



Computational Science:
Computational Methods in Engineering

Multiple Integrals



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Problem Setup

Suppose there is a function $f(x, y)$ with two independent variables x and y .

How is a double integral calculated?

$$\int_{x=a}^{x=b} \int_{y=c}^{y=d} f(x, y) dx dy = ?$$

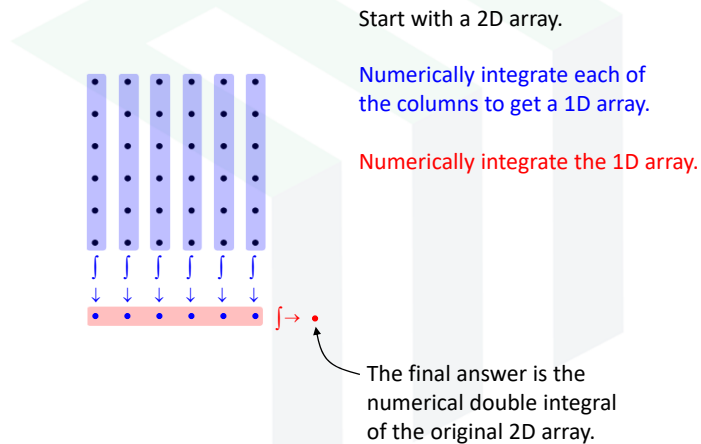
Think of this as an “integral of integrals.”

$$\int_{x=a}^{x=b} \left(\int_{y=c}^{y=d} f(x, y) dy \right) dx = ?$$

Evaluate the inside integral for each point of the outside integral.

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Illustration of Numerical Double Integration



Discrete Double Integration

This is very easy using discrete integration. The discrete equation is

$$\int_{x=a}^{x=b} \int_{y=c}^{y=d} f(x, y) dx dy \approx \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(a + m\Delta x, c + n\Delta y) \Delta x \Delta y$$

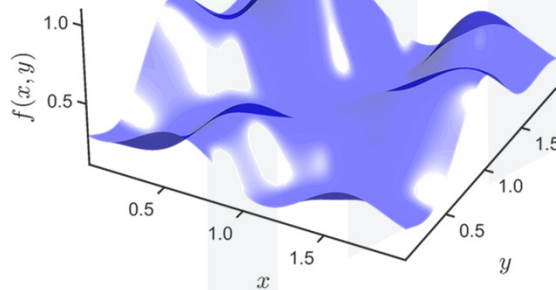
$$\Delta x = \frac{b - a}{M}$$

$$\Delta y = \frac{d - c}{N}$$

The MATLAB code to do this is very simple...

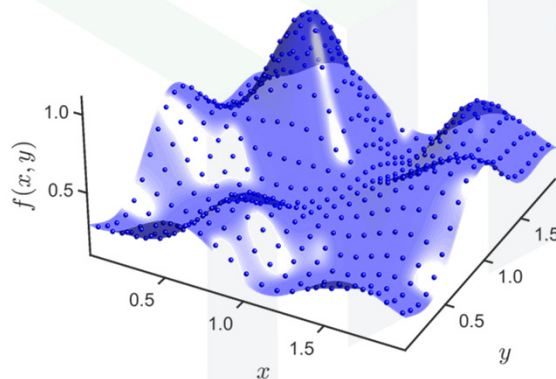
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dx = (b - a) / M;
dy = (d - c) / N;
I = sum(f(:)) * dx * dy;
```

Visualizing Discrete Double Integration



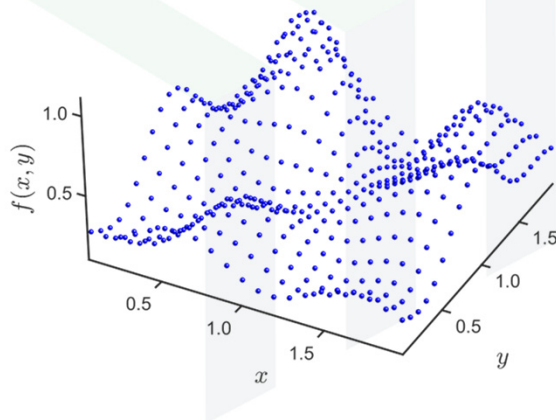
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Visualizing Discrete Double Integration



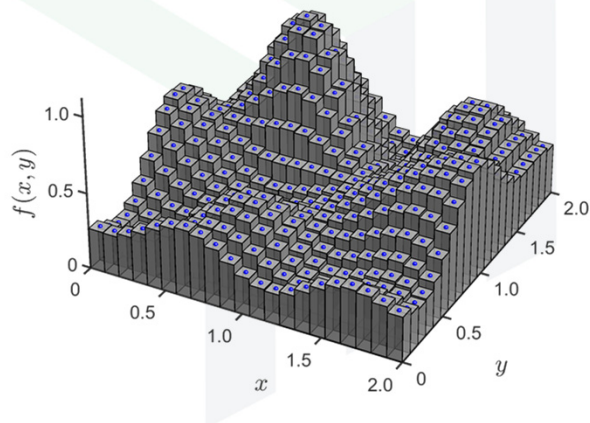
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Visualizing Discrete Double Integration



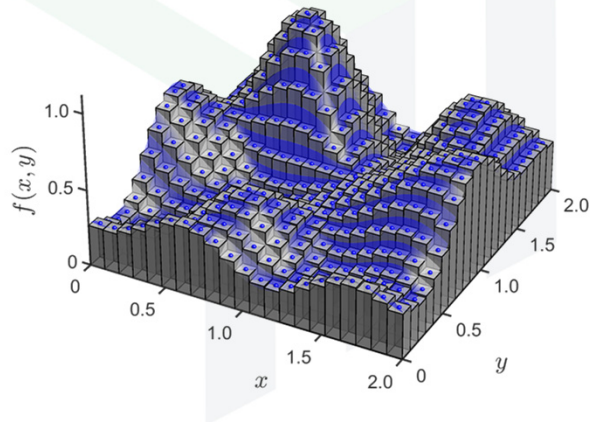
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Visualizing Discrete Double Integration



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Visualizing Discrete Double Integration



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