Computational Science:
Computational Methods in Engineering

Matrix Operations

Outline

- Matrix Arithmetic
- Matrix Algebra

Matrix Arithmetic

Matrix Arithmetic (1 of 4)

Addition:

$$[A]+[B]=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}+\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}=\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \\ a_{31}+b_{31} & a_{32}+b_{32} & a_{33}+b_{33} \end{bmatrix}$$

Subtraction:

$$[A]-[B]=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}-\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}=\begin{bmatrix} a_{11}-b_{11} & a_{12}-b_{12} & a_{13}-b_{13} \\ a_{21}-b_{21} & a_{22}-b_{22} & a_{23}-b_{23} \\ a_{31}-b_{31} & a_{32}-b_{32} & a_{33}-b_{33} \end{bmatrix}$$

Matrix Arithmetic (2 of 4)

Multiplication by a Scalar:

$$s[A] = s\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} sa_{11} & sa_{12} & sa_{13} \\ sa_{21} & sa_{22} & sa_{23} \\ sa_{31} & sa_{32} & sa_{33} \end{bmatrix}$$

Multiplication by a Matrix

$$[A][B] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} & \# & \# \\ \# & \# & \# \\ \# & \# & \# \end{bmatrix}$$

$$[A][x] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{bmatrix}$$

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Matrix Arithmetic (3 of 4)

Matrix Transpose:

$$[A]^T = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$a'_{ij} = a_{ji}$$

Animation of Transpose Operation

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} & a_{41} \\ a_{12} & a_{22} & a_{32} & a_{42} \\ a_{13} & a_{23} & a_{33} & a_{43} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix}$$

Hermitian Transpose:

$$[A]^H = \left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^T\right)^* = \begin{bmatrix} a_{11}^* & a_{21}^* & a_{31}^* \\ a_{12}^* & a_{22}^* & a_{32}^* \\ a_{13}^* & a_{23}^* & a_{33}^* \end{bmatrix}$$

$$a'_{ij} = a_{ji}^*$$

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Matrix Arithmetic (4 of 4)

Determinants:

$\det[A]$ Think of this as the “magnitude” or “volume” of a matrix.

Matrix Inverse:

$$[A]^{-1}[A]=[I]$$

Matrix Division:

$[A]^{-1}[B]$ predivide `A\B`

$[B][A]^{-1}$ postdivide `B/A`

While both expressions divide by $[A]$, these do not give the same answer.

Matrix Multiplication:

$[A][B]$ $[A]$ premultiplies $[B]$

$[B][A]$ $[A]$ postmultiplies $[B]$

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Matrix Algebra

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Matrix Algebra (1 of 3)

Commutative Laws

$$[A]+[B]=[B]+[A]$$
$$[A][B]\neq[B][A]$$

$[A][B] = [B][A]$ only when $[A]$ and $[B]$ are diagonal matrices.

Associative Laws

$$([A]+[B])+[C]=[A]+([B]+[C])$$
$$([A][B])[C]=[A]([B][C])$$

Matrix Inverses and Transposes

$$[A]^{-1}[A]=[A][A]^{-1}=[I]$$
$$\left([A]^{-1}\right)^{-1}=[A]$$
$$([A][B])^{-1}=[B]^{-1}[A]^{-1}$$
$$\left([A]^{T}\right)^{-1}=\left([A]^{-1}\right)^{T}$$
$$\left([A]^{T}\right)^{T}=[A]$$
$$([A]+[B])^{T}=[A]^{T}+[B]^{T}$$
$$([A][B])^{T}=[B]^{T}[A]^{T}$$

Distributive Laws

$$([A]+[B])[C]=[A][C]+[B][C]$$
$$[A]([B]+[C])=[A][B]+[A][C]$$

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Matrix Algebra (2 of 3)

Addition with a Scalar

$$\alpha+[A]=\text{doesn't make sense}$$

$$\alpha[I]+[A]=\begin{bmatrix}\alpha+a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & \alpha+a_{22} & \cdots & a_{2n}\\ \vdots & \vdots & \ddots & \\ a_{n1} & a_{n2} & & \alpha+a_{nn}\end{bmatrix}$$

Multiplication with a Scalar

$$\alpha([A]+[B])=\alpha[A]+\alpha[B]$$
$$\alpha([A][B])=(\alpha[A])[B]=[A](\alpha[B])$$

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Matrix Algebra (3 of 3)

Operations with Special Matrices

$$[0][A]=[A][0]=[0]$$
$$[I][A]=[A][I]=[A]$$
$$[0]+[A]=[A]+[0]=[A]$$
$$[A]-[A]=[0]$$

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Example of Matrix Algebra

Simplify the Following Equation

$$\left(\mathbf{C}^{-1}\mathbf{A}\right)^{-1}+\mathbf{D}=\mathbf{BC}+\mathbf{D}$$

Step 1 – Subtract $\mathbf{D}$ from both sides

$$\left(\mathbf{C}^{-1}\mathbf{A}\right)^{-1}+\mathbf{D}-\mathbf{D}=\mathbf{BC}+\mathbf{D}-\mathbf{D}$$
$$\left(\mathbf{C}^{-1}\mathbf{A}\right)^{-1}+\mathbf{0}=\mathbf{BC}+\mathbf{0}$$
$$\left(\mathbf{C}^{-1}\mathbf{A}\right)^{-1}=\mathbf{BC}$$

Step 2 – Inverse both sides

$$\left\{\left(\mathbf{C}^{-1}\mathbf{A}\right)^{-1}\right\}^{-1}=\left\{\mathbf{BC}\right\}^{-1}$$
$$\mathbf{C}^{-1}\mathbf{A}=\mathbf{C}^{-1}\mathbf{B}^{-1}$$

Step 3 – Premultiply both sides by $\mathbf{C}$.

$$\mathbf{CC}^{-1}\mathbf{A}=\mathbf{CC}^{-1}\mathbf{B}^{-1}$$
$$\mathbf{IA}=\mathbf{IB}^{-1}$$
$$\mathbf{A}=\mathbf{B}^{-1}$$

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