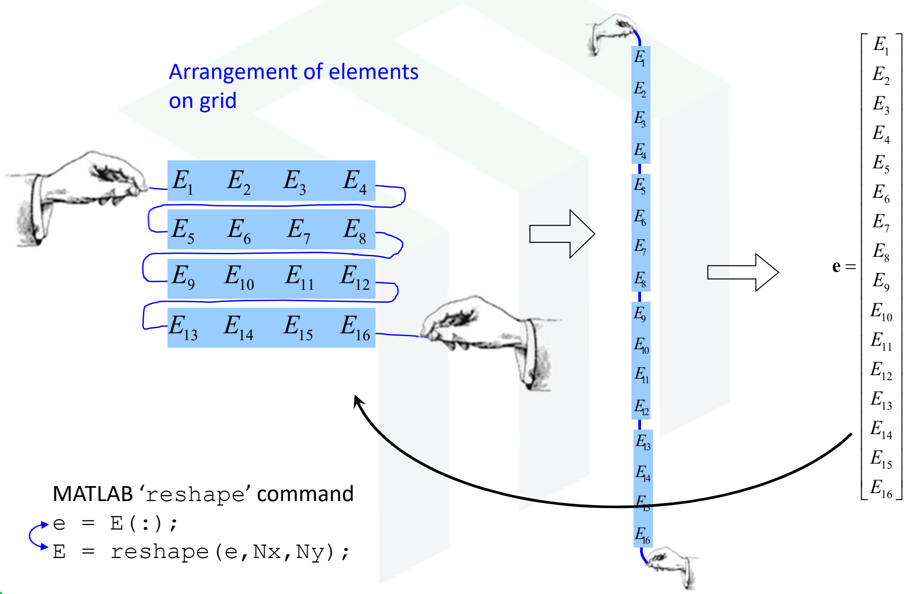
Slide 5

Putting Functions into Column Vectors

Arrangement of elements on grid



MATLAB 'reshape' command

```
e = E(:);
E = reshape(e,Nx,Ny);
```

EMPossible

Slide 6

Finite-Difference Approximations on a Collocated Grid

Derivatives in the x direction

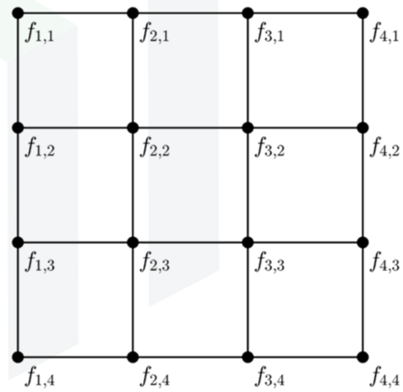
$$\frac{\partial f_{i,j}}{\partial x} \cong \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x}$$

$$\frac{\partial^2 f_{i,j}}{\partial x^2} \cong \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{(\Delta x)^2}$$

Derivatives in the y direction

$$\frac{\partial f_{i,j}}{\partial y} \cong \frac{f_{i,j+1} - f_{i,j-1}}{2\Delta y}$$

$$\frac{\partial^2 f_{i,j}}{\partial y^2} \cong \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{(\Delta y)^2}$$



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Finite-Difference Approximations on a Staggered Grid

Right-Handed Derivatives of $f(x)$

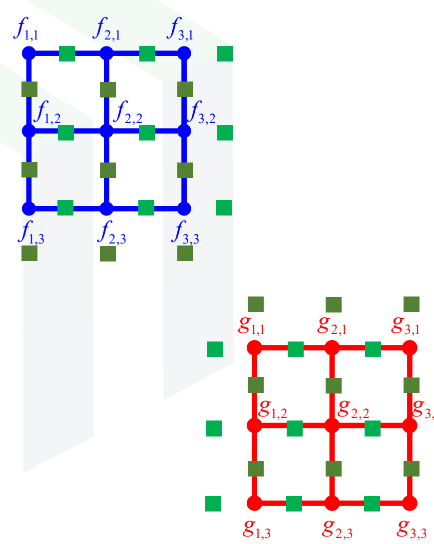
$$\frac{\partial f_{i,j}}{\partial x} \cong \frac{f_{i+1,j} - f_{i,j}}{\Delta x}$$

$$\frac{\partial f_{i,j}}{\partial y} \cong \frac{f_{i,j+1} - f_{i,j}}{\Delta y}$$

Left-Handed Derivatives of $g(x)$

$$\frac{\partial g_{i,j}}{\partial x} \cong \frac{g_{i,j} - g_{i-1,j}}{\Delta x}$$

$$\frac{\partial g_{i,j}}{\partial y} \cong \frac{g_{i,j} - g_{i,j-1}}{\Delta y}$$



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